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Faculty of Computer Science, Institute of Artificial Intelligence, Computational Logic Group

Finite-Cliquewidth Sets of Existential Rules

Toward a General Criterion for Decidable yet Highly Expressive Querying // ICDT 2023, Ioannina, 29.03.2023

Introduction: Existential Rules

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An ***ontology*** $(\mathcal{D}, \mathcal{R})$ consists of a database $\mathcal{D}$ and a set of rules $\mathcal{R}$.

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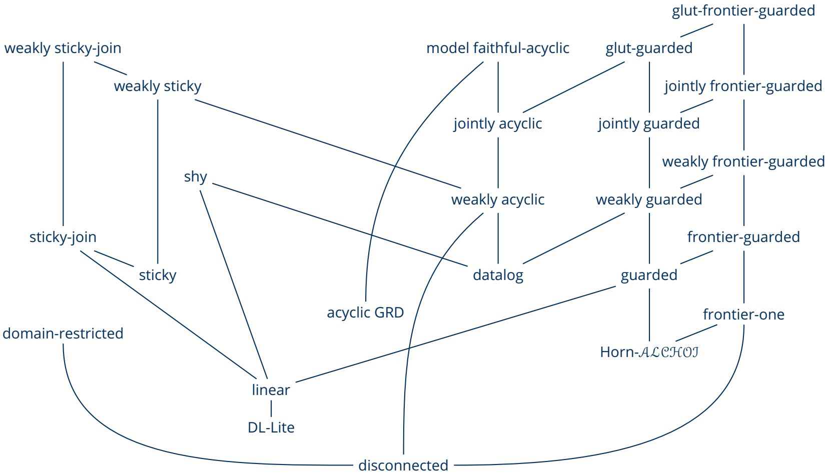
Problem: Undecidable!

Solution: Restricting rulesets.

Zoo of Decidable Existential Rules



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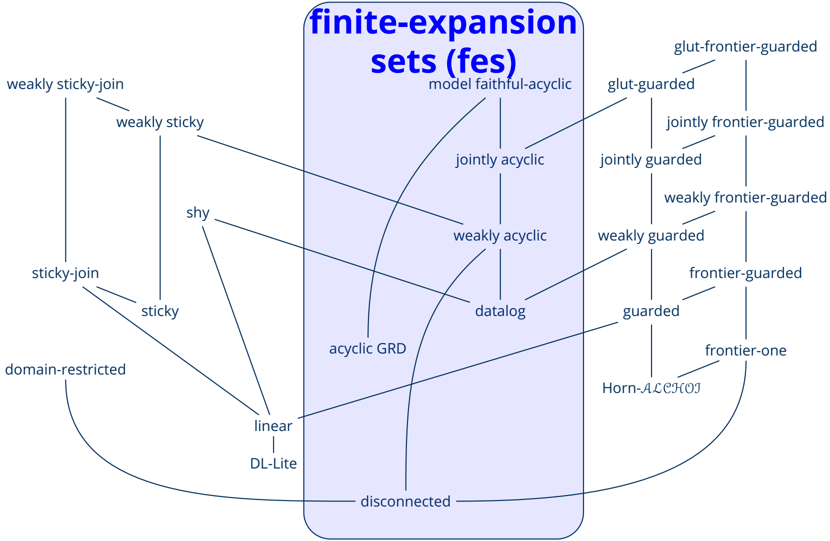

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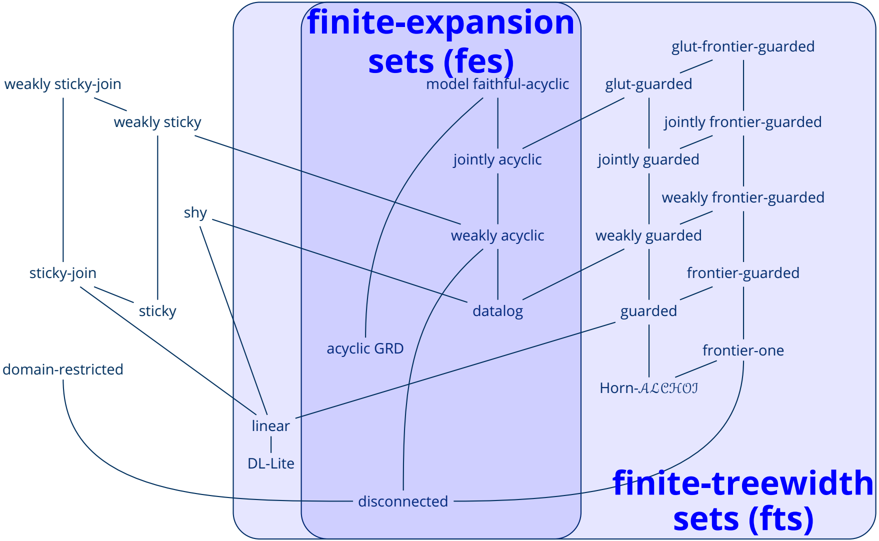

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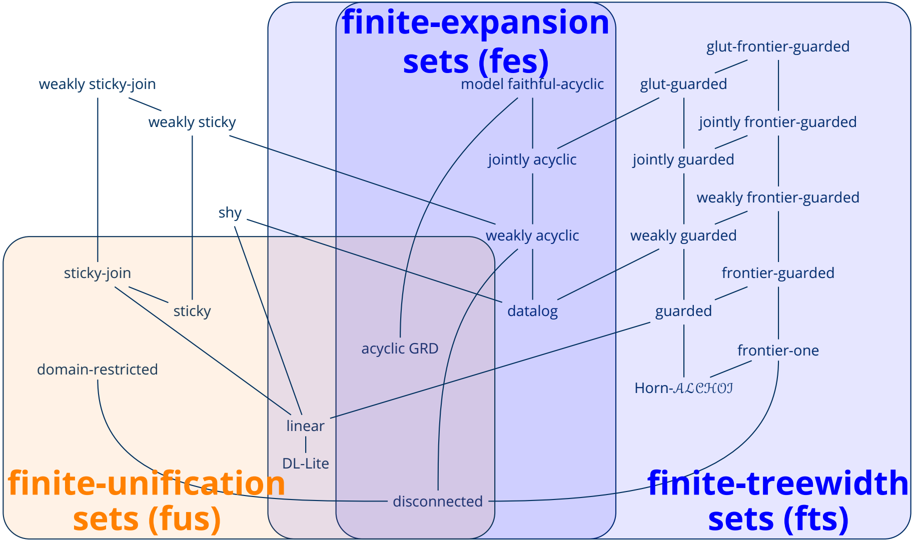

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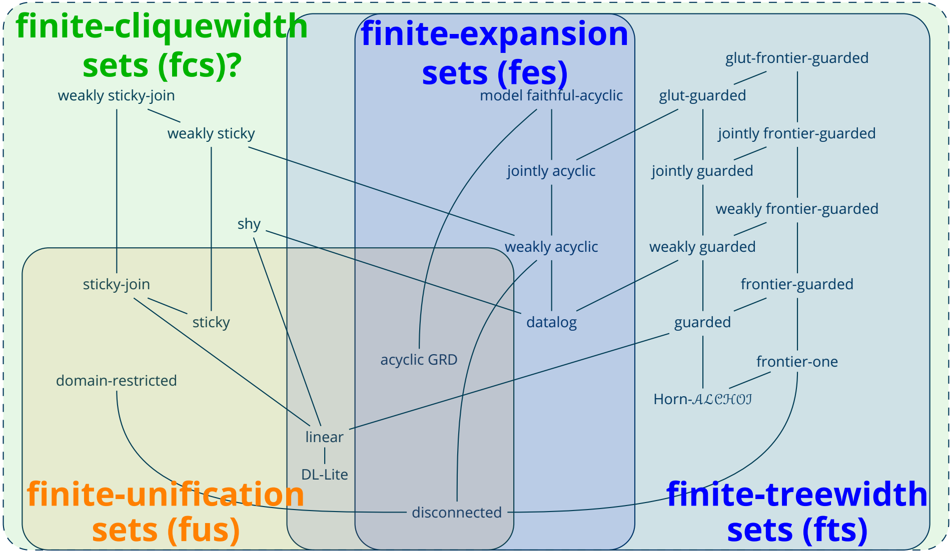

Example: fts, neither fes nor fus

$\forall x, y. A(x) \wedge E(x, y) \rightarrow A(y)$ $\forall x. A(x) \rightarrow \exists y. E(x, y)$

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Example: Concept Products, fus but not fts

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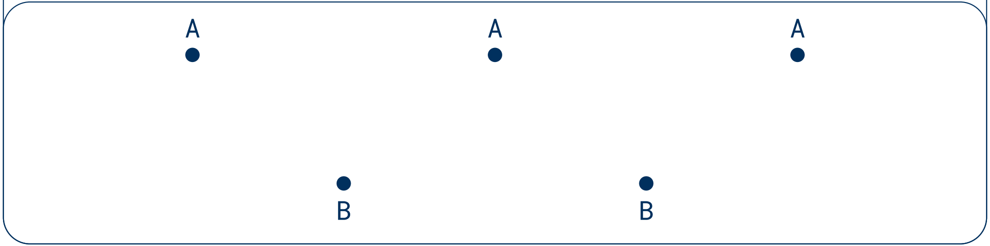

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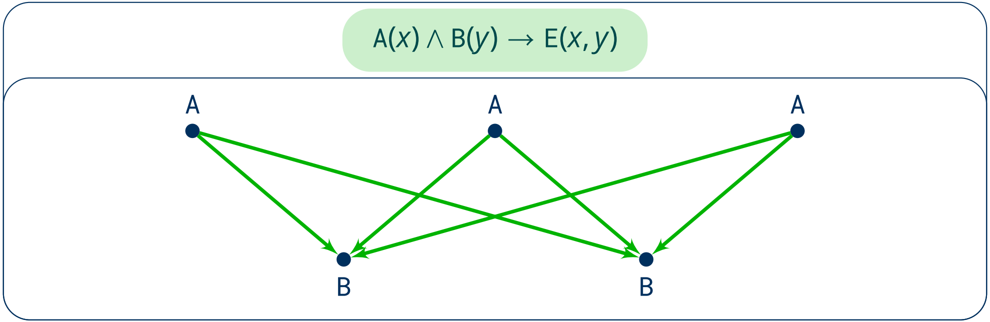

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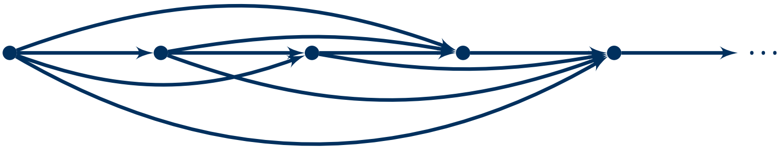

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In this talk: Will be introduced by example for finite and infinite instances in the binary and in the higher arity setting.

Our Contributions

We adapt cliquewidth for **infinite** instances in the **higher arity** setting.

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Take a peek into the paper!

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Cliquewidth: A finite and binary example

expression

instance

syntax tree

# of colors used

cliquewidth

Cliquewidth: A finite and binary example

expression

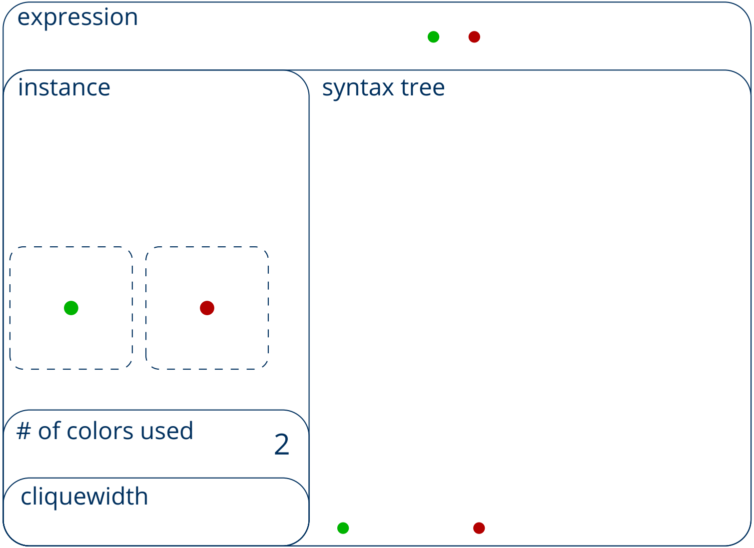
instance

syntax tree

# of colors used 1

cliquewidth

Cliquewidth: A finite and binary example



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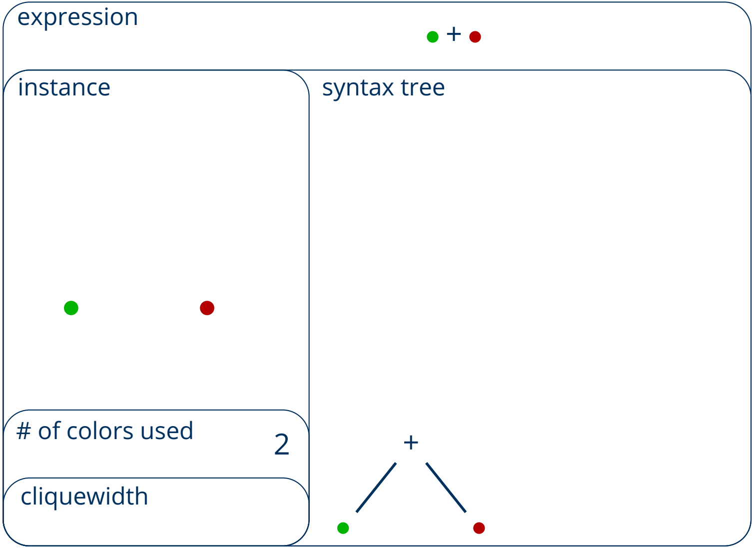

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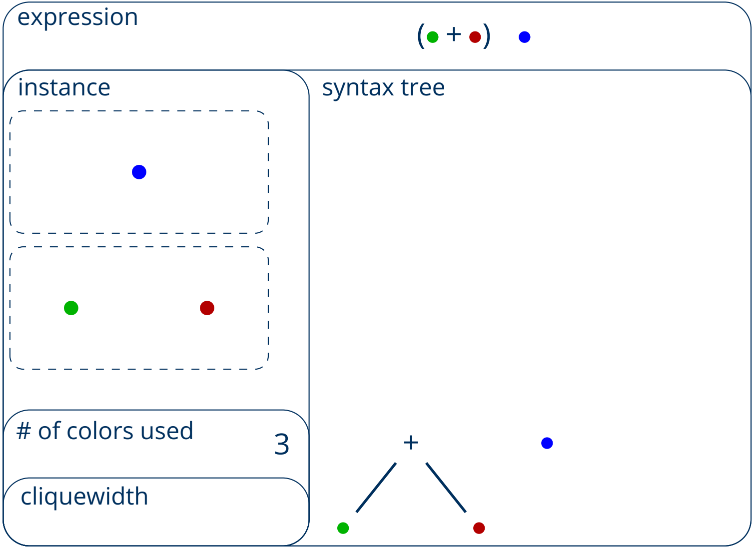

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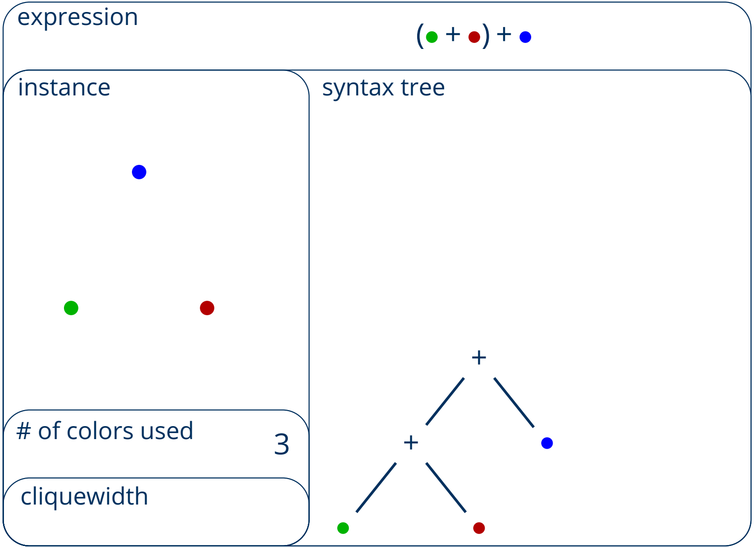

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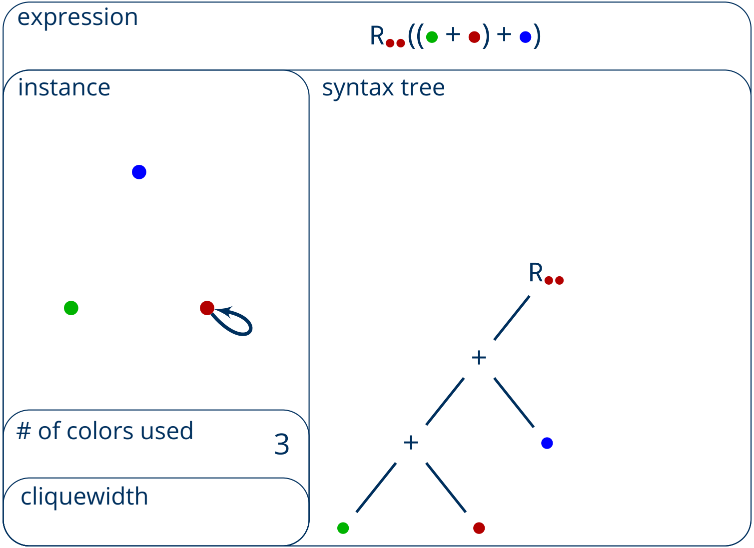

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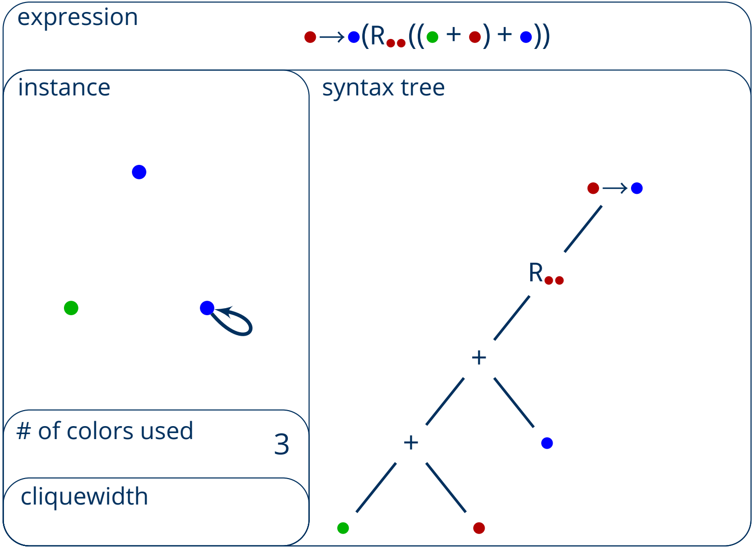

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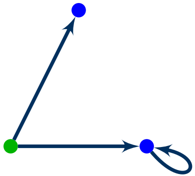
expression

R●●(●→●(R●●((● + ●) + ●)))

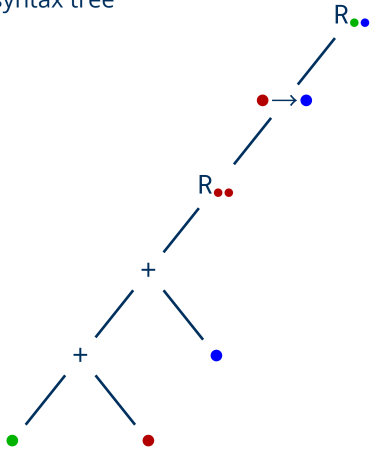
instance



# of colors used 3

cliquewidth

syntax tree



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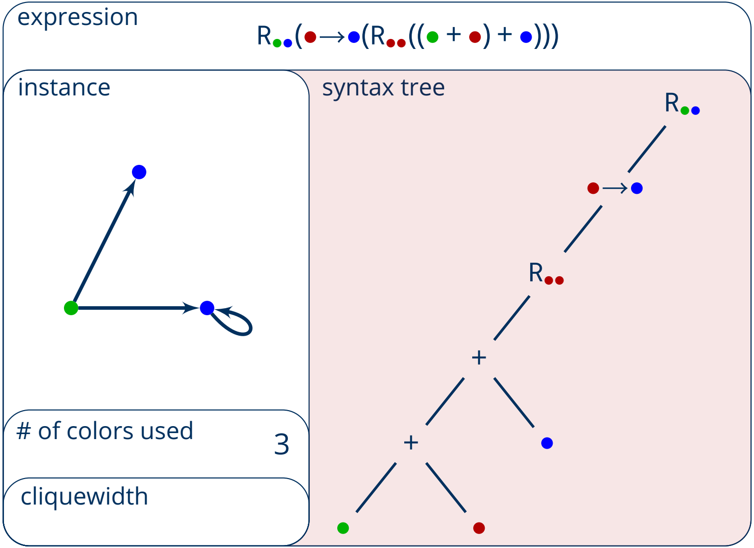

A finite, binary, and non-optimal example

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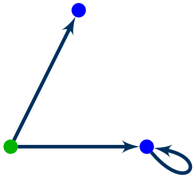

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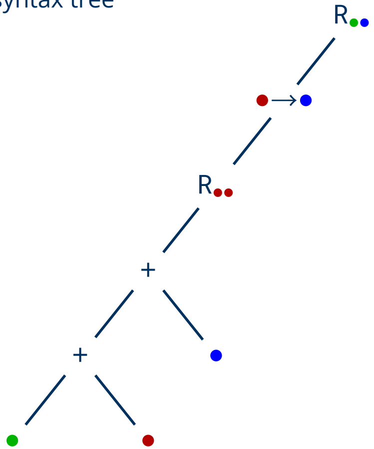

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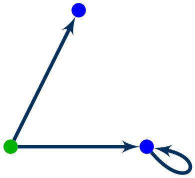
cliquewidth ≤ 3

A finite, binary, and optimal example

expression

R••((R••(• + •)) + •)

instance



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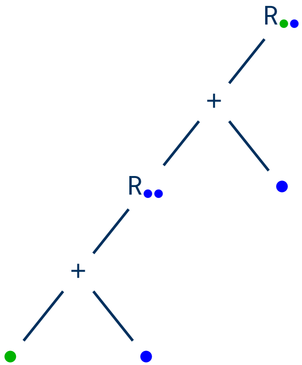

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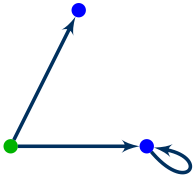
instance



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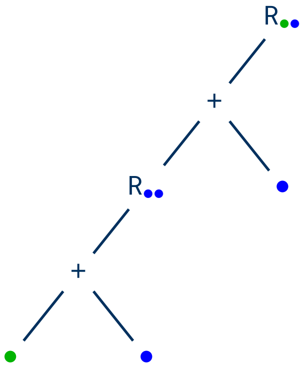

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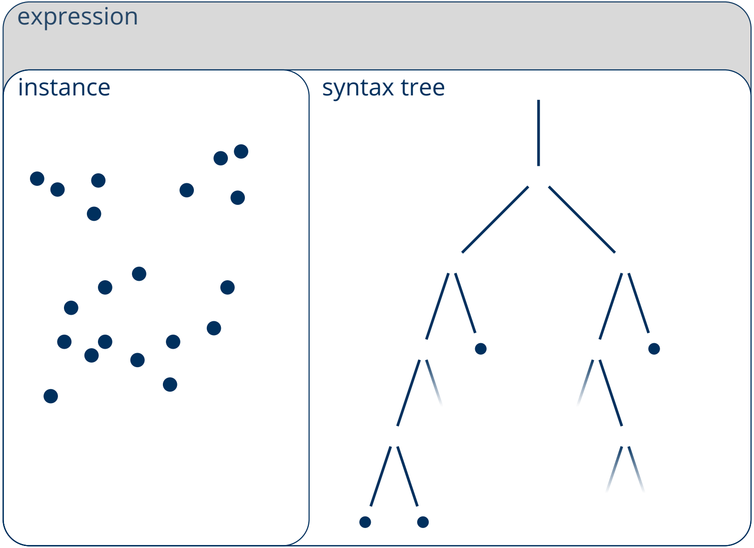
cliquewidth = 2

What about the infinite case?



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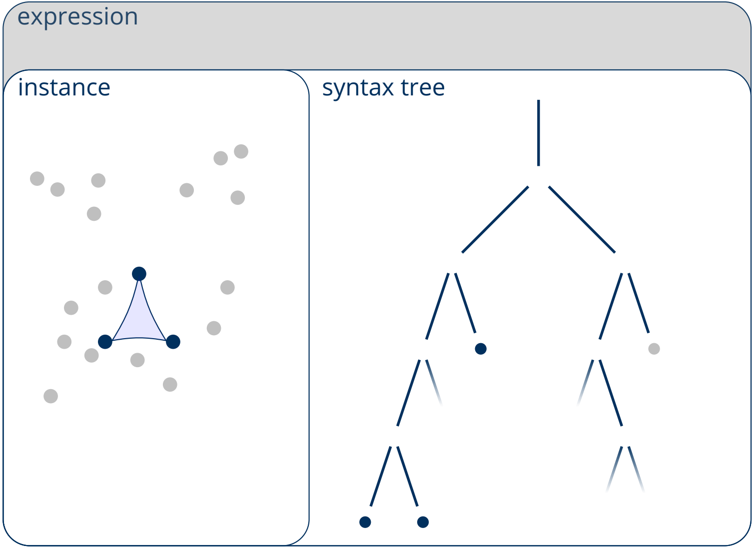

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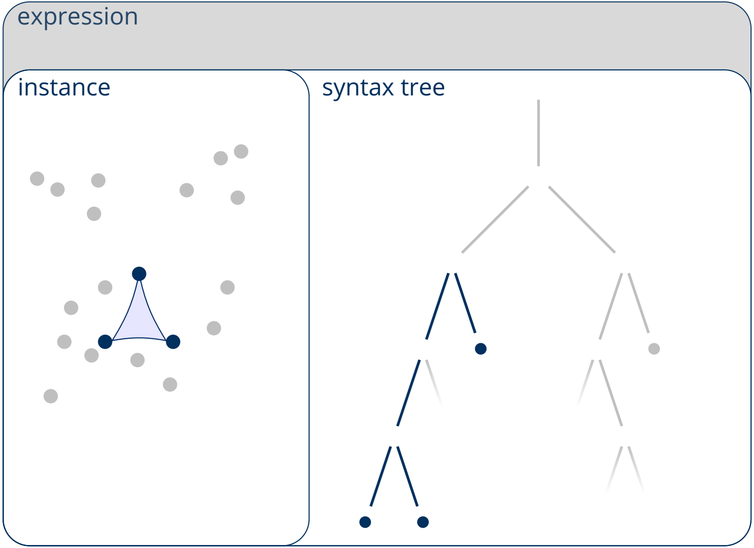

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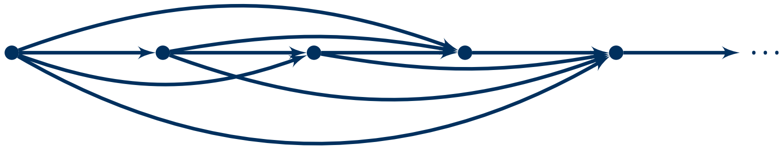

An (infinite) instance of finite Cliquewidth

Transitive chain

$\mathbb{N}$ with $\mathrm{R}(n, m)$ iff $n < m$ has a cliquewidth of 2.

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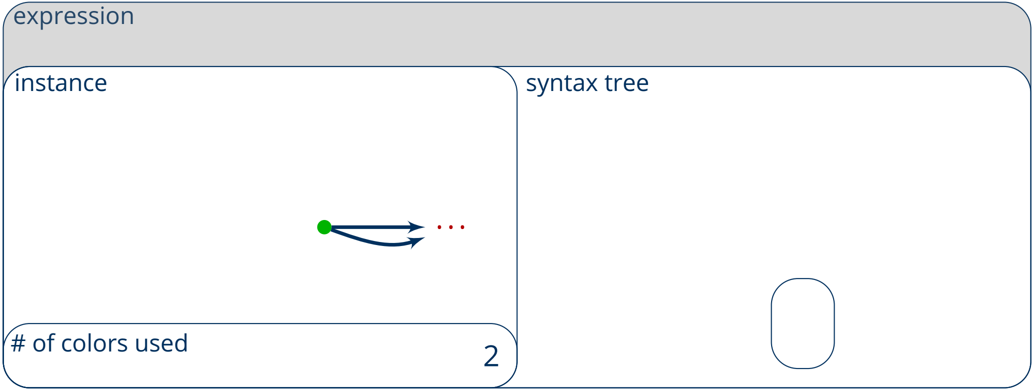

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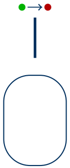

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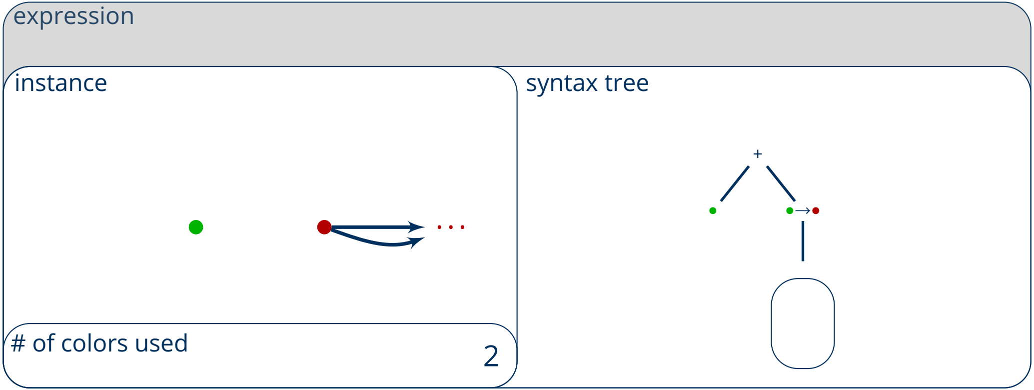

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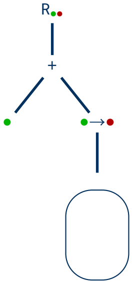

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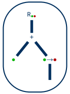

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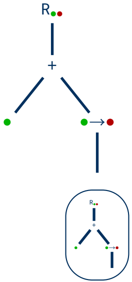

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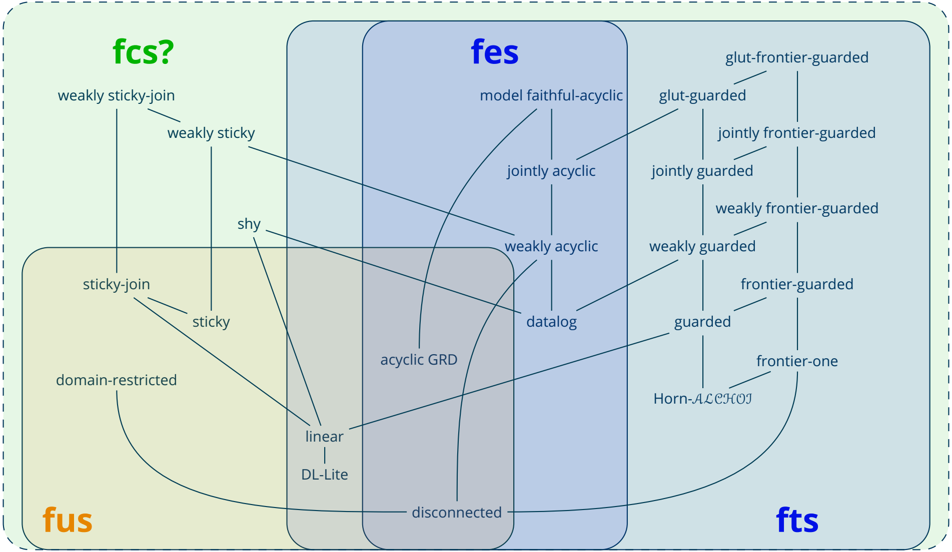
Idea: Constrain universal models by demanding finite cliquewidth.

Reminder: Zoo of Decidable Existential Rules



Finite cliquewidth and Monadic Second Order Logic

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Theorem

Determining if a given MSO-formula Φ has a model $\mathcal{I}$ with cliquewidth $\leq n$ is decidable for a fixed $n \in \mathbb{N}$.

Deciding query entailment

Definition (fcs)

A ruleset $\mathcal{R}$ is a ***finite-cliquewidth set*** (or *is fcs*), if for any database $\mathcal{D}$ there exists a universal model $\mathcal{U}$ for $(\mathcal{D}, \mathcal{R})$ of finite cliquewidth.

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Corollary

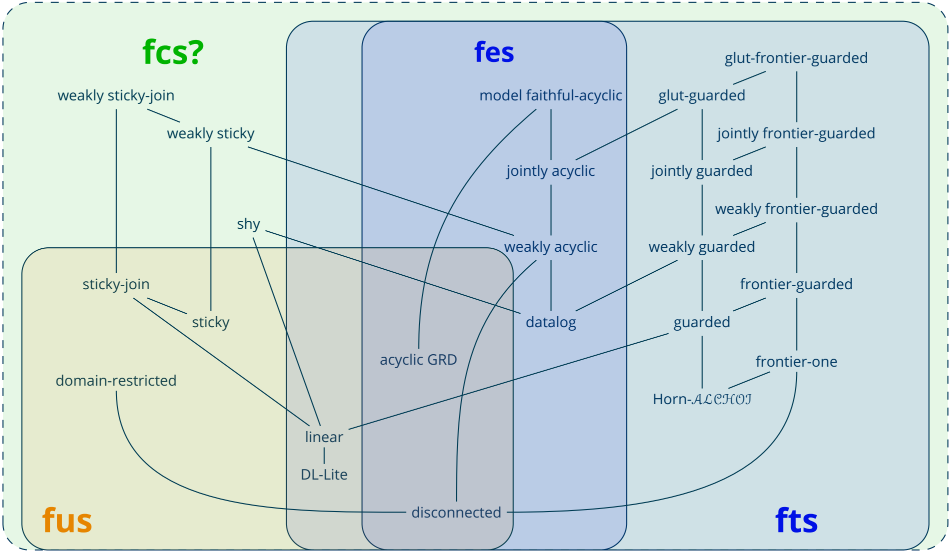
For arbitrary databases $\mathcal{D}$, if $\mathcal{R}$ is fcs and q a query expressible in MSO and Datalog, then query entailment is decidable.

Results: Binary single-head fus is fcs

Theorem

Any fus ruleset of single-headed rules over a binary signature is fcs.

Summary



fcs?
fes
fus
fts
weakly sticky-join
weakly sticky
shy
sticky-join
sticky
domain-restricted
linear
DL-Lite
acyclic GRD
disconnected
model faithful-acyclic
jointly acyclic
weakly acyclic
datalog
glut-frontier-guarded
glut-guarded
jointly frontier-guarded
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weakly frontier-guarded
weakly guarded
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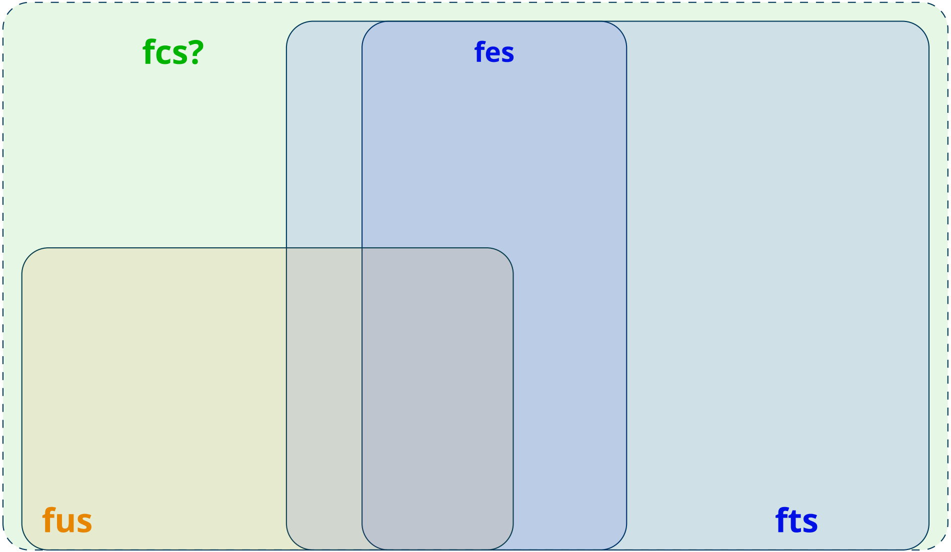
guarded
frontier-one
Horn-$\mathcal{ALCHOI}$

Summary: The binary picture



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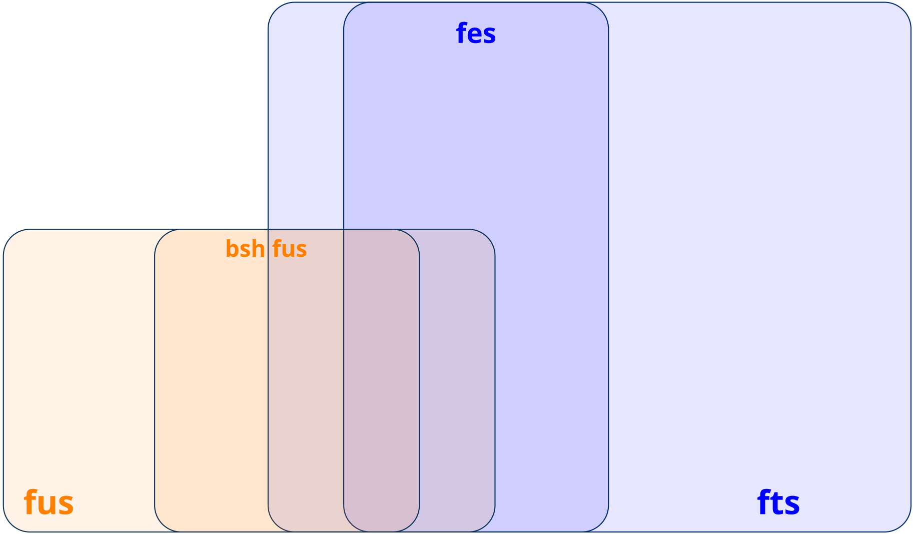
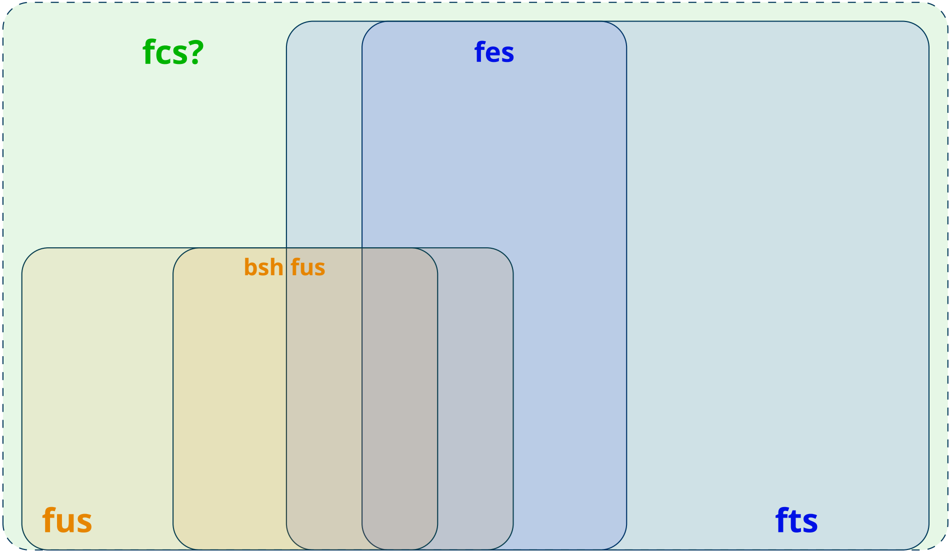
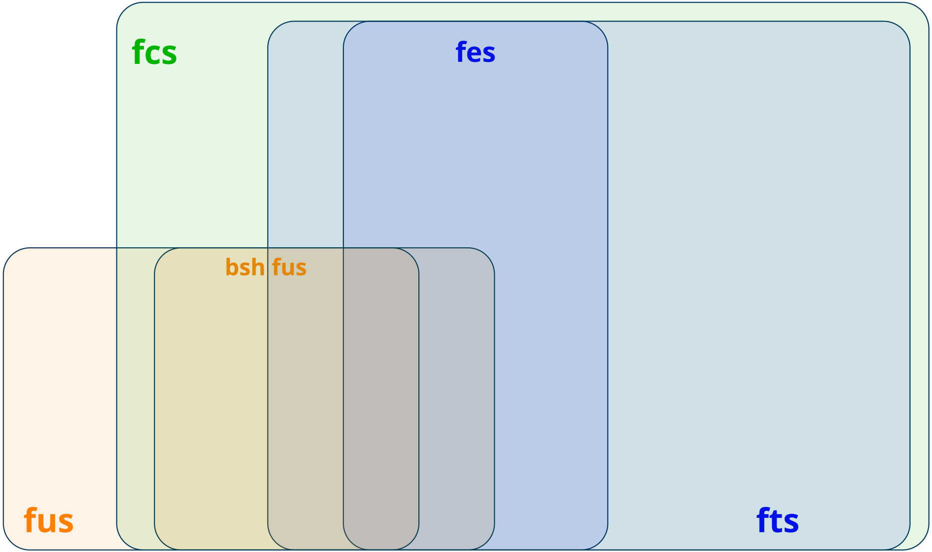


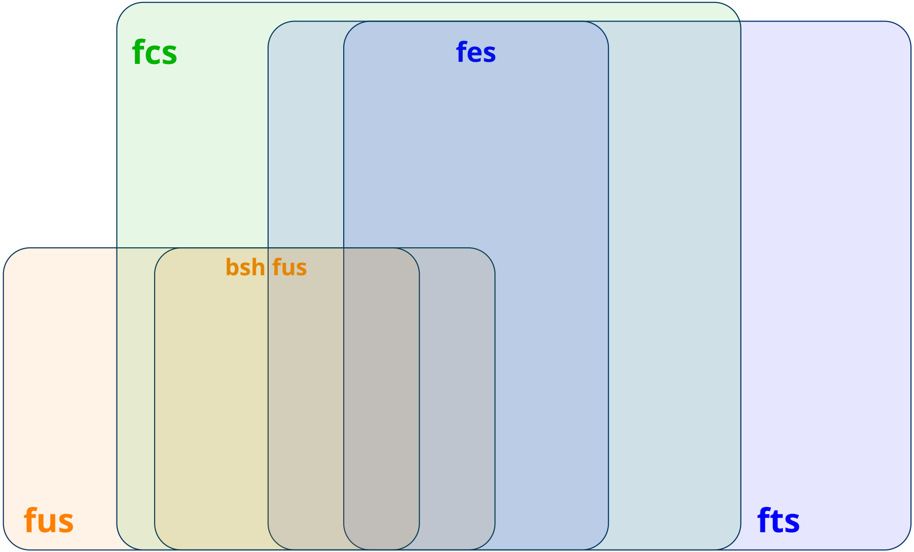
Summary: The binary picture



Summary: The binary picture



Summary: The higher arity case



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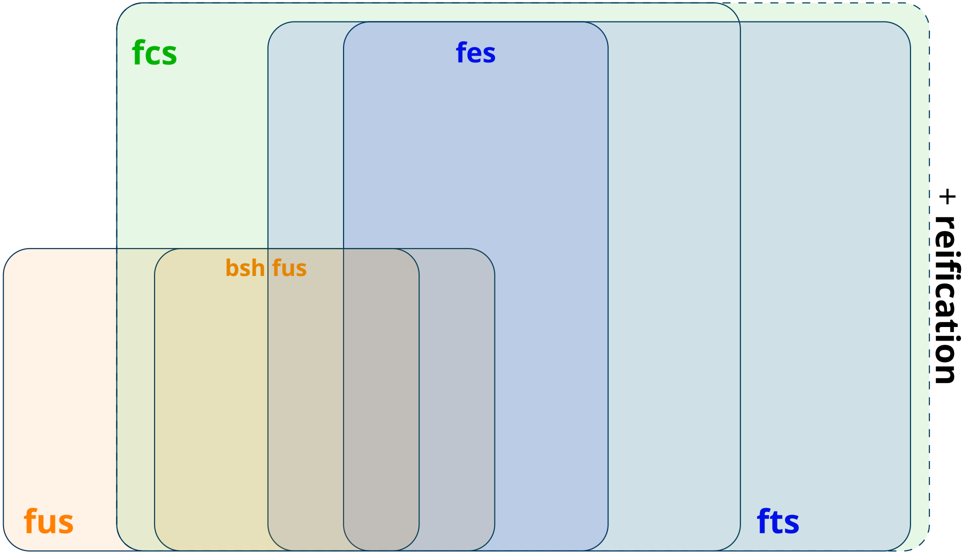

We adapt cliquewidth for **infinite** instances in the **higher arity** setting.

↓

Based on cliquewidth, we define **fcs** as a class of **decidable** existential rules.

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Thank you for your attention!