

General Acyclicity and Cyclicity Notions for the Disjunctive Skolem Chase

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AAAI, February 2023



International Center
for Computational Logic



Chasing a Universal Model Set

Motorcycle(x) → HasEngine(x) ∨ ∃z.NeedsEngine(x, z)

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NeedsEngine(x, y) → HasEngine(x) ∧ Engine(y)

Engine(x) → VTwin(x) ∨ InlineFour(x)

Chasing a Universal Model Set

$Motorcycle(x) \rightarrow HasEngine(x) \vee \exists z.NeedsEngine(x, z)$

$NeedsEngine(x, y) \rightarrow HasEngine(x) \wedge Engine(y)$

$Engine(x) \rightarrow VTwin(x) \vee InlineFour(x)$

$Engine(x) \rightarrow \exists z.Motorcycle(z) \wedge NeedsEngine(z, x)$

Chasing a Universal Model Set

$Motorcycle(x) \rightarrow HasEngine(x) \vee NeedsEngine(x, f_z^1(x))$

$NeedsEngine(x, y) \rightarrow HasEngine(x) \wedge Engine(y)$

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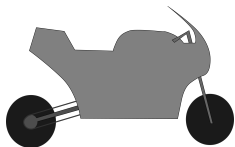
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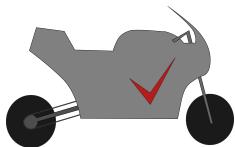
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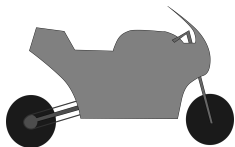
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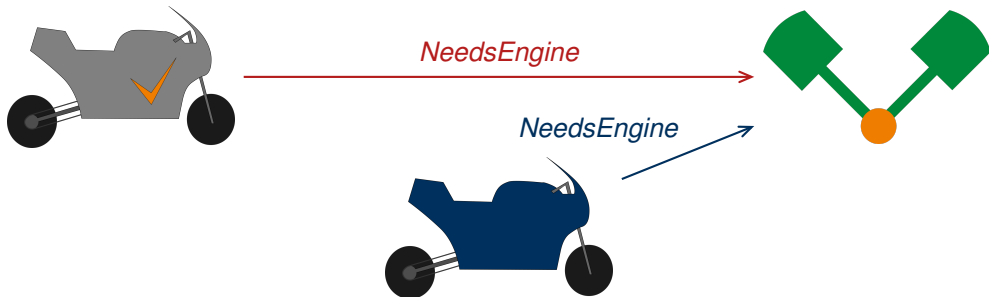
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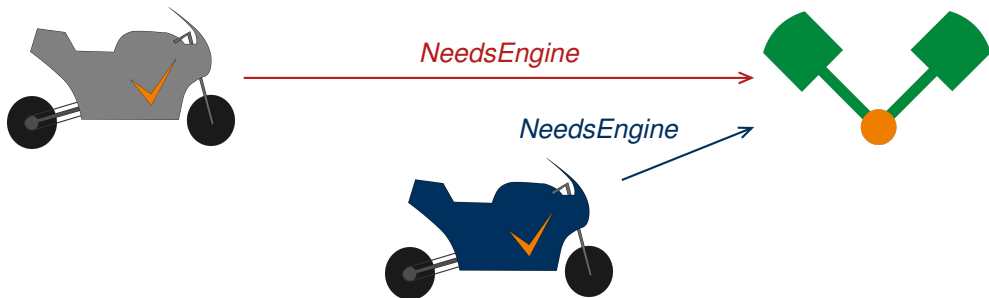
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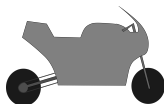
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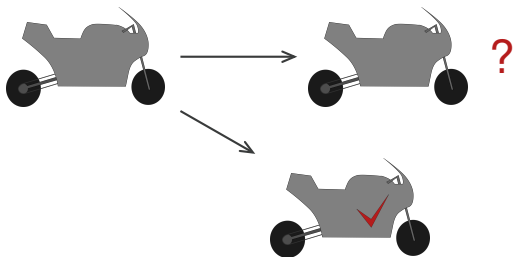
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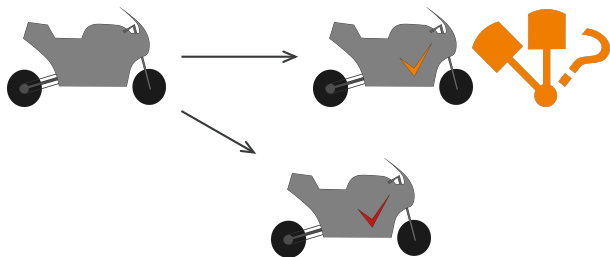
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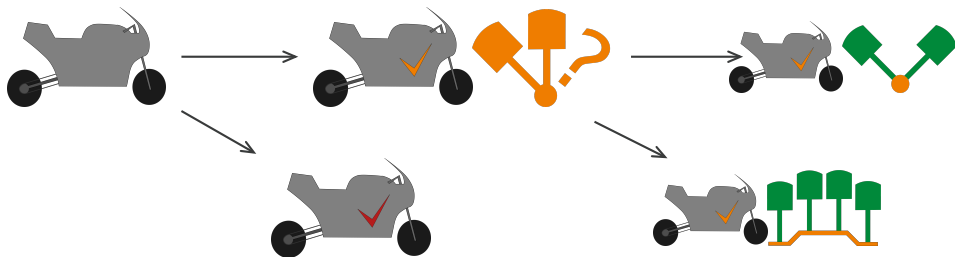
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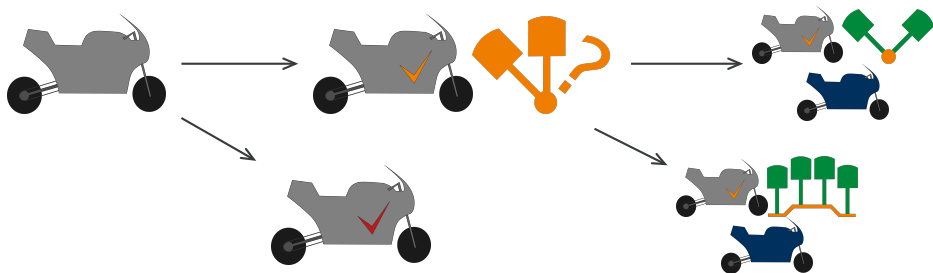
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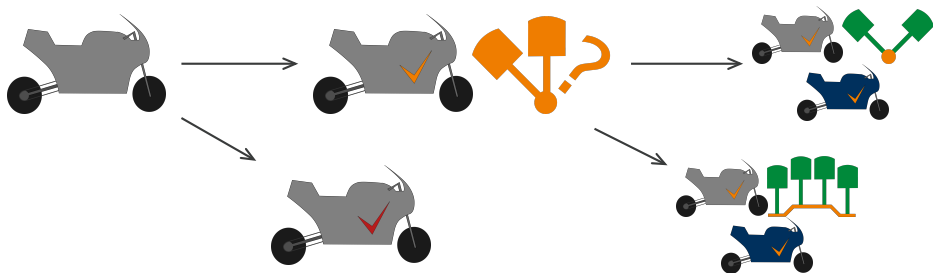
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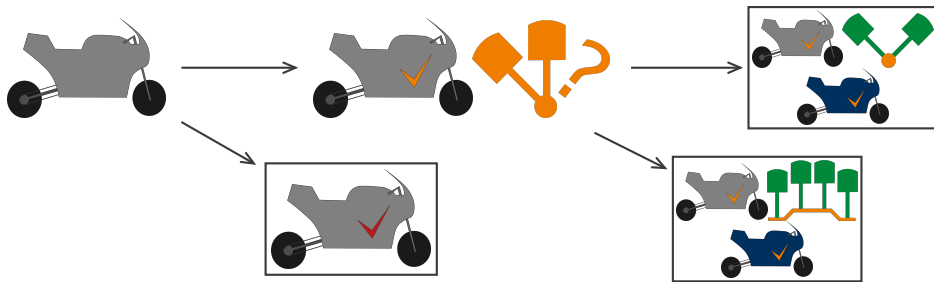
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The **disjunctive skolem chase** yields a **universal model set**. [Bourhis et al., 2016]

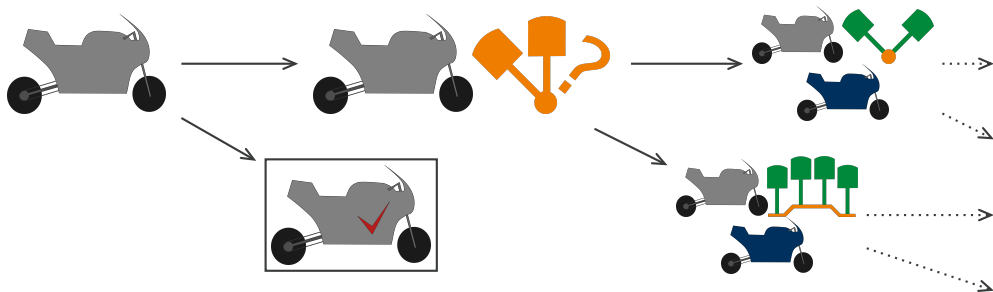
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Model Faithful Acyclicity [Cuenca Grau et al., 2013]

Motorcycle(x) \rightarrow *HasEngine*(x) \vee *NeedsEngine*($x, f_z^1(x)$)

NeedsEngine(x, y) \rightarrow *HasEngine*(x) \wedge *Engine*(y)

Engine(x) \rightarrow *VTwin*(x) \vee *InlineFour*(x)

Engine(x) \rightarrow *Motorcycle*($f_z^4(x)$) \wedge *NeedsEngine*($f_z^4(x), x$)

Model Faithful Acyclicity [Cuenca Grau et al., 2013]

$Motorcycle(x) \rightarrow HasEngine(x) \not\ll \wedge NeedsEngine(x, f_z^1(x))$

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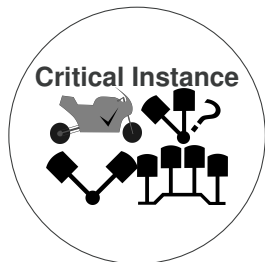
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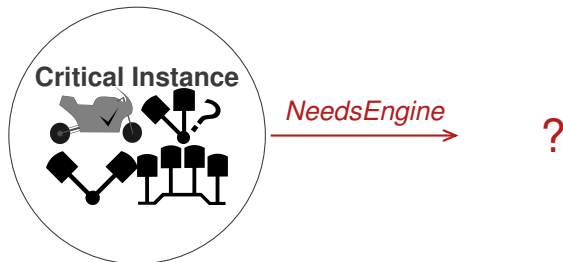
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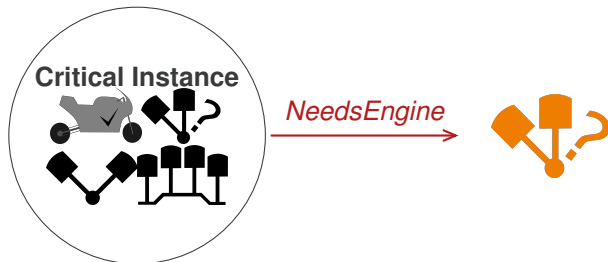
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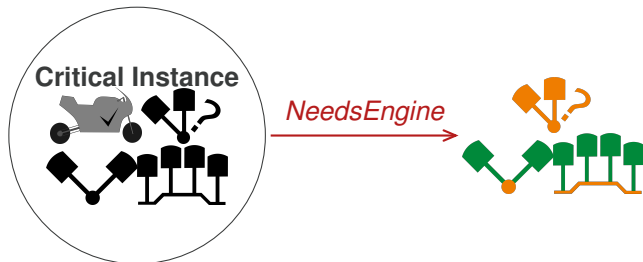
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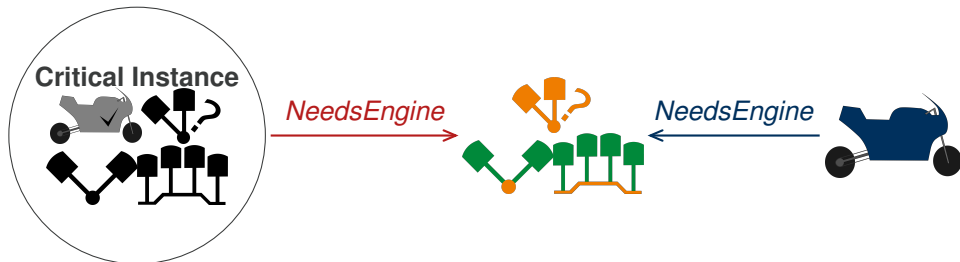
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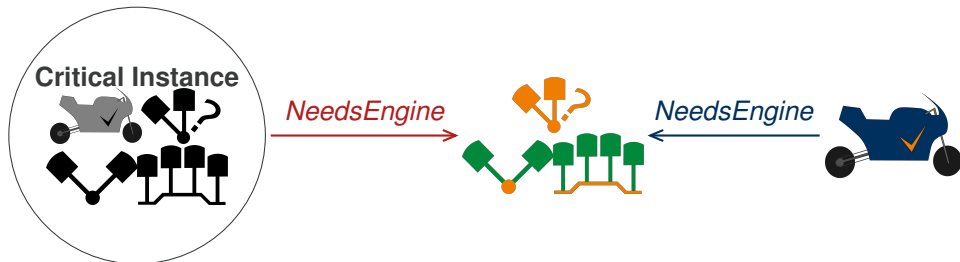
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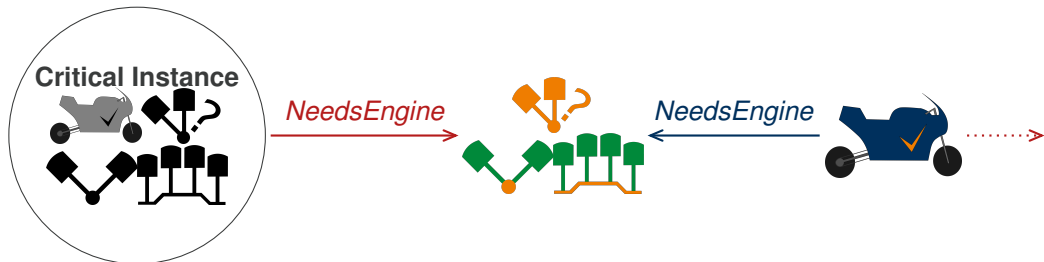
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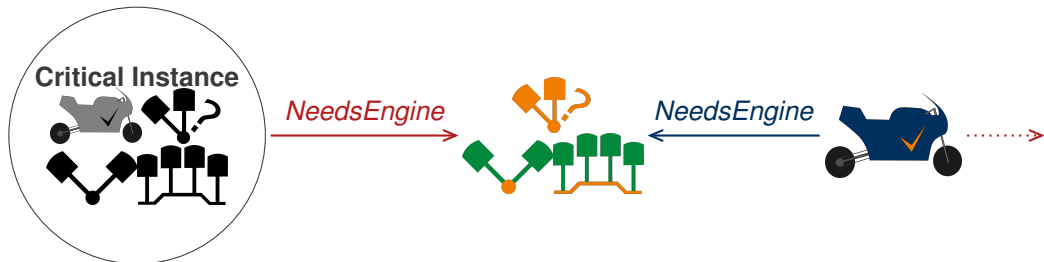
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The rule set is **not MFA** even though it is **terminating**.

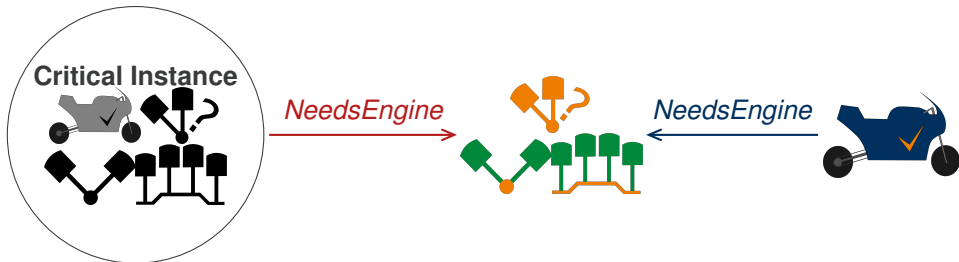
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Underapproximation



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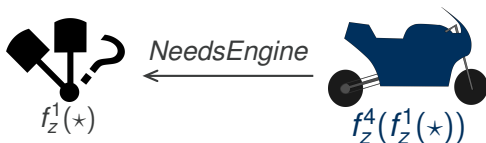
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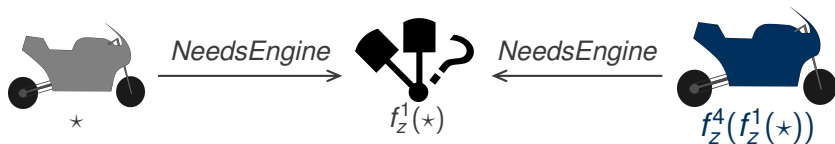
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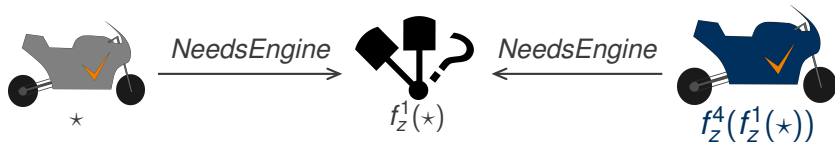
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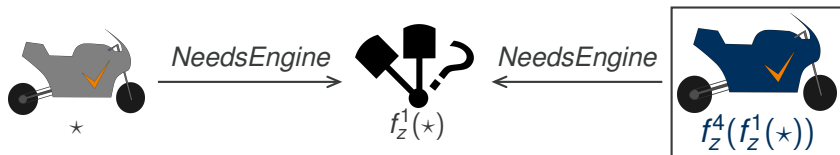
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Underapproximation



The **rule application** is **blocked**.

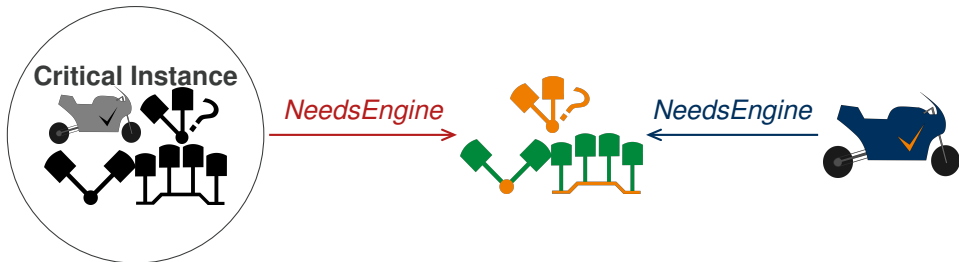
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The **rule application** is **blocked**. The rule set is **DMFA!**

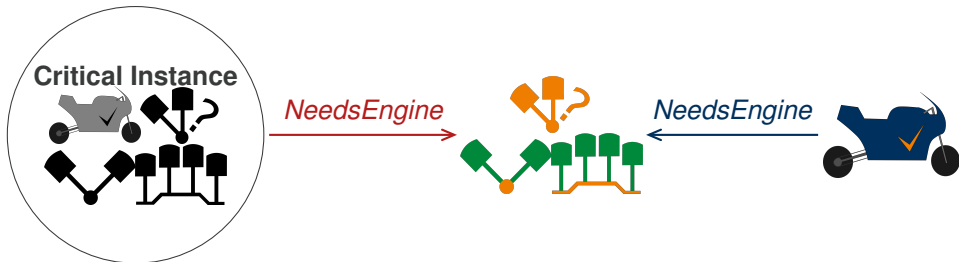
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(DMFA is essentially RMFA [Carral et al., 2017] with a different notion of applicability.)

Model Faithful Cyclicity [Carral et al., 2017]

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Rule Instance



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Rule Instance



The rule set is **not MFC** even though it is **non-terminating**.

Disjunctive Model Faithful Cyclicity

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$NeedsEngine(x, y) \rightarrow \cancel{HasEngine(x)} \wedge Engine(y)$

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Disjunctive Model Faithful Cyclicity

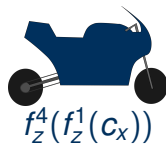
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Overapproximation



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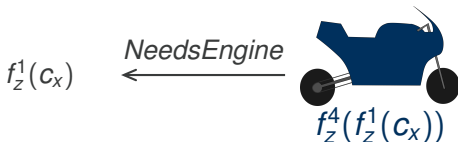
$$\text{Motorcycle}(x) \rightarrow \text{HasEngine}(x) \vee \underline{\text{NeedsEngine}(x, f_z^1(x))}$$

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Overapproximation



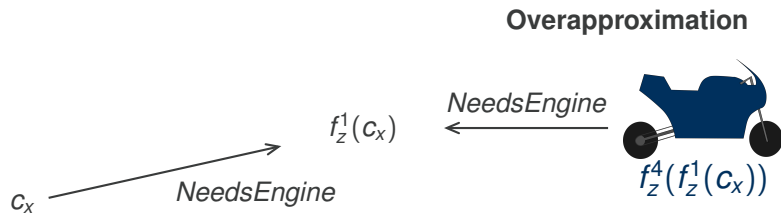
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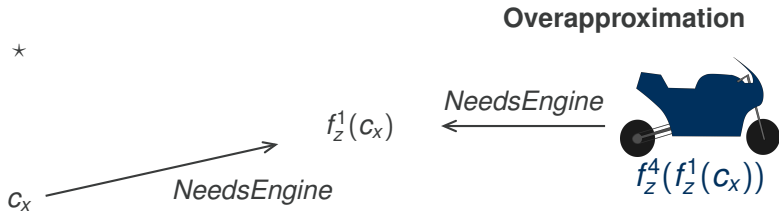
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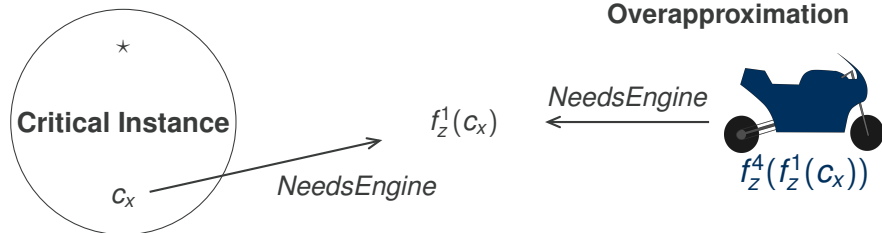
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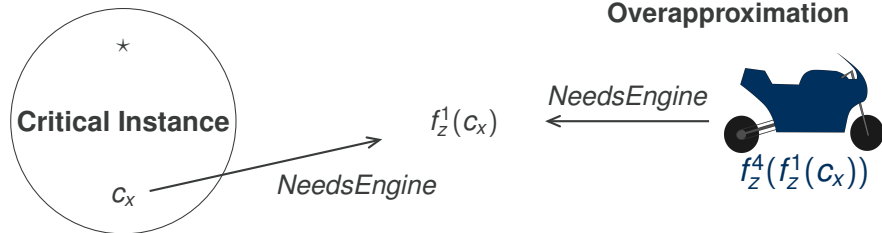
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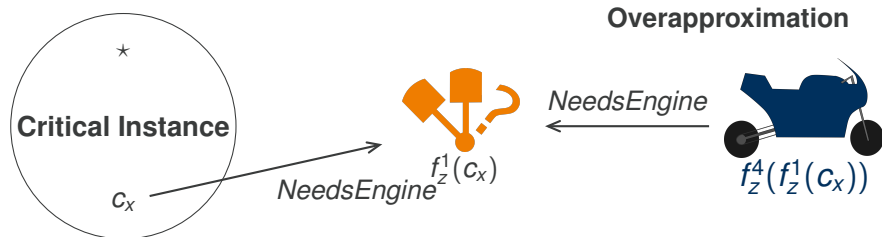
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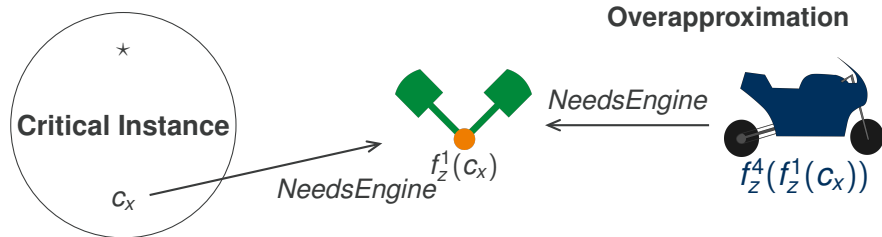
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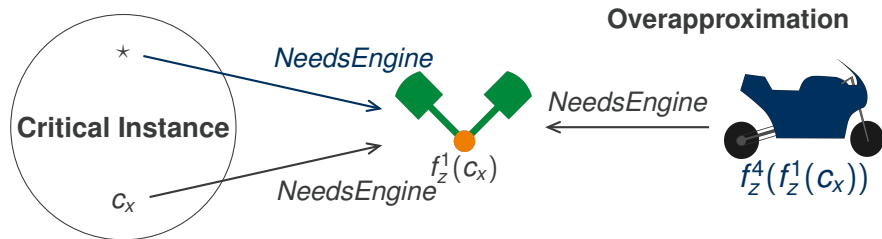
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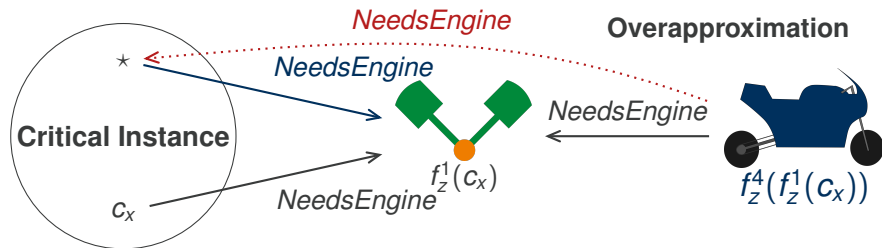
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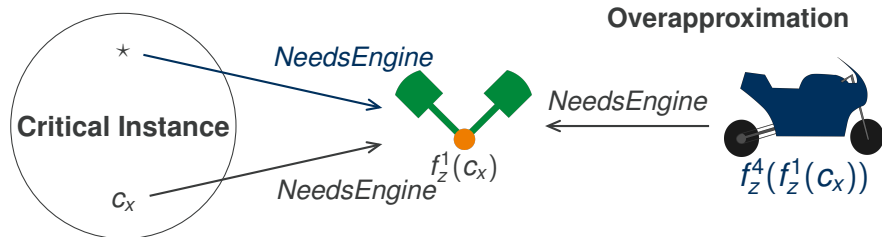
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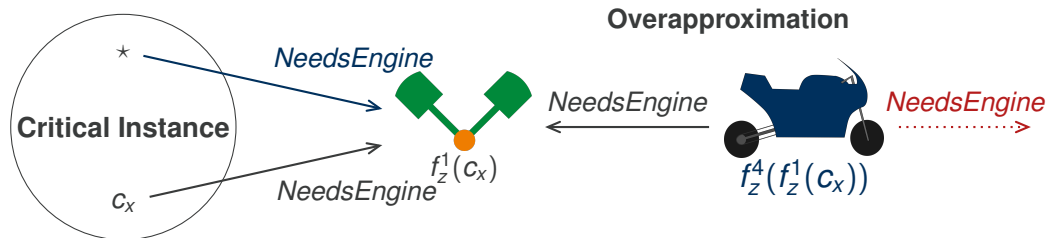
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The **rule application** is **unblockable**. The rule set is **DMFC!**

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The **rule application** is **unblockable**. The rule set is **DMFC!**
(The idea of DMFC is close to RMFC [Carral et al., 2017].)

Evaluation Results

```
cat my-ruleset | docker run --rm -i registry.gitlab.com/m0nstr/dmfa-checker  
-t [non_]termination -cv skolem -disj [-depth 2]
```

	# \exists	# tot.	# fin.	MFA	DMFA	DMFA ²	MFC	DMFC ^s
OXFD	1–19	37	36	21	28	28	4	8
	20–99	18	17	3	3	3	10	14
	100+	82	26	4	6	6	14	19
	1+	137	79	28 (35%)	37 (46%)	37 (46%)	28 (35%)	41 (51%)
ORE15	1–19	103	98	51	66	66	18	31
	20–99	119	105	32	33	35	54	69
	100–999	278	219	5	6	119	89	100
	1–999	500	422	88 (20%)	105 (24%)	220 (52%)	161 (38%)	200 (47%)
MOWL	1–19	1361	1283	676	725	732	173	515
	20–99	894	740	104	114	121	301	610
	100–299	448	254	25	25	111	103	143
	1–299	2703	2277	805 (35%)	864 (37%)	964 (42%)	577 (25%)	1268 (55%)

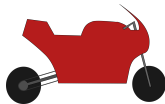
Conclusion & Outlook

Summary:





- **DMFA** and **DMFC** are novel sufficient conditions for **disjunctive skolem chase** termination and non-termination, respectively, inspired by (R)MFA and (R)MFC.
- **DMFA** and **DMFC** are significantly more general than **MFA** and **MFC**.

Future Work:




- Port improvements for **DMFC** to the **restricted chase** including additional ideas.
- Look into computing the **disjunctive skolem chase** with **ASP**.
- Look into different normalization and **translation** techniques from **OWL** into **rules**.
- Investigate if similar **(non-)termination conditions** can be used in **other areas** of knowledge representation and reasoning, e.g. automated theorem proving.







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