Dresden University of Technology

Computational Logic ∴ Group

Dr. Hannes Strass

Dr. Sarah Gaggl

Algorithmic Game Theory

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Exercises 11

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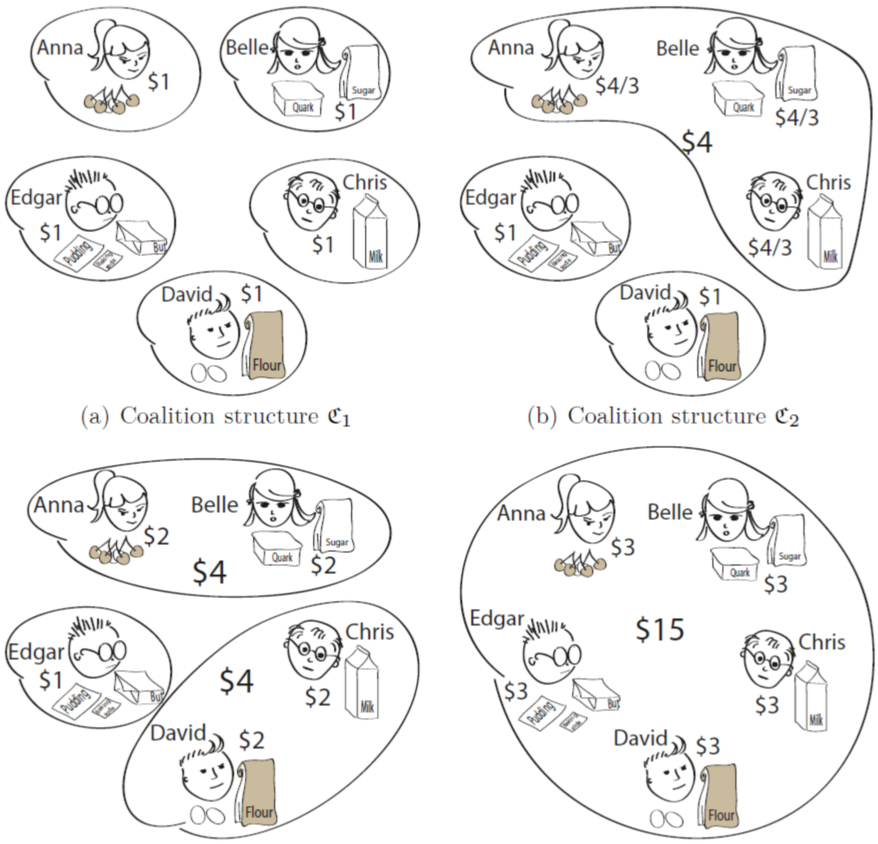
Problem 1.
Consider the following four coalition structures:



(a) Coalition structure $\mathfrak{C}_1$

(b) Coalition structure $\mathfrak{C}_2$

(c) Coalition structure $\mathfrak{C}_3$

(d) Coalition structure $\mathfrak{C}_4$

Write down the four coalition structures formally. Then, write down the amount each player receives for each outcome. Finally, argue from the coalitions displayed here whether (a) the characteristic function is monotonic, and (b) the game is superadditive.

Problem 2.
Consider the cooperative game $G = (P, v)$ where three players together can obtain \$1 to share, any two players can obtain α, and one player by herself can obtain zero. Then

$$\begin{aligned} v(\{1,2,3\}) &= 1, \\ v(\{1,2\}) = v(\{1,3\}) = v(\{2,3\}) &= \alpha, \\ v(\{1\}) = v(\{2\}) = v(\{3\}) = v(\emptyset) &= 0 \end{aligned}$$

What is the core of this game? When is the core empty? When is the core non-empty?

Problem 3.
Consider the 3-person game in which no single player can generate any surplus on her own:

$$\begin{aligned} &v(\{1\}) = 0, v(\{2\}) = 0, v(\{3\}) = 0, \\ &v(\{1,2\}) = 7, v(\{1,3\}) = 6, v(\{2,3\}) = 5, \\ &v(\{1,2,3\}) = 8 \end{aligned}$$

Is the core non-empty? If not, what is the cost of stability/the value of the least core?

Problem 4.
Let $G(P, v)$ be a cooperative game with transferable utility.

We say that player $i \in P$ is a *dummy* iff for all $C \subseteq P$, we have $v(C \cup \{i\}) = v(C)$.

Prove that if $i \in P$ is a dummy and $\mathbf{a} \in Core(G)$, then $a_i = 0$.