Exercise 3.1:
Consider the following program $P$:

\[
\begin{align*}
&\text{double}(0,0). \\
&\text{double}(s(X),s(s(Y))) :- \text{double}(X,Y).
\end{align*}
\]

a) Give the Herbrand universe $HU_F$ and the Herbrand base $HB_{\Pi,F}$ determined by $P$.

b) Give two models of $P$.

c) Consider the following interpretations $I_1$ and $I_2$. For each case specify whether the given interpretation satisfies $P$ or not. Justify your answer.

- $I_1 : D_{I_1} = \mathbb{N}, 0_{I_1} = 1, s(t)_{I_1} = 2 \times t_{I_1}, double_{I_1} = \{(a,a^2) \mid a \geq 1\}$

- $I_2 : D_{I_2} = \mathbb{N}, 0_{I_2} = 0, s(t)_{I_2} = (2 \times t_{I_2}) + 1,$
  \[double_{I_2} = \{(0,0)\} \cup \{(a,a^2 - a + 1) \mid a \geq 1\}\]

Exercise 3.2:
Consider the following program which specifies the descendant relation which is the relation of being a child of, or a child of a child of, or a child of a child of a child of, \ldots

\[
\begin{align*}
&\text{descend}(X,Y) :- \text{child}(X,Y). \\
&\text{descend}(X,Y) :- \text{child}(X,Z), \text{descend}(Z,Y).
\end{align*}
\]

With the input database

\[
\begin{align*}
&\text{child}(anne, bridget). \\
&\text{child}(bridget, caroline). \\
&\text{child}(caroline, donna). \\
&\text{child}(donna, emily).
\end{align*}
\]

Give the search tree for the query: ?- descend(anne, donna).
Exercise 3.3:
Consider the program from Exercise 3.2. What happens if we change the order of the rules and goals. What is the result of the queries \(-\) descend\((X,Y)\), \(-\) descend\((anne, emily)\). and \(-\) descend\((anne, bridget)\).

\[\text{a)} \quad \text{descend}(X,Y) : - \text{child}(X,Z), \text{descend}(Z,Y).
\]
\[\text{descend}(X,Y) : - \text{child}(X,Y).\]

\[\text{b)} \quad \text{descend}(X,Y) : - \text{descend}(Z,Y), \text{child}(X,Z).
\]
\[\text{descend}(X,Y) : - \text{child}(X,Y).\]

\[\text{c)} \quad \text{descend}(X,Y) : - \text{child}(X,Y).
\]
\[\text{descend}(X,Y) : - \text{descend}(Z,Y), \text{child}(X,Z).\]

Exercise 3.4:
Consider the following program for addition.

\[
\begin{align*}
\text{add}(0,Y,Y). \\
\text{add}(\text{s}(X),Y,\text{s}(Z)) : - \text{add}(X,Y,Z).
\end{align*}
\]

Give the search tree and instantiations for the query:
\(-\) add\((\text{s}(\text{s}(\text{s}(0))), \text{s}(\text{s}(0)), R).\)