Exercise Sheet 3: First-order Query Complexity
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Exercise 3.1. Three decision problems related to query answering have been introduced in the lecture:

- Boolean query entailment,
- query answering, and
- the query emptiness problem.

Show that these problems are equivalent, i.e., if we have an algorithm that solves any of these problems, then we can also use it to solve the others.

Exercise 3.2. It was shown in the lecture that joins can be computed in logarithmic space. Outline algorithms that implement

1. selection, and
2. projection,

in logarithmic space.

Exercise 3.3. Expressions of relational algebra under named perspective can be translated into Boolean circuits, in a similar fashion to the translation illustrated for FO queries in the lecture. Show how each operator of relational algebra gives rise to a corresponding circuit by describing the circuits for the following expressions:

1. $\sigma_{n=c}(R)$ where c is a constant
2. $\sigma_{n=m}(R)$ where m is an attribute
3. $\pi_{a_1,\ldots,a_n}(R)$
4. $R \bowtie S$
5. $\delta_{a_1,\ldots,a_n \to b_1,\ldots,b_n} R$
6. $R - S$
7. $R \cup S$
8. $R \cap S$

Exercise 3.4. Decide whether the following statements are true or false:

1. The combined complexity of a query language is at least as high as its data complexity.
2. The query complexity of a query language is at least as high as its data complexity.

If true, explain why, otherwise give a counter-example.

Exercise 3.5. It was claimed in the lecture that the composition of two functions that can each be computed in LogSpace can also be computed in LogSpace. How can this be achieved, considering the fact that the output of one LogSpace function may already require more than logarithmic space?

Exercise 3.6. Is the question "$\mathrm{P} = \mathrm{NP}$?" decidable? Explain your answer by either showing that there is an algorithm that correctly answers this question, or by showing that such an algorithm cannot exist.