Satisfiability Checking and Conjunctive Query Answering in Description Logics with Global and Local Cardinality Constraints

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Abstract

We introduce and investigate the expressive description logic (DL) $\mathcal{ALCSCC}^{++}$, in which the global and local cardinality constraints introduced in previous papers can be mixed. On the one hand, we prove that this does not increase the complexity of satisfiability checking and other standard inference problems. On the other hand, the satisfiability problem becomes undecidable if inverse roles are added to the languages. In addition, even without inverse roles, conjunctive query entailment in this DL turns out to be undecidable. We prove that decidability of querying can be regained if global and local constraints are not mixed and the global constraints are appropriately restricted. The latter result is based on a model pumping argument, and it reduces query entailment to ABox consistency in the restricted setting, i.e., to ABox consistency w.r.t. restricted cardinality constraints in $\mathcal{ALCSCC}$, for which we can show an ExpTime upper bound.

B: change slightly

1 Introduction

Description Logics (DLs) [6] are a well-investigated family of logic-based knowledge representation languages, which are frequently used to formalize ontologies for application domains such as biology and medicine [11]. To define the important notions of such an application domain as formal concepts, DLs state necessary and sufficient conditions for an individual to belong to a concept. These conditions can be Boolean combinations of atomic properties required for the individual (expressed by concept names) or properties that refer to relationships with other individuals and their properties (expressed as role restrictions). Using an example from [7], the concept of a motor vehicle can be formalized by the concept description

$$\mathit{Vehicle} \sqcap \exists \mathit{part}.\mathit{Motor},$$

which uses the concept names *Vehicle* and *Motor* and the role name *part* as well as the concept constructors conjunction ($\sqcap$) and existential restriction ($\exists r.C$). The concept inclusion (CI)

$$\mathit{Motor\text{-}vehicle} \sqsubseteq \mathit{Vehicle} \sqcap \exists \mathit{part}.\mathit{Motor}$$