Agenda

1. Introduction
2. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
3. Local Search, Stochastic Hill Climbing, Simulated Annealing
4. Tabu Search
5. Answer-set Programming (ASP)
6. Constraint Satisfaction (CSP)
7. Structural Decomposition Techniques (Tree/Hypertree Decompositions)
8. Evolutionary Algorithms/ Genetic Algorithms
Main Idea

- A memory forces the search to explore new areas of the search space
- Memorize solutions that have been examined recently. They become tabu points in next steps
- Tabu search is deterministic
Tabu Search and SAT

- SAT problem with $n = 8$ variables
- Initial (random) assignment $x = (0, 1, 1, 1, 0, 0, 0, 1)$
- Evaluation function: weighted sum of number of satisfied clauses. Weights depend on the number of variables in the clause
- Maximize evaluation function (i.e. we’re trying to satisfy all clauses)
- Random assignment provides $eval(x) = 27$
- Neighborhood of $x$ consists of 8 solutions. Evaluate them and select best
- At this stage, it is the same as hill-climbing
- Suppose flipping 3rd variable generates best evaluation ($eval(x') = 31$)
- Memory keeps track of actions
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Answer the Following Questions

1. What is stored in memory (think of SAT as an example)?
2. How can we escape local optima with help of the memory?
Recency-based Memory

- Index of flipped variable + time when it was flipped
- **Differentiate between older and more recent flips**
- SAT: time stamp for each position of solution vector $M$ (initialized to 0)
- Value of time stamp provides information on recency of flip at position

**Memory Vector**

$$M(i) = j \text{ (when } j \neq 0)$$

$j$ is most recent iteration when $i$-th bit was flipped
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Memory Vector

$M(i) = j$ (when $j \neq 0$)

$j$ is most recent iteration when $i$-th bit was flipped

Assume information is stored for at most 5 iterations.

Alternative Interpretation

$M(i) = j$ (when $j \neq 0$)

$i$-th bit was flipped $5 - j$ iterations ago
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- Index of flipped variable + time when it was flipped
- Differentiate between older and more recent flips
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**Alternative Interpretation**

\[ M(i) = j \text{ (when } j \neq 0) \]

\( i \)-th bit was flipped \( 5 - j \) iterations ago

**Example**

\[
\begin{array}{cccccccc}
0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Memory after one iteration. 3rd bit is **tabu** for next 5 iterations.
Different Interpretations

1st Variant

- Stores iteration number of most recent flip
- Requires a current iteration counter $t$ which is compared with memory values
- If $t - M(i) > 5$ forget
- Only requires updating a single entry, and increase the counter
- **Used in most implementations**
## Different Interpretations

### 1st Variant
- Stores iteration number of most recent flip
- Requires a current iteration counter $t$ which is compared with memory values
- If $t - M(i) > 5$ forget
- Only requires updating a single entry, and increase the counter
- **Used in most implementations**

### 2nd Variant
- Values are interpreted as number of iterations for which a position is **not available**
- **All** nonzero entries are decreased by one **at every iteration**
Example ctd.

- Initial assignment $x = (0, 1, 1, 1, 0, 0, 0, 1)$
- After 4 additional iterations $M$:

  
  $\begin{array}{ccccccc}
  3 & 0 & 1 & 5 & 0 & 4 & 2 & 0
  \end{array}$

- Most recent flip $M(4) = 5$
- Current solution: $x = (1, 1, 0, 0, 0, 1, 1, 1)$ with $\text{eval}(x) = 33$
Example ctd.

- Initial assignment \( x = (0, 1, 1, 1, 0, 0, 0, 1) \)
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  \[
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  \end{array}
  \]

- Most recent flip \( M(4) = 5 \)
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**Neighborhood of \( x \)**

\[
\begin{align*}
  x_1 &= (0, 1, 0, 0, 0, 1, 1, 1) \\
  x_2 &= (1, 0, 0, 0, 0, 1, 1, 1) \\
  x_3 &= (1, 1, 1, 0, 0, 1, 1, 1) \\
  x_4 &= (1, 1, 0, 1, 0, 1, 1, 1) \\
  x_5 &= (1, 1, 0, 0, 1, 1, 1, 1) \\
  x_6 &= (1, 1, 0, 0, 0, 0, 1, 1) \\
  x_7 &= (1, 1, 0, 0, 0, 0, 1, 0) \\
  x_8 &= (1, 1, 0, 0, 0, 1, 1, 0)
\end{align*}
\]
Example ctd.

- Initial assignment $x = (0, 1, 1, 0, 0, 0, 1)$
- After 4 additional iterations $M$:

  $M = \begin{bmatrix} 3 & 0 & 1 & 5 & 0 & 4 & 2 & 0 \end{bmatrix}$

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- Current solution: $x = (1, 1, 0, 0, 1, 1, 1)$ with $eval(x) = 33$

**Neighborhood of $x$**

- $x_1 = (0, 1, 0, 0, 0, 1, 1, 1)$
- $x_2 = (1, 0, 0, 0, 0, 1, 1, 1)$
- $x_3 = (1, 1, 1, 0, 0, 1, 1, 1)$
- $x_4 = (1, 1, 0, 1, 0, 1, 1, 1)$
- $x_5 = (1, 1, 0, 0, 1, 1, 1, 1)$
- $x_6 = (1, 1, 0, 0, 0, 0, 1, 1)$
- $x_7 = (1, 1, 0, 0, 0, 1, 0, 1)$
- $x_8 = (1, 1, 0, 0, 0, 1, 1, 0)$

**TABU**, best evaluation $eval(x_5) = 32$, decrease!
Example ctd.

- Current solution: $\mathbf{x} = (1, 1, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$
- New solution: $\mathbf{x}_5 = (1, 1, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}_5) = 32$

<table>
<thead>
<tr>
<th>3</th>
<th>0</th>
<th>1</th>
<th>5</th>
<th>0</th>
<th>4</th>
<th>2</th>
<th>0</th>
</tr>
</thead>
</table>

changes to:

| 2 | 0 | 0 | 4 | 5 | 3 | 1 | 0 |
Example ctd.

- Current solution: \( x = (1, 1, 0, 0, 0, 1, 1, 1) \) with \( \text{eval}(x) = 33 \)
- New solution: \( x_5 = (1, 1, 0, 0, 1, 1, 1, 1) \) with \( \text{eval}(x_5) = 32 \)

changes to:

\[
\begin{array}{cccccccc}
3 & 0 & 1 & 5 & 0 & 4 & 2 & 0 \\
\end{array}
\]

Policy might be too restrictive

- What if tabu neighbor \( x_6 \) provides excellent evaluation score?
- Make search more flexible: override tabu classification if solution is outstanding

\( \implies \text{aspiration criterion} \)
Frequency-based Memory

- Operates over a longer horizon
- SAT: vector $H$ serves as long-term memory.
  - Initialized to 0, at any stage of the search

$$H(i) = j$$

interpreted as: during last $h$ (horizon) iterations, the $i$-th bit was flipped $j$ times
- Usually horizon is large
- After 100 iterations with $h = 50$, long-term memory $H$ might have the following values

```
5 7 11 3 9 8 1 6
```
- Shows distribution of moves throughout the last 50 iterations

Diversity of Search

Frequency-based memory provides information about which flips have been under-represented or not represented.

⇒ we can diversify the search by exploring these possibilities
Use of Long-term Memory

### Special Circumstances

- Situations where **all non-tabu moves lead to worse solution**
- To make a meaningful decision about which direction to explore next
- Typically: **most frequent moves are less attractive**
- Value of evaluation score is decreased by some **penalty measure** that depends on frequency, final score implies the winner
Example SAT

- Assume value of current solution is $eval(x) = 35$
- Non-tabu flips 2, 3 and 7 have values 30, 33, 31
- None of tabu moves provides value greater than 37 (highest value so far)
  $\implies$ we can’t apply aspiration criterion
Example SAT

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- Non-tabu flips 2, 3 and 7 have values 30, 33, 31
- None of tabu moves provides value greater than 37 (highest value so far) $\implies$ we can’t apply aspiration criterion
- Frequency based-memory and evaluation function for new solution $x'$ is

$$eval(x') - penalty(x')$$

- $penalty(x') = 0.7 \times H(i)$, where 0.7 coefficient, $H(i)$ value from long-term memory $H$:

  | 7  | for solution created by flipping 2nd bit |
  | 11 | for solution created by flipping 3nd bit |
  | 1  | for solution created by flipping 7nd bit |
Example SAT

- Assume value of current solution is \( \text{eval}(x) = 35 \)
- Non-tabu flips 2, 3 and 7 have values 30, 33, 31
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\[
\text{eval}(x') - \text{penalty}(x')
\]

- \( \text{penalty}(x') = 0.7 \times H(i) \), where 0.7 coefficient, \( H(i) \) value from long-term memory \( H \):
  - 7 for solution created by flipping 2nd bit
  - 11 for solution created by flipping 3rd bit
  - 1 for solution created by flipping 7th bit

- New scores are:

\[
\begin{align*}
30 - 0.7 \times 7 &= 25.1 & \text{2nd bit} \\
33 - 0.7 \times 11 &= 25.3 & \text{3rd bit} \\
31 - 0.7 \times 1 &= 30.3 & \text{7th bit}
\end{align*}
\]
Example SAT

- Frequency based-memory and evaluation function for new solution \( x' \) is
  \[
  eval(x') - penalty(x')
  \]

- \( penalty(x') = 0.7 \times H(i) \), where 0.7 coefficient, \( H(i) \) value from long-term memory \( H \):
  
<table>
<thead>
<tr>
<th>Bit</th>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.7</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>0.7</td>
<td>1</td>
</tr>
</tbody>
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  for solution created by flipping 2nd bit
  for solution created by flipping 3nd bit
  for solution created by flipping 7nd bit

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  \end{align*}
  \]
Further Options to Diversify Search

We might add additional rules:

- **Aspiration by default:** select the oldest of all considered
- **Aspiration by search direction:** memorize whether or not the performed moves generated any improvement
- **Aspiration by influence:** measures the degree of change of the new solution
  a) in terms of the distance between old and new solution
  b) change in solution’s feasibility, if we deal with a constraint problem
    - **Intuition:** particular move has a larger influence if a larger step was made from old to new solution
Groupwork

Questions

1. How "close" were your answers to the presented information?
2. Which information was (un)expected?
Summary

- Simulated annealing and tabu search are both design to escape local optima
- Tabu search makes uphill moves only when it is stuck in local optima
- Simulated annealing can make uphill moves at any time
- Simulated annealing is stochastic, tabu search is deterministic
- Compared to classic algorithms, both work on complete solutions. One can halt them at any iteration and obtain a possible solution
- Both have many parameters to worry about
References

Zbigniew Michalewicz and David B. Fogel. 