1 Set Theory

Exercise 1 (a piece of recapitulation)
Given the following hints and the universe $M := \{1, 2, 3, 4, 5, 6, 7, 8\}$, compute the sets $A, B, C$:

(a) $A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$
(b) $B \cup C = \{1, 2, 4, 6, 8\}$
(c) $A \cup C = \{1, 2, 3, 4, 5, 7, 8\}$
(d) $A \cap B = \{2\}$
(e) $B \cap C = \{2, 4, 8\}$
(f) $A \cap C = \{2\}$

2 Logic

Exercise 2 (repetition first-order logic)
Formalize the following statements for natural numbers $a, b, c$, using only multiplication ("·"), equality ("=") and natural numbers ("0","1","2",...) besides the usual logical symbols ("¬","∧","∨","→","↔","∀","∃", variables and parentheses):

(i) $a$ divides $b$
(ii) $a$ is odd.
(iii) $a$ is common divisor of $b$ and $c$
(iv) $a$ is the gcd of $b$ and $c$.
(v) $a$ is a square number.
(vi) $a$ is a prime number.

3 Derivation Operators and Formal Concepts

Exercise 3 (line diagram)

a) Recall: how is the derivation operator $(·)'$ defined?

b) Let $K = (G, M, I)$ be a formal context and let $A, B \subseteq G$. Prove the following statements:

1. $A \subseteq B$ implies $B' \subseteq A'$
2. $A \subseteq A''$
3. $A' = A'''$
4. For $C \in G$ and $D \in M$ holds: $(C, D)$ is a formal concept if and only if there is some $E \subseteq G$ such that $C = E''$ and $D = E'$. 
## 4 Formal Concept Analysis

### Exercise 4 (Formal Context)

Regard the following formal context \( K \), given as a cross table:

<table>
<thead>
<tr>
<th></th>
<th>needs water to live</th>
<th>lives in water</th>
<th>lives on land</th>
<th>needs chlorophyll to produce food</th>
<th>two seed leaves</th>
<th>one seed leaf</th>
<th>can move around</th>
<th>has limbs</th>
<th>suckles its offspring</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leech</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bream</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frog</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spike-Weed</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reed</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bean</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maize</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Specify the following sets:

(i) \( \{ \text{Bean} \}' \)

(ii) \( \{ \text{lives on land} \}' \)

(iii) \( \{ \text{two seed leaves} \}'' \)

(iv) \( \{ \text{Frog, Maize} \}' \)

(v) \( \{ \text{needs chlorophyll to produce food, can move around} \}' \)

(vi) \( \{ \text{lives in water, lives on land} \}' \)

(vii) \( \{ \text{needs chlorophyll to produce food, can move around} \}'' \)

b) Extend \( K \) with both an object and an attribute.