

Exercise 2: First-Order Queries

Database Theory

2023-04-18

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Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
...
Dogma	Smith	Damon
Dogma	Smith	Affleck
Dogma	Smith	Morissette
Dogma	Smith	Smith

Venues

Cinema	Address	Phone
UFA	St. Petersburger Str. 24	4825825
Schauburg	Königsbrücker Str. 55	8032185
CinemaxX	Hüblerstr. 8	3158910
...

Program

Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films		
Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig
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Dogma	Smith	Damon
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Dogma	Smith	Smith

Venues		
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UFA	St. Petersburger Str. 24	4825825
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Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
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Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

1. Who is the director of "The Imitation Game"?

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
The Internet's Own Boy	Knappenberger	Berners-Lee			
...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

1. Who is the director of "The Imitation Game"?

$$\exists y_A. \text{Films}(\text{"The Imitation Game"}, x_D, y_A)[x_D]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
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Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

1. Who is the director of "The Imitation Game"?

$$\exists y_A. \text{Films}(\text{"The Imitation Game"}, x_D, y_A)[x_D]$$

2. Which cinemas feature "The Imitation Game"?

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

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Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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Program		
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Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

1. Who is the director of "The Imitation Game"?

$$\exists y_A. \text{Films}(\text{"The Imitation Game"}, x_D, y_A)[x_D]$$

2. Which cinemas feature "The Imitation Game"?

$$\exists y_T. \text{Program}(x_C, \text{"The Imitation Game"}, y_T)[x_C]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

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Title	Director	Actor	Cinema	Address	Phone
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Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

3. What are the address and phone number of "Schauburg"?

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hüblerstr. 8	3158910
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The Internet's Own Boy	Knappenberger	Swartz			
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Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

3. What are the address and phone number of "Schauburg"?

$$\text{Venues}(\text{"Schauburg"}, x_A, x_P)[x_A, x_P]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

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Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

3. What are the address and phone number of "Schauburg"?

$$\text{Venues}(\text{"Schauburg"}, x_A, x_P)[x_A, x_P]$$

4. *Boolean query:* Is a film directed by "Smith" playing in some cinema?

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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Dogma	Smith	Damon			
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Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

3. What are the address and phone number of "Schauburg"?

$$\text{Venues}(\text{"Schauburg"}, x_A, x_P)[x_A, x_P]$$

4. *Boolean query:* Is a film directed by "Smith" playing in some cinema?

$$\exists y_T, y_A, y_C, z_T. \text{Films}(y_T, \text{"Smith"}, y_A) \wedge \text{Program}(y_C, y_T, z_T)$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

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Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
...
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The Internet's Own Boy	Knappenberger	Lessig
The Internet's Own Boy	Knappenberger	Berners-Lee
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Dogma	Smith	Damon
Dogma	Smith	Affleck
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Dogma	Smith	Smith

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CinemaxX	The Imitation Game	19:30

Solution.

- List the pairs of persons such that the first directed the second in a film, and vice versa.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

- List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\exists y_T, z_T. \text{Films}(y_T, x_D, x_A) \wedge \text{Films}(z_T, x_A, x_D)[x_D, x_A]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

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Program		
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Schauburg	The Imitation Game	19:30
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\exists y_T, z_T. \text{Films}(y_T, x_D, x_A) \wedge \text{Films}(z_T, x_A, x_D)[x_D, x_A]$$

6. List the names of directors who have acted in a film they directed.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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Dogma	Smith	Affleck			
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Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

5. List the pairs of persons such that the first directed the second in a film, and vice versa.

$$\exists y_T, z_T. \text{Films}(y_T, x_D, x_A) \wedge \text{Films}(z_T, x_A, x_D)[x_D, x_A]$$

6. List the names of directors who have acted in a film they directed.

$$\exists y_T. \text{Films}(y_T, x_D, x_D)[x_D]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films

Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
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The Internet's Own Boy	Knappenberger	Lessig
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Dogma	Smith	Damon
Dogma	Smith	Affleck
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Cinema	Address	Phone
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Schauburg	The Imitation Game	19:30
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CinemaxX	The Imitation Game	19:30

Solution.

- Always return {Title \mapsto "Apocalypse Now", Director \mapsto "Coppola"} as the answer.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
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CinemaxX	The Imitation Game	19:30

Solution.

7. Always return {Title \mapsto "Apocalypse Now", Director \mapsto "Coppola"} as the answer.

$\{\text{DirectedBy}(\text{"Apocalypse Now"}, \text{"Coppola"})\}$

Note: FO queries always use the unnamed perspective.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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CinemaxX	The Imitation Game	19:30

Solution.

7. Always return $\{\text{Title} \mapsto \text{"Apocalypse Now"}, \text{Director} \mapsto \text{"Coppola"}\}$ as the answer.

$$\{\text{DirectedBy}(\text{"Apocalypse Now"}, \text{"Coppola"})\}$$

Note: FO queries always use the unnamed perspective.

8. Find the actors cast in at least one film by "Smith".

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
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CinemaxX	The Imitation Game	19:30

Solution.

7. Always return $\{\text{Title} \mapsto \text{"Apocalypse Now"}, \text{Director} \mapsto \text{"Coppola"}\}$ as the answer.

$$\{\text{DirectedBy}(\text{"Apocalypse Now"}, \text{"Coppola"})\}$$

Note: FO queries always use the unnamed perspective.

8. Find the actors cast in at least one film by "Smith".

$$\exists y_T. \text{Films}(y_T, \text{"Smith"}, x_A)[x_A]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films		
Title	Director	Actor
The Imitation Game	Tyldum	Cumberbatch
The Imitation Game	Tyldum	Knightley
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Program		
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Schauburg	The Imitation Game	19:30
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

9. Find the actors cast in every film by "Smith."

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

9. Find the actors cast in every film by "Smith."

$$\exists y_T, y_D. (\text{Films}(y_T, y_D, x_A) \wedge \forall z_T, z_A. (\text{Films}(z_T, \text{"Smith"}, z_A) \rightarrow \text{Films}(z_T, \text{"Smith"}, x_A))) [x_A]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
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CinemaxX	The Imitation Game	19:30

Solution.

9. Find the actors cast in every film by "Smith."

$$\exists y_T, y_D. (\text{Films}(y_T, y_D, x_A) \wedge \forall z_T, z_A. (\text{Films}(z_T, \text{"Smith"}, z_A) \rightarrow \text{Films}(z_T, \text{"Smith"}, x_A))) [x_A]$$

10. Find the actors cast only in films by "Smith."

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

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Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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Cinema	Title	Time
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CinemaxX	The Imitation Game	19:30

Solution.

9. Find the actors cast in every film by "Smith."

$$\exists y_T, y_D. (\text{Films}(y_T, y_D, x_A) \wedge \forall z_T, z_A. (\text{Films}(z_T, \text{"Smith"}, z_A) \rightarrow \text{Films}(z_T, \text{"Smith"}, x_A))) [x_A]$$

10. Find the actors cast only in films by "Smith."

$$\exists y_T, y_D. (\text{Films}(y_T, y_D, x_A) \wedge \forall z_T. \exists z_D. (\text{Films}(z_T, z_D, x_A) \rightarrow z_D \approx \text{"Smith"})) [x_A]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hüblerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
The Internet's Own Boy	Knappenberger	Berners-Lee			
...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

11. Find all pairs of actors who act together in at least one film.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hüblerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
The Internet's Own Boy	Knappenberger	Berners-Lee			
...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

11. Find all pairs of actors who act together in at least one film.

$$\exists y_T, y_D. \text{Films}(y_T, y_D, x_A) \wedge \text{Films}(y_T, y_D, x_{A'}) \wedge x_A \neq x_{A'} [x_A, x_{A'}]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hüblerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
The Internet's Own Boy	Knappenberger	Berners-Lee			
...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

11. Find all pairs of actors who act together in at least one film.

$$\exists y_T, y_D. \text{Films}(y_T, y_D, x_A) \wedge \text{Films}(y_T, y_D, x_{A'}) \wedge x_A \neq x_{A'} [x_A, x_{A'}]$$

12. Find all pairs of actors cast in exactly the same films.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
...	CinemaxX	Hüblerstr. 8	3158910
The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
The Internet's Own Boy	Knappenberger	Berners-Lee			
...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

11. Find all pairs of actors who act together in at least one film.

$$\exists y_T, y_D. \text{Films}(y_T, y_D, x_A) \wedge \text{Films}(y_T, y_D, x_{A'}) \wedge x_A \neq x_{A'} [x_A, x_{A'}]$$

12. Find all pairs of actors cast in exactly the same films.

$$\exists y_T, y_D. \left(\text{Films}(y_T, y_D, x_A) \wedge \exists z_T, z_D. \text{Films}(z_T, z_D, x_{A'}) \wedge \forall w, v. \left(\text{Films}(w, v, x_A) \leftrightarrow \text{Films}(w, v, x_{A'}) \right) \right) [x_A, x_{A'}]$$

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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The Internet's Own Boy	Knappenberger	Swartz
The Internet's Own Boy	Knappenberger	Lessig			
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...			
Dogma	Smith	Damon			
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Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

- Find the directors such that every actor is cast in one of their films.

Exercise 1

Exercise. Express the queries from Exercise 1.1 as domain-independent FO-queries.

Films			Venues		
Title	Director	Actor	Cinema	Address	Phone
The Imitation Game	Tyldum	Cumberbatch	UFA	St. Petersburger Str. 24	4825825
The Imitation Game	Tyldum	Knightley	Schauburg	Königsbrücker Str. 55	8032185
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The Internet's Own Boy	Knappenberger	Swartz
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...			
Dogma	Smith	Damon			
Dogma	Smith	Affleck			
Dogma	Smith	Morissette			
Dogma	Smith	Smith			

Program		
Cinema	Title	Time
Schauburg	The Imitation Game	19:30
Schauburg	Dogma	20:45
UFA	The Imitation Game	22:45
CinemaxX	The Imitation Game	19:30

Solution.

13. Find the directors such that every actor is cast in one of their films.

$$\exists y_T, y_A. \left(\text{Films}(y_T, x_D, y_A) \wedge \forall z_T, z_D, z_A. \left(\text{Films}(z_T, z_D, z_A) \rightarrow \exists w_T. \text{Films}(w_T, x_D, z_A) \right) \right) [x_D]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- ▶ If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- ▶ if $q = \delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}. (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;
Assumption: A_1, \dots, A_n in $\delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n}$ are written in attribute order; B_1, \dots, B_n may be in arbitrary order.
- ▶ if $q = \pi_{A_1, \dots, A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$
with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}. \varphi_{q'}$;
- ▶ if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- ▶ if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg \varphi_{q_2}$.

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- ▶ If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- ▶ if $q = \delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}. (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;
Assumption: A_1, \dots, A_n in $\delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n}$ are written in attribute order; B_1, \dots, B_n may be in arbitrary order.
- ▶ if $q = \pi_{A_1, \dots, A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$ with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}. \varphi_{q'}$;
- ▶ if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- ▶ if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg \varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B. R(x_A, y_B)[x_A]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- ▶ If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- ▶ if $q = \delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}. (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;
Assumption: A_1, \dots, A_n in $\delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n}$ are written in attribute order; B_1, \dots, B_n may be in arbitrary order.
- ▶ if $q = \pi_{A_1, \dots, A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$ with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}. \varphi_{q'}$;
- ▶ if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- ▶ if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg \varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B. R(x_A, y_B)[x_A]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A. R(y_A, x_B)[x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- ▶ If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- ▶ if $q = \delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}. (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;
Assumption: A_1, \dots, A_n in $\delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n}$ are written in attribute order; B_1, \dots, B_n may be in arbitrary order.
- ▶ if $q = \pi_{A_1, \dots, A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$ with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}. \varphi_{q'}$;
- ▶ if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- ▶ if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg \varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B. R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A. R(y_A, x_B)[x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- ▶ If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- ▶ if $q = \delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}. (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;
Assumption: A_1, \dots, A_n in $\delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n}$ are written in attribute order; B_1, \dots, B_n may be in arbitrary order.
- ▶ if $q = \pi_{A_1, \dots, A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$ with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}. \varphi_{q'}$;
- ▶ if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- ▶ if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg \varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B. R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A. R(y_A, x_B)[x_B]$$

$$\varphi_{\pi_A(R) \bowtie \pi_B(R)}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)}[x_A, x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- ▶ If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- ▶ if $q = \delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}. (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;
Assumption: A_1, \dots, A_n in $\delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n}$ are written in attribute order; B_1, \dots, B_n may be in arbitrary order.
- ▶ if $q = \pi_{A_1, \dots, A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$ with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}. \varphi_{q'}$;
- ▶ if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- ▶ if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg \varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B. R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

$$\varphi_{(\delta_{B,A \rightarrow A,B}(R))}[x_A, x_B] = \exists y_B, y_A. (x_A \approx y_B) \wedge (x_B \approx y_A)$$

$$\wedge R(y_A, y_B)[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A. R(y_A, x_B)[x_B]$$

$$\varphi_{\pi_A(R) \bowtie \pi_B(R)}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)}[x_A, x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- ▶ If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- ▶ if $q = \delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}. (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;
Assumption: A_1, \dots, A_n in $\delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n}$ are written in attribute order; B_1, \dots, B_n may be in arbitrary order.
- ▶ if $q = \pi_{A_1, \dots, A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$ with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}. \varphi_{q'}$;
- ▶ if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- ▶ if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg \varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B. R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

$$\varphi_{(\delta_{B,A \rightarrow A,B}(R))}[x_A, x_B] = \exists y_B, y_A. (x_A \approx y_B) \wedge (x_B \approx y_A)$$

$$\wedge R(y_A, y_B)[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A. R(y_A, x_B)[x_B]$$

$$\varphi_{\pi_A(R) \bowtie \pi_B(R)}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)}[x_A, x_B]$$

$$\varphi_{R \bowtie (\delta_{B,A \rightarrow A,B}(R))}[x_A, x_B] = \varphi_R \wedge \varphi_{(\delta_{B,A \rightarrow A,B}(R))}[x_A, x_B]$$

Exercise 2

Exercise. Let $R[A, B]$ be a table. Express the following RA_{named} query as a DI_{unnamed} query:

$$q[A, B] = (\pi_A(R) \bowtie \pi_B(R)) - (R \bowtie (\delta_{B,A \rightarrow A,B}(R)))$$

Solution.

Definition (Lecture 2, Slide 19/20, excerpt)

For an RA query $q[a_1, \dots, a_n]$, let $\varphi_q[x_{a_1}, \dots, x_{a_n}]$ be the DI query defined as follows:

- ▶ If $q = R$ with signature $R[A_1, \dots, A_n]$, then $\varphi_q = R(x_{A_1}, \dots, x_{A_n})[x_{A_1}, \dots, x_{A_n}]$;
- ▶ if $q = \delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n} q'$, then $\varphi_q = \exists y_{B_1}, \dots, y_{B_n}. (x_{A_1} \approx y_{B_1}) \wedge \dots \wedge (x_{A_n} \approx y_{B_n}) \wedge \varphi_{q'}[y_{A_1}, \dots, y_{A_n}]$;
Assumption: A_1, \dots, A_n in $\delta_{B_1, \dots, B_n \rightarrow A_1, \dots, A_n}$ are written in attribute order; B_1, \dots, B_n may be in arbitrary order.
- ▶ if $q = \pi_{A_1, \dots, A_n}(q')$ for a subquery $q'[B_1, \dots, B_m]$ with $\{B_1, \dots, B_m\} = \{A_1, \dots, A_n\} \cup \{C_1, \dots, C_k\}$, then $\varphi_q = \exists x_{C_1}, \dots, x_{C_k}. \varphi_{q'}$;
- ▶ if $q = q_1 \bowtie q_2$, then $\varphi_q = \varphi_{q_1} \wedge \varphi_{q_2}$; and
- ▶ if $q = q_1 - q_2$, then $\varphi_q = \varphi_{q_1} \wedge \neg \varphi_{q_2}$.

$$\varphi_{\pi_A(R)}[x_A] = \exists y_B. R(x_A, y_B)[x_A]$$

$$\varphi_R[x_A, x_B] = R(x_A, x_B)[x_A, x_B]$$

$$\varphi_{(\delta_{B,A \rightarrow A,B}(R))}[x_A, x_B] = \exists y_B, y_A. (x_A \approx y_B) \wedge (x_B \approx y_A)$$

$$\wedge R(y_A, y_B)[x_A, x_B]$$

$$\varphi_{q[A,B]}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)} \wedge \neg (\varphi_R \wedge \varphi_{(\delta_{B,A \rightarrow A,B}(R))})[x_A, x_B]$$

$$\varphi_{\pi_B(R)}[x_B] = \exists y_A. R(y_A, x_B)[x_B]$$

$$\varphi_{\pi_A(R) \bowtie \pi_B(R)}[x_A, x_B] = \varphi_{\pi_A(R)} \wedge \varphi_{\pi_B(R)}[x_A, x_B]$$

$$\varphi_{R \bowtie (\delta_{B,A \rightarrow A,B}(R))}[x_A, x_B] = \varphi_R \wedge \varphi_{(\delta_{B,A \rightarrow A,B}(R))}[x_A, x_B]$$

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Exercise. It was stated in the lecture (Lecture 2, slide 17) that query mappings under named perspective can be translated into query mappings under unnamed perspective. Specify this translation.

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- ▶ Let \mathbb{D}_n and \mathbb{D}_u be the sets of all database instances over a named and an unnamed perspective, respectively, and let \mathbb{T}_n and \mathbb{T}_u be the sets of all database tables over a named and an unnamed perspective.

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- ▶ Consider a query mapping $M[q] : \mathbb{D}_n \rightarrow \mathbb{T}_n$.

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- ▶ Consider a query mapping $M[q] : \mathbb{D}_n \rightarrow \mathbb{T}_n$.
- ▶ Define $\nu : \mathbb{T}_n \rightarrow \mathbb{T}_u$ as the function taking named database tables $R[A_1, \dots, A_n]$ to unnamed database tables $R^{\mathcal{J}}$, such that attribute A_i is mapped to column i :

$$\nu(R^{\mathcal{J}}) = \{ \langle r(A_1), \dots, r(A_n) \rangle \mid (r : \{A_1, \dots, A_n\} \rightarrow \mathbf{dom}) \in R^{\mathcal{J}} \}$$

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$$\nu(R^{\mathcal{I}}) = \{ \langle r(A_1), \dots, r(A_n) \rangle \mid (r : \{A_1, \dots, A_n\} \rightarrow \mathbf{dom}) \in R^{\mathcal{I}} \}$$

- ▶ Conversely, let $\mu : \mathbb{D}_u \rightarrow \mathbb{D}_n$ be the function taking unnamed database instances \mathcal{J} to named database instances \mathcal{I} , by mapping each table $R^{\mathcal{J}}$ to a named table taking attribute A_i from column i :

$$\mu(\mathcal{J}) = \left\{ \left\{ \{A_1 \mapsto a_1, \dots, A_n \mapsto a_n\} \mid \langle a_1, \dots, a_n \rangle \in R^{\mathcal{J}} \right\} \mid R \in \mathcal{J} \right\}$$

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- ▶ Then $\nu \circ M[q] \circ \mu : \mathbb{D}_u \rightarrow \mathbb{T}_u$ is the required translation of $M[q] : \mathbb{D}_n \rightarrow \mathbb{T}_n$.

$$\begin{array}{ccc} \mathbb{D}_n & \xrightarrow{M[q]} & \mathbb{T}_n \\ \mu \uparrow & & \downarrow \nu \\ \mathbb{D}_u & \xrightarrow{\nu \circ M[q] \circ \mu} & \mathbb{T}_u \end{array}$$

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Exercise. Complete the proof that $RA_{\text{named}} \sqsubseteq DI_{\text{unnamed}}$ by showing that the results of the transformation are (a) *domain independent* and (b) equivalent to the input query. In each case, show that the claimed property holds true for each case of the recursive construction under the assumption (induction hypothesis) that it has been established for all subqueries. Use the mappings from the previous exercise to compare named and unnamed results.

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Exercise. Consider a binary predicate R and the AD_{unnamed} query

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Exercise. Complete the constructions for the proof of $AD \sqsubseteq RA$ given in the lecture.

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1. Assume that the database schema consists of tables R_1, \dots, R_ℓ with table schemata $R_i[a_1^i, \dots, a_{|R_i|}^i]$. Let q be the query and define

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1. Define the relational algebra expression $E_{a, \mathbf{adom}}$, such that $E_{a, \mathbf{adom}}(I) = \{\{a \mapsto c\} \mid c \in \mathbf{adom}(I, q)\}$ (assume that the query and the database schema are known).
2. Define the expressions E_φ for $\varphi = \varphi_1 \vee \varphi_2$ and $\varphi = \forall y. \psi$ in terms of expressions that have already been defined in the lecture.
3. Give a direct definition for the expression E_φ for $\varphi = \varphi_1 \vee \varphi_2 \equiv \neg(\neg\varphi_1 \wedge \neg\varphi_2)$.

Solution.

1. Assume that the database schema consists of tables R_1, \dots, R_ℓ with table schemata $R_i[a_1^i, \dots, a_{|R_i|}^i]$. Let q be the query and define

$$E_{a, \mathbf{adom}} = \left(\bigcup_{i=1}^{\ell} \bigcup_{j=1}^{|R_i|} \delta_{a_j^i \rightarrow a}(\pi_{a_j^i}(R_i)) \right) \cup \{\{a \mapsto c\} \mid c \in \mathbf{adom}(q)\}.$$

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Exercise 7

Exercise. Use the function `rr` from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{\text{SID}}, y_{\text{Stop}}, y_{\text{To}}. (\text{Stops}(y_{\text{SID}}, y_{\text{Stop}}, \text{"true"}) \wedge \text{Connect}(y_{\text{SID}}, y_{\text{To}}, x_{\text{Line}})) [x_{\text{Line}}]$
2. $\varphi_2 = \neg \text{Lines}(x, \text{"bus"}) [x]$
3. $\varphi_3 = (\text{Connect}(x_1, \text{"42"}, \text{"85"}) \vee \text{Connect}(\text{"57"}, x_2, \text{"85"})) [x_1, x_2]$
4. $\varphi_4 = \forall y. p(x, y) [x]$
5. $\varphi_5 = \exists x. (((p(x) \rightarrow q(c)) \rightarrow p(x)) \rightarrow p(x))$

Which of these queries is a safe-range query? Which of the queries is domain independent?

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Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

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Definition (Lecture 2, Slide 26)

The set $rr(\varphi)$ of **range-restricted variables** of φ in **Safe-Range Normal Form** is defined recursively:

$$\begin{aligned} rr(R(t_1, \dots, t_n)) &= \{x \mid x \text{ is a variable among the } t_1, \dots, t_n\} & rr(x \approx a) &= \{x\} \\ rr(\varphi_1 \wedge \varphi_2) &= \begin{cases} rr(\varphi_1) \cup \{x, y\} & \text{if } \varphi_2 = (x \approx y) \text{ and } \{x, y\} \cap rr(\varphi_1) \neq \emptyset \\ rr(\varphi_1) \cup rr(\varphi_2) & \text{otherwise} \end{cases} & rr(x \approx y) &= \emptyset \\ rr(\exists y. \psi) &= \begin{cases} rr(\psi) \setminus \{y\} & \text{if } y \in rr(\psi) \\ \text{throw new NotSafeException}() & \text{if } y \notin rr(\psi) \end{cases} & rr(\varphi_1 \vee \varphi_2) &= rr(\varphi_1) \cap rr(\varphi_2) \\ & & rr(\neg \psi) &= \emptyset \quad \text{if } rr(\psi) \text{ is defined} \end{aligned}$$

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Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$rr(\varphi_1) = \{x_{Line}\} \quad rr(\varphi_2) = \emptyset \quad rr(\varphi_3) = \emptyset \quad rr(SNRF(\varphi_4)) = \text{Exception}$$

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Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$rr(\varphi_1) = \{x_{Line}\} \quad rr(\varphi_2) = \emptyset \quad rr(\varphi_3) = \emptyset \quad rr(SNRF(\varphi_4)) = \text{Exception}$$

Definition (Lecture 2, Slide 26)

The set $rr(\varphi)$ of **range-restricted variables** of φ in **Safe-Range Normal Form** is defined recursively:

$$\begin{aligned} rr(R(t_1, \dots, t_n)) &= \{x \mid x \text{ is a variable among the } t_1, \dots, t_n\} & rr(x \approx a) &= \{x\} \\ rr(\varphi_1 \wedge \varphi_2) &= \begin{cases} rr(\varphi_1) \cup \{x, y\} & \text{if } \varphi_2 = (x \approx y) \text{ and } \{x, y\} \cap rr(\varphi_1) \neq \emptyset \\ rr(\varphi_1) \cup rr(\varphi_2) & \text{otherwise} \end{cases} & rr(x \approx y) &= \emptyset \\ rr(\exists y. \psi) &= \begin{cases} rr(\psi) \setminus \{y\} & \text{if } y \in rr(\psi) \\ \text{throw new NotSafeException}() & \text{if } y \notin rr(\psi) \end{cases} & rr(\varphi_1 \vee \varphi_2) &= rr(\varphi_1) \cap rr(\varphi_2) \\ & & rr(\neg \psi) &= \emptyset \quad \text{if } rr(\psi) \text{ is defined} \end{aligned}$$

Exercise 7

Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{SID}, y_{Stop}, y_{To}. (\text{Stops}(y_{SID}, y_{Stop}, \text{"true"}) \wedge \text{Connect}(y_{SID}, y_{To}, x_{Line})) [x_{Line}]$
2. $\varphi_2 = \neg \text{Lines}(x, \text{"bus"}) [x]$
3. $\varphi_3 = (\text{Connect}(x_1, \text{"42"}, \text{"85"}) \vee \text{Connect}(\text{"57"}, x_2, \text{"85"})) [x_1, x_2]$
4. $\varphi_4 = \forall y. p(x, y) [x] = \neg \exists y. \neg p(x, y) [x]$
5. $\varphi_5 = \exists x. (((p(x) \rightarrow q(c)) \rightarrow p(x)) \rightarrow p(x)) = \exists x. (((\neg p(x) \vee q(c)) \wedge \neg p(x)) \vee p(x))$

Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$rr(\varphi_1) = \{x_{Line}\} \quad rr(\varphi_2) = \emptyset \quad rr(\varphi_3) = \emptyset \quad rr(SNRF(\varphi_4)) = \text{Exception} \quad rr(SNRF(\varphi_5)) = \text{Exception}$$

Definition (Lecture 2, Slide 26)

The set $rr(\varphi)$ of **range-restricted variables** of φ in **Safe-Range Normal Form** is defined recursively:

$$\begin{aligned} rr(R(t_1, \dots, t_n)) &= \{x \mid x \text{ is a variable among the } t_1, \dots, t_n\} & rr(x \approx a) &= \{x\} \\ rr(\varphi_1 \wedge \varphi_2) &= \begin{cases} rr(\varphi_1) \cup \{x, y\} & \text{if } \varphi_2 = (x \approx y) \text{ and } \{x, y\} \cap rr(\varphi_1) \neq \emptyset \\ rr(\varphi_1) \cup rr(\varphi_2) & \text{otherwise} \end{cases} & rr(x \approx y) &= \emptyset \\ rr(\exists y. \psi) &= \begin{cases} rr(\psi) \setminus \{y\} & \text{if } y \in rr(\psi) \\ \text{throw new NotSafeException}() & \text{if } y \notin rr(\psi) \end{cases} & rr(\varphi_1 \vee \varphi_2) &= rr(\varphi_1) \cap rr(\varphi_2) \\ & & rr(\neg \psi) &= \emptyset \quad \text{if } rr(\psi) \text{ is defined} \end{aligned}$$

Exercise 7

Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{SID}, y_{Stop}, y_{To}. (\text{Stops}(y_{SID}, y_{Stop}, \text{"true"}) \wedge \text{Connect}(y_{SID}, y_{To}, x_{Line})) [x_{Line}]$
2. $\varphi_2 = \neg \text{Lines}(x, \text{"bus"}) [x]$
3. $\varphi_3 = (\text{Connect}(x_1, \text{"42"}, \text{"85"}) \vee \text{Connect}(\text{"57"}, x_2, \text{"85"})) [x_1, x_2]$
4. $\varphi_4 = \forall y. p(x, y) [x] = \neg \exists y. \neg p(x, y) [x]$
5. $\varphi_5 = \exists x. (((p(x) \rightarrow q(c)) \rightarrow p(x)) \rightarrow p(x)) = \exists x. (((\neg p(x) \vee q(c)) \wedge \neg p(x)) \vee p(x))$

Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$rr(\varphi_1) = \{ x_{Line} \} \quad rr(\varphi_2) = \emptyset \quad rr(\varphi_3) = \emptyset \quad rr(SNRF(\varphi_4)) = \text{Exception} \quad rr(SNRF(\varphi_5)) = \text{Exception}$$

Definition (Lecture 2, Slide 27)

An FO query $q = \varphi[x_1, \dots, x_n]$ is a **safe-range query** if $rr(SNRF(\varphi)) = \{ x_1, \dots, x_n \}$.

Exercise 7

Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{SID}, y_{Stop}, y_{To}. (\text{Stops}(y_{SID}, y_{Stop}, \text{"true"}) \wedge \text{Connect}(y_{SID}, y_{To}, x_{Line})) [x_{Line}]$
2. $\varphi_2 = \neg \text{Lines}(x, \text{"bus"}) [x]$
3. $\varphi_3 = (\text{Connect}(x_1, \text{"42"}, \text{"85"}) \vee \text{Connect}(\text{"57"}, x_2, \text{"85"})) [x_1, x_2]$
4. $\varphi_4 = \forall y. p(x, y) [x] = \neg \exists y. \neg p(x, y) [x]$
5. $\varphi_5 = \exists x. (((p(x) \rightarrow q(c)) \rightarrow p(x)) \rightarrow p(x)) = \exists x. (((\neg p(x) \vee q(c)) \wedge \neg p(x)) \vee p(x))$

Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$\begin{aligned} rr(\varphi_1) &= \{x_{Line}\} & rr(\varphi_2) &= \emptyset & rr(\varphi_3) &= \emptyset & rr(SNRF(\varphi_4)) &= \text{Exception} & rr(SNRF(\varphi_5)) &= \text{Exception} \\ & \text{SR, DI} & & & & & & & & \end{aligned}$$

Definition (Lecture 2, Slide 27)

An FO query $q = \varphi[x_1, \dots, x_n]$ is a **safe-range query** if $rr(SNRF(\varphi)) = \{x_1, \dots, x_n\}$.

Exercise 7

Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{SID}, y_{Stop}, y_{To}. (\text{Stops}(y_{SID}, y_{Stop}, \text{"true"}) \wedge \text{Connect}(y_{SID}, y_{To}, x_{Line})) [x_{Line}]$
2. $\varphi_2 = \neg \text{Lines}(x, \text{"bus"}) [x]$
3. $\varphi_3 = (\text{Connect}(x_1, \text{"42"}, \text{"85"}) \vee \text{Connect}(\text{"57"}, x_2, \text{"85"})) [x_1, x_2]$
4. $\varphi_4 = \forall y. p(x, y) [x] = \neg \exists y. \neg p(x, y) [x]$
5. $\varphi_5 = \exists x. (((p(x) \rightarrow q(c)) \rightarrow p(x)) \rightarrow p(x)) = \exists x. (((\neg p(x) \vee q(c)) \wedge \neg p(x)) \vee p(x))$

Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$\begin{array}{llllll} rr(\varphi_1) = \{ x_{Line} \} & rr(\varphi_2) = \emptyset & rr(\varphi_3) = \emptyset & rr(SNRF(\varphi_4)) = \text{Exception} & rr(SNRF(\varphi_5)) = \text{Exception} \\ SR, DI & \text{not SR, not DI} & & & & \end{array}$$

Definition (Lecture 2, Slide 27)

An FO query $q = \varphi[x_1, \dots, x_n]$ is a **safe-range query** if $rr(SNRF(\varphi)) = \{x_1, \dots, x_n\}$.

Exercise 7

Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{SID}, y_{Stop}, y_{To}. (\text{Stops}(y_{SID}, y_{Stop}, \text{"true"}) \wedge \text{Connect}(y_{SID}, y_{To}, x_{Line})) [x_{Line}]$
2. $\varphi_2 = \neg \text{Lines}(x, \text{"bus"}) [x]$
3. $\varphi_3 = (\text{Connect}(x_1, \text{"42"}, \text{"85"}) \vee \text{Connect}(\text{"57"}, x_2, \text{"85"})) [x_1, x_2]$
4. $\varphi_4 = \forall y. p(x, y) [x] = \neg \exists y. \neg p(x, y) [x]$
5. $\varphi_5 = \exists x. (((p(x) \rightarrow q(c)) \rightarrow p(x)) \rightarrow p(x)) = \exists x. (((\neg p(x) \vee q(c)) \wedge \neg p(x)) \vee p(x))$

Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$\begin{array}{llllll} rr(\varphi_1) = \{ x_{Line} \} & rr(\varphi_2) = \emptyset & rr(\varphi_3) = \emptyset & rr(SNRF(\varphi_4)) = \text{Exception} & rr(SNRF(\varphi_5)) = \text{Exception} \\ SR, DI & \text{not SR, not DI} & \text{not SR, not DI} & & \end{array}$$

Definition (Lecture 2, Slide 27)

An FO query $q = \varphi[x_1, \dots, x_n]$ is a **safe-range query** if $rr(SNRF(\varphi)) = \{x_1, \dots, x_n\}$.

Exercise 7

Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{SID}, y_{Stop}, y_{To}. (\text{Stops}(y_{SID}, y_{Stop}, \text{"true"}) \wedge \text{Connect}(y_{SID}, y_{To}, x_{Line})) [x_{Line}]$
2. $\varphi_2 = \neg \text{Lines}(x, \text{"bus"}) [x]$
3. $\varphi_3 = (\text{Connect}(x_1, \text{"42"}, \text{"85"}) \vee \text{Connect}(\text{"57"}, x_2, \text{"85"})) [x_1, x_2]$
4. $\varphi_4 = \forall y. p(x, y) [x] = \neg \exists y. \neg p(x, y) [x]$
5. $\varphi_5 = \exists x. (((p(x) \rightarrow q(c)) \rightarrow p(x)) \rightarrow p(x)) = \exists x. (((\neg p(x) \vee q(c)) \wedge \neg p(x)) \vee p(x))$

Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$\begin{array}{llllll} rr(\varphi_1) = \{ x_{Line} \} & rr(\varphi_2) = \emptyset & rr(\varphi_3) = \emptyset & rr(SNRF(\varphi_4)) = \text{Exception} & rr(SNRF(\varphi_5)) = \text{Exception} \\ SR, DI & \text{not SR, not DI} & \text{not SR, not DI} & \text{not SR, not DI} & \end{array}$$

Definition (Lecture 2, Slide 27)

An FO query $q = \varphi[x_1, \dots, x_n]$ is a **safe-range query** if $rr(SRNF(\varphi)) = \{x_1, \dots, x_n\}$.

Exercise 7

Exercise. Use the function rr from the lecture to compute the set of range-restricted variables for the following queries:

1. $\varphi_1 = \exists y_{SID}, y_{Stop}, y_{To}. (\text{Stops}(y_{SID}, y_{Stop}, \text{"true"}) \wedge \text{Connect}(y_{SID}, y_{To}, x_{Line})) [x_{Line}]$
2. $\varphi_2 = \neg \text{Lines}(x, \text{"bus"}) [x]$
3. $\varphi_3 = (\text{Connect}(x_1, \text{"42"}, \text{"85"}) \vee \text{Connect}(\text{"57"}, x_2, \text{"85"})) [x_1, x_2]$
4. $\varphi_4 = \forall y. p(x, y) [x] = \neg \exists y. \neg p(x, y) [x]$
5. $\varphi_5 = \exists x. (((p(x) \rightarrow q(c)) \rightarrow p(x)) \rightarrow p(x)) = \exists x. (((\neg p(x) \vee q(c)) \wedge \neg p(x)) \vee p(x))$

Which of these queries is a safe-range query? Which of the queries is domain independent?

Solution.

$$\begin{array}{ccccccc} rr(\varphi_1) = \{ x_{Line} \} & rr(\varphi_2) = \emptyset & rr(\varphi_3) = \emptyset & rr(SNRF(\varphi_4)) = \text{Exception} & rr(SNRF(\varphi_5)) = \text{Exception} \\ SR, DI & \text{not SR, not DI} & \text{not SR, not DI} & \text{not SR, not DI} & \text{not SR, DI} \end{array}$$

Definition (Lecture 2, Slide 27)

An FO query $q = \varphi[x_1, \dots, x_n]$ is a **safe-range query** if $rr(SNRF(\varphi)) = \{x_1, \dots, x_n\}$.