EXERCISE 1

Science of Computational Logic

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Problem 1.1

In the lectures the following example from Description Logics was presented:

$$
\begin{aligned}
\mathcal{K}_T: \quad \text{woman} &\sqsubseteq \text{person},\\
\text{man} &\sqsubseteq \text{person},\\
\text{mother} &= \text{woman} \sqcap \exists \text{child} : \text{person},\\
\text{father} &= \text{man} \sqcap \exists \text{child} : \text{person},\\
\text{parent} &= \text{mother} \sqcup \text{father},\\
\text{grandparent} &= \text{parent} \sqcap \exists \text{child} : \text{parent},\\
\text{father\_without\_son} &= \text{father} \sqcap \forall \text{child} : \neg \text{man}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{K}_A: \quad & \text{parent}(\text{carl}),\ \text{parent}(\text{conny}),\\
& \text{child}(\text{conny}, \text{joe}),\ \text{child}(\text{conny}, \text{carl}),\\
& \text{man}(\text{joe}),\ \text{man}(\text{carl}),\ \text{woman}(\text{conny}).
\end{aligned}
$$

Are the following consequences valid? **Justify** your answers.

1. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{grandparent}(\text{conny})$
2. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{father}(\text{carl})$
3. $\mathcal{K}_T \cup \mathcal{K}_A \models \text{father\_without\_son}(\text{carl})$

Problem 1.2

Prove that $F \sqsubseteq G \ \equiv \ F \sqcap \neg G = \bot$

Problem 1.3

Show that grandparent $\sqsubseteq_{\mathcal{K}_T}$ parent by reducing subsumption into concept satisfiability, where $\mathcal{K}_T$ is the T-Box from Problem 1.1.

Problem 1.4

Is the concept (father $\sqcap$ mother) satisfiable w.r.t. $\mathcal{K}_T$ from Problem 1.1?

Problem 1.5

1. Which generalized concept axioms must be added to prevent that a person is female and male?

2. Is there a single generalized concept axiom that prevents that a person is female and male?

Problem 1.6

Give an equivalent concept of $(\text{woman} \sqcap \exists\text{child.person})$ without using the constructors $\sqcap$ and $\exists r.C$

Problem 1.7

Prove the following:

If $(\forall r.C)(a) \in \mathcal{A}$, and $r(a,b) \in \mathcal{A}$, then $\mathcal{A} \models C(b)$.

Problem 1.8

Prove the following:

If $(\exists r.C)(a) \in \mathcal{A}$, $\mathcal{A}$ is satisfiable, and b is a Skolem constant, then $\mathcal{A} \cup \{r(a,b), C(b)\}$ is satisfiable as well.

Problem 1.9

Let $\mathcal{A}$ be an ABox. Proof or refute the following claims:

1. If $\mathcal{A}$ contains only elements of the form $r(a,b)$ where r is a role name and a,b are individual names, then $\mathcal{A}$ is satisfiable.
2. If $\mathcal{A}$ contains only elements of the form $A(x)$ where A is a concept name and a is an individual name,then $\mathcal{A}$ is satisfiable.
3. If $\mathcal{A}$ contains only elements of the form $A(x)$ or $\neg A(x)$, where A is an atomic concept name and x an individual name, then $\mathcal{A}$ is satisfiable.