Exercise Sheet 1: Getting to Know Graphs

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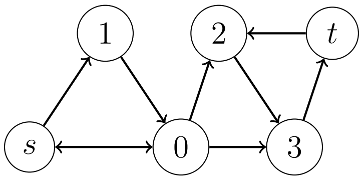
Exercise 1.1. Show that the number of vertices of odd degree is even in every simple graph.

Exercise 1.2. Consider the following graph:



1. Is the graph connected?
2. Find all directed paths from vertex s to vertex t of length at most 6.
3. Is the graph strongly connected? If not, which edge(s) would need to be added to make it strongly connected?
4. Find the strongly connected components (i.e., the maximal induced subgraphs that are strongly connected) of the graph.

Exercise 1.3. A *bipartite graph* is a simple graph $G = \langle V, E \rangle$, where V can be partitioned into two sets X, Y (i.e., $X \cup Y = V$, and $X \cap Y = \varnothing$), such that every edge $\{ a, b \} \in E$ coincides with both X and Y, i.e., $\{ a, b \} \cap X \neq \varnothing$ and $\{ a, b \} \cap Y \neq \varnothing$.

Show that the following are equivalent:

1. $G = \langle V, E \rangle$ is bipartite.
2. G is *2-colourable*, i.e., there is a map $c : V \to \{ 0, 1 \}$ such that no two adjacent vertices a, b have the same colour, i.e., $c(a) \neq c(b)$ for all $\{ a, b \} \in E$.
3. G does not contain a cycle $v_1 \xrightarrow{e_1} v_2 \xrightarrow{e_2} \cdots \xrightarrow{e_{n-1}} v_n \xrightarrow{e_n} v_1$ of odd length.

Exercise 1.4. Write a program that reads a directed graph from a file in the format of Exercise 0.2 and prints out the graph in METIS graph format:

$$\begin{array}{ccccc} n & m & & & \\ v_1 & n_1^1 & n_1^2 & \cdots & n_1^{d_1} \\ v_2 & n_2^1 & n_2^2 & \cdots & n_2^{d_2} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ v_n & n_n^1 & n_n^2 & \cdots & n_n^{d_n} \end{array}$$

The first line consists of two integers n and m, separated by a space, where n is the number of vertices, and m is the total number of edges. Each of the following lines specifies the neighbours $n_i^1, n_i^2, \ldots, n_i^{d_i}$ of vertex v_i.

As an example, the directed star S_3 would be encoded as:

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4 3
0 1 2 3
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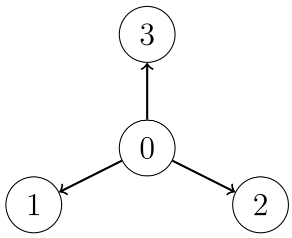

Exercise 1.5. A triangle in a directed graph is a simple directed path $v_1 \xrightarrow{e_1} v_2 \xrightarrow{e_2} v_3 \xrightarrow{e_3} v_1$.

Write a program that reads a directed graph G from a file in the format of Exercise 1.4 and prints out the number of triangles in G. How does the runtime of your program scale with the size of the input graph?