Review

**SPARQL:**
- ... is the W3C-standard for querying RDF graphs
- ... at its core relies on basic graph patterns (BGPs)
- ... returns sequences or multi-sets of partial functions (“solutions”)

**Wikidata:**
- ... is a large, free knowledge graph & open community
- ... can be viewed as a document-centric or graph-based database
- ... provides an RDF-mapping, linked-data exports, and the SPARQL-based Wikidata query service (WDQS)

In this lecture: many more SPARQL features
Projection and Solution Set Modifiers
From patterns to queries

**SELECT clauses**

- specify the bindings that get returned (projection = removal of some bindings from results)
- may define additional results computed by functions
- may define additional results computed by aggregates

**Example 7.1:** Find cities and their population densities:

```
SELECT ?city (?population/?area AS ?populationDensity)
WHERE {
  ?city rdf:type eg:city ;
  eg:population ?population ;
  eg:areaInSqkm ?area .
}
```
Projection and Duplicates

Projection can increase the multiplicity of solutions

**Definition 7.2:** The projection of a solutions mapping $\mu$ to a set of variables $V$ is the restriction of the partial function $\mu$ to variables in $V$. The projection of a solution sequence is the set of all projections of its solution mappings, ordered by the first occurrence of each projected solution mapping.

The cardinality of a solution mapping $\mu$ in a solution $\Omega$ is the sum of the cardinalities of all mappings $\nu \in \Omega$ that project to the same mapping $\mu$.

**Note:** This definition also works if additional results are defined by functions or aggregates. Solution mappings are extended first by adding the bound variables, and then subjected to projection.

The keyword `DISTINCT` can be used after `SELECT` to remove duplicate solutions (=to set multiplicity of any element in the result to 1)
Solution set modifiers

SPARQL supports several expressions after the query’s WHERE clause:

- **ORDER BY** defines the desired order of results
  - Can be followed by several expressions (separated by space)
  - May use order modifiers `ASC()` (default) or `DESC()`

- **LIMIT** defines a maximal number of results

- **OFFSET** specifies the index of the first result within the list of all results
  
  Note: Both **LIMIT** and **OFFSET** should only be used on explicitly ordered results

**Example 7.3:** In Wikidata, find the largest German cities, rank 6 to 15:

```sparql
SELECT ?city ?population
WHERE {
  ?city wdt:P31 wd:Q515 ; # instance of city
      wdt:P17 wd:Q183 ; # county Germany
      wdt:P1082 ?population # get population
}
ORDER BY DESC(?population) OFFSET 5 LIMIT 10
```
Groups, Union, Minus, Optional, Subqueries
Groups

So far, all of our queries had a single pattern consisting of

- triple patterns
- property path patterns
- filters

When introducing further features, we will often have to group them: this is done with braces `{ ... }`

**Terminology**: A query part within braces is called a group graph pattern in SPARQL.

We were already using group graph patterns in all queries: the part after `WHERE` is one

Semantically, results of juxtaposed group graph patterns are combined using Join.
The **UNION** operator allows us to obtain the union of the results of two group graph patterns.

**Example 7.4:** In Wikidata, find everybody who is a composer by occupation or who has composed something:

```
SELECT ?person
WHERE {
  { ?person wdt:P106 wd:Q36834 } # ?person occupation: composer
  UNION
}
```

**UNION** produces the union of results and adds up multiplicities

~ using **DISTINCT** might be necessary
Semantics of SPARQL queries

SPARQL query features are defined by corresponding query algebra operations that produce results (i.e., multisets of solution mappings).

We already encountered some such operations:

- \( \text{eval}_G \) produced results for BGPs and property path patterns
- \( \text{Join} \) computed the natural join of two results

Definition 7.5:
Given a filter expression \( \phi \), a multiset \( M \) of solution mappings, and an RDF graph \( G \), we define the multiset:

\[
\text{Filter}(\phi, M, G) = \{ \mu | \mu \in M \text{ and } \phi \text{ evaluates to true for } \mu \text{ (over } G) \}
\]

with the cardinality of a solution mapping \( \mu \):

\[
\text{card} \text{Filter}(\phi, M, G)(\mu) = \text{card} M(\mu).
\]
Semantics of SPARQL queries

SPARQL query features are defined by corresponding query algebra operations that produce results (i.e., multisets of solution mappings).

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- $eval_G$ produced results for BGPs and property path patterns
- Join computed the natural join of two results

We omitted the according operation for \textbf{FILTER} so far. It is simple; we just need to take into account that the meaning of some filter expressions (e.g., \textbf{NOT EXISTS}) depends on the given RDF graph:

\textbf{Definition 7.5: } Given a filter expression $\varphi$, a multiset $M$ of solution mappings, and an RDF graph $G$, we define the multiset

$$\text{Filter}(\varphi, M, G) = \{\mu \mid \mu \in M \text{ and } \varphi \text{ evaluates to true for } \mu \text{ (over } G)\}$$

with the cardinality of a solution mapping $\mu$ defined as $\text{card}_{\text{Filter}(\varphi, M, G)}(\mu) = \text{card}_M(\mu)$.
The semantics of \textbf{UNION} is defined by the operation \text{Union}(M_1, M_2) that computes the union of two multisets $M_1$ and $M_2$ of solution mappings:

\textbf{Definition 7.6:} Given multisets $M_1$ and $M_2$ of solution mappings, we define the multiset

\[
\text{Union}(M_1, M_2) = \{ \mu \mid \mu \in M_1 \text{ or } \mu \in M_2 \}
\]

with the cardinality of a solution mapping $\mu$ defined as

\[
\text{card}_{\text{Union}(M_1, M_2)}(\mu) = \text{card}_{M_1}(\mu) + \text{card}_{M_2}(\mu).
\]
The **MINUS** operator allows us to remove the results of one group graph pattern from the results of another.

**Example 7.7:** In Wikidata, find living people who are composers by occupation:

```sparql
SELECT ?person
WHERE {
  {?person wdt:P106 wd:Q36834 } # ?person occupation: composer
  MINUS
  {?person wdt:P570 [] } # ?person date of death: some value
}
```

Similar results can often be achieved with **FILTER NOT EXISTS**, but the two are used differently:

**MINUS** and **FILTER NOT EXISTS** behave differently, e.g., when applied to a group graph patterns that do not share any variables.
The semantics of **MINUS** is defined by the operation $\text{Minus}(M_1, M_2)$ that computes the set difference of two results $M_1$ and $M_2$:

**Definition 7.8:** Given multisets $M_1$ and $M_2$ of solution mappings, we define the multiset

$$\text{Minus}(M_1, M_2) = \{ \mu | \mu \in M_1 \text{ and for all } \mu' \in M_2 : \mu \text{ and } \mu' \text{ are not compatible or have disjoint domains: } \text{dom}(\mu) \cap \text{dom}(\mu') = \emptyset \}$$

with the cardinality of a mapping $\mu$ defined as $\text{card}_{\text{Minus}(M_1, M_2)}(\mu) = \text{card}_{M_1}(\mu)$.

**Recall:** mappings $\mu_1$ and $\mu_2$ are **compatible** if $\mu_1(x) = \mu_2(x)$ for all variable names $x \in \text{dom}(\mu_1) \cap \text{dom}(\mu_2)$

**Note:** $\text{Minus}(M_1, M_2)$ does not depend on cardinalities of mappings in $M_2$. 

Markus Krötzsch, 27th Nov 2018
The **OPTIONAL** operator is used to extend solution mappings with additional, optional information.

**Example 7.9:** In Wikidata, find composers, and, optionally, their spouses:

```
SELECT ?person ?spouse
WHERE {
  ?person wdt:P106 wd:Q36834 # ?person occupation: composer
}
```

Solutions for queries with **OPTIONAL** may leave some query variables unbound (people without spouses in the example).

**Note:** Like **FILTER**, **OPTIONAL** patterns are used inside one group graph pattern, together with triple patterns etc.
What does the following query mean?

**Example 7.10:**

```
SELECT ?person ?spouse
WHERE {
  ?person wdt:P106 wd:Q36834 ; # ?person occupation: composer
      wdt:P569 ?bd . # ?person date of birth: ?bd

  OPTIONAL {
    FILTER (year(?bd)=year(?bd2)) # born in same year
  }
}
```
Optional and filters

What does the following query mean?

Example 7.10:

```
SELECT ?person ?spouse
WHERE {
  ?person wdt:P106 wd:Q36834 ; # ?person occupation: composer
  wdt:P569 ?bd . # ?person date of birth: ?bd

  OPTIONAL {
    FILTER (year(?bd)=year(?bd2)) # born in same year
  }
}
```

SPARQL: “Composers, and, optionally, their spouses that were born in the same year.”
The semantics of \texttt{OPTIONAL} is defined by the operation $\text{LeftJoin}(M_1, M_2, \varphi, G)$ that augments solutions in $M_1$ with compatible solutions in $M_2$ if this combination satisfies the filter condition $\varphi$ (w.r.t. graph $G$):

\textbf{Definition 7.11}: Given multisets $M_1$ and $M_2$ of solution mappings, a filter expression $\varphi$, and an RDF graph $G$, we define the multiset

$$\text{LeftJoin}(M_1, M_2, \varphi, G) = \text{Filter}(\varphi, \text{Join}(M_1, M_2), G) \cup \{\mu_1 \in M_1 \mid \text{for all } \mu_2 \in M_2 : \mu_1 \text{ incompatible } \mu_2 \text{ or } \varphi \text{ evaluates to false on } \mu_1 \uplus \mu_2 \text{ (over } G)\}$$

with the cardinality of each mapping $\mu$ being its cardinality in $\text{Filter}(\varphi, \text{Join}(M_1, M_2), G)$ (in case $\mu \in \text{Filter}(\varphi, \text{Join}(M_1, M_2), G)$) or in $M_1$ (in case $\mu \notin \text{Filter}(\varphi, \text{Join}(M_1, M_2), G)$).

Note that only one of the two cases can occur.

\textbf{Recall}: mappings $\mu_1$ and $\mu_2$ are \textit{compatible} if $\mu_1(x) = \mu_2(x)$ for all variable names $x \in \text{dom}(\mu_1) \cap \text{dom}(\mu_2)$.
Subqueries are used to use results of other queries within queries, typically to achieve results that cannot be accomplished using other patterns.

**Example 7.12:** In Wikidata, find universities located in one of the 15 largest German cities:

```
SELECT DISTINCT ?university ?city
WHERE {
  { SELECT DISTINCT ?city ?population
    WHERE {
      SELECT DISTINCT ?city ?population
      WHERE {
        ?city wdt:P31/wdt:P279* wd:Q515 ; # instance of: city
        wdt:P17 wd:Q183 ; # country: Germany
        wdt:P1082 ?population . # population: ?population
      } ORDER BY DESC(?population) LIMIT 15 # get top 15 by ?population
    }
    ?university wdt:P31/wdt:P279* wd:Q3918 ; # instance of: university
    wdt:P131+ ?city . # located in+: ?city
  }
}
```
Semantics of subqueries

The semantics of subqueries does not require any special operator: the result multiset of the subquery is simply used like the result of any other (sub) group graph pattern.

Notes:

- The order of results from subqueries is not conveyed to the enclosing query (subqueries return multisets, not sequences).
- The use of `ORDER BY` is still meaningful to select top-$k$ results by some ordering.
- Only selected variable names are part of the subquery result; other variables might be hidden from the enclosing query.
Values and Bind
Defining own values

It is often useful to add bindings to results that do not come directly from the database:

- Predefine batches of (tuples of) constants $\leadsto$ VALUES
- Define derived values by applying functions to query results $\leadsto$ BIND

Both constructs behave slightly differently.
**Values**

VALUES is used to inject pre-defined result multisets into the query evaluation.

**Example 7.13:** In Wikidata, find people who are composers, or musicians, or who play some instrument:

```sparql
SELECT DISTINCT ?item
WHERE {
    VALUES (?predicate ?value) { # define values for two variables
        ( wdt:P106 wd:Q36834 ) # occupation / composer
        ( wdt:P106 wd:Q639669 ) # occupation / musician
        ( wdt:P1303 UNDEF ) # instrument played / any
    }
    ?item ?predicate ?value ;
}
```

The VALUES expression defines three solution mappings, two of which are defined for variable names predicate and value, and one defined for predicate only.

**Note:** One may leave away the (...) if values are given for just one variable.
VALUES behaves just like a subquery with the specified result.

- As with subqueries, order does not matter.
- The special value UNDEF is used to signify that a variable should be unbound for a solution mapping.
- Otherwise, only IRIs or literals can be used in VALUES – especially no functions.

In practice, the most important use of VALUES is to encode batch queries that ask for many possible options in a single query. Using this to ask about, say, 100 possible values in one query is much more efficient than sending 100 small queries or using nested UNION with 100 possibilities.
Bind

**BIND** is used to assign a computed value to a variable.

**Example 7.14:** Find cities and their population densities:

```
SELECT ?city ?populationDensity
WHERE {
  ?city rdf:type eg:city ;
    eg:population ?population ;
    eg:areaInSqkm ?area .
  **BIND** (?population/?area AS ?populationDensity)
}
```

**BIND** can be used instead of expression assignments with **AS** in **SELECT**

However, variables assigned with **BIND** can already be used in the query pattern, but not before they were assigned.

Assignments of constants to variables are better realised with **VALUES**, which can be used before or after other patterns using the variable.
Semantics of \textbf{BIND}

The semantics of \textbf{BIND} is defined by the operation $\text{Extend}(M, v, \varphi)$ that computes the extension of solution mappings in $M$ by assigning the output of expression $\varphi$ to variable name $v$.

\begin{definition}
Consider a variable name $v$ and an expression $\varphi$. Given a solution mapping $\mu$ such that $v \notin \text{dom}(\mu)$, we define an extended mapping

\[
\text{Extend}(\mu, v, \varphi) = \begin{cases} 
\mu \cup \{v \mapsto \text{eval}(\mu(\varphi))\} & \text{if eval}(\mu(\varphi)) \text{ is not "error"} \\
\mu & \text{if eval}(\mu(\varphi)) \text{ is "error"}
\end{cases}
\]

Given a multiset $M$ of solution mappings, we define $\text{Extend}(M, v, \varphi) = \{\text{Extend}(\mu, v, \varphi) \mid \mu \in M\}$, where the cardinalities of extended mappings are the same as in $M$.
\end{definition}

\textbf{Notation}: $\text{eval}(\mu(\varphi))$ denotes evaluation of the expression obtained from $\varphi$ by replacing variables by their values in $\mu$. 
Summary: SPARQL algebra

We have already encountered a number of operators for extending results:

- **Join**\((M_1, M_2)\): join compatible mappings from \(M_1\) and \(M_2\)
- **Filter**\((\varphi, M, G)\): remove from multiset \(M\) all mappings for which \(\varphi\) does not evaluate to EBV “true”
- **Union**\((M_1, M_2)\): compute the union of mappings from multisets \(M_1\) and \(M_2\)
- **Minus**\((M_1, M_2)\): remove from multiset \(M_1\) all mappings compatible with a non-empty mapping in \(M_2\)
- **LeftJoin**\((M_1, M_2, \varphi, G)\): extend mappings from \(M_1\) by compatible mappings from \(M_2\) when filter condition is satisfied; keep remaining mappings from \(M_1\) unchanged
- **Extend**\((M, v, \varphi)\): extend all mappings from \(M\) by assigning \(v\) the value of \(\varphi\).

SPARQL also defines operators for solution set modifiers, which work on lists of mappings (“ordered multisets”):

- **OrderBy**\((L, \text{condition})\): sort list by a condition
- **Slice**\((L, \text{start}, \text{length})\): apply limit and offset modifiers

Further operators exist, e.g., **Distinct**\((L)\).
Aggregates
Grouping and aggregates

**Aggregate functions** compute values from multisets of solution mappings (rather than from individual mappings)

**Grouping** is used to split a multiset of solutions into several multisets based on some key that is computed for each solution

---

**Example 7.16:** In Wikidata, find the ten most common professions of people born in Dresden:

```sql
SELECT ?job (COUNT(?person) as ?count)
WHERE {
  ?person wdt:P19 wd:Q1731 ; # born in: Dresden
  wdt:P106 ?job . # occupation: ?job
} GROUP BY ?job
ORDER BY DESC(?count) LIMIT 10
```

Note: we can select non-aggregate terms used for grouping (since they are the same across the whole group!).

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SPARQL aggregate functions

SPARQL offers several aggregate functions:

- **COUNT**: count the sum of all multiplicities of solutions
- **SUM**: sum up numeric values
- **AVG**: compute the average of numeric values
- **MIN/MAX**: compute the minimum/maximum (over any type of term)
- **SAMPLE**: non-deterministically get one value from all values (no probability distribution implied)
- **GROUP_CONCAT**: concatenate string values into one large string (in any order)

All aggregate functions receive one expression as parameter, e.g., \( \text{SUM}(?\text{population}) \) or \( \text{MIN}(?\text{birthdate}) \).

All aggregates optionally accept `DISTINCT` before the parameter to indicate that duplicates should be eliminated from the multiset of expression results before applying the aggregate.
The keyword `HAVING` is used to specify a filter condition on mappings produced by aggregation:

**Example 7.17:** In Wikidata, find all professions of more than 100 people born in Dresden:

```
SELECT ?job (COUNT(?person) as ?count)
WHERE {
  ?person wdt:P19 wd:Q1731 ; # born in: Dresden
  wdt:P106 ?job . # occupation: ?job
} GROUP BY ?job
HAVING (COUNT(?person) > 100)
```
Semantics of grouping

The semantics of **GROUP BY** is defined by the operation $\text{Group}(\Phi, M)$ that computes a mapping from keys (that we group by) to multisets (that are the groups of solution mappings).

**Definition 7.18:** Consider a list of expressions $\Phi = \langle \varphi_1, \ldots, \varphi_n \rangle$. Given a solution mapping $\mu$, we define $\Phi(\mu)$ as the list $\langle \varphi_1(\mu), \ldots, \varphi_n(\mu) \rangle$ of values obtained by evaluating expressions for the bindings of $\mu$.

Given a multiset $M$ of solution mappings, we define

$$
\text{Group}(\Phi, M) = \{ \Phi(\mu) \mapsto \{ \mu' \in M \mid \Phi(\mu') = \Phi(\mu) \} \mid \mu \in M \}
$$

where the cardinality of each solution within the sub-multisets is the same as its cardinality in $M$.

**Note:** We can group by multiple expressions, hence the list $\Phi$ rather than a single expression only (example: **GROUP BY** ?occupation year(?date) would group by two expressions, where one is derived using a function)
Semantics of aggregate functions

Results that include aggregate function values are computed as follows:

- An aggregate function takes as input a mapping of the form 
  \( \{ k_1 \mapsto M_1, \ldots, k_\ell \mapsto M_\ell \} \) from keys \( k_i \) to multiset \( M_i \) and produces a new mapping 
  \( \{ k_1 \mapsto v_1, \ldots, k_\ell \mapsto v_\ell \} \) from keys to values.

- If several aggregates are selected in the query, they are joined by combining value assignments for the same key into a single solution mapping.

The formal definition in SPARQL is rather general (hence more complicated) to allow for extension points where tools can add support for more complex aggregates.
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- An aggregate function takes as input a mapping of the form
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- If several aggregates are selected in the query, they are joined by combining value assignments for the same key into a single solution mapping.

The formal definition in SPARQL is rather general (hence more complicated) to allow for extension points where tools can add support for more complex aggregates.

**Example 7.19:** In Wikidata, find the most common professions of people born in Dresden, together with average birth years of people with this job:

```
SELECT ?job (COUNT(?person) as ?count) (AVG(year(?bdate)) as ?aYear)
WHERE {
  ?person wdt:P19 wd:Q1731 ; # born in: Dresden
  wdt:P106 ?job ; # occupation: ?job
  wdt:P569 ?bdate . # date of birth: ?bdate
} GROUP BY ?job ORDER BY DESC(?count)
```
Summary

Solutions set modifiers define standard operations on result sets

Important SPARQL query operators are \texttt{UNION, MINUS, OPTIONAL, BIND, and VALUES}

The semantics of SPARQL is defined using algebraic operators

Aggregates are used to obtain answers that combine several solutions.

What's next?

- Further background on SPARQL complexity and semantics
- Other graph models and their query languages
- Other aspects of graph analysis and management