Exercise 3.1:
Consider the following program $P$:

$$
double(0,0).
\text{double}(s(X), s(s(Y))) :- double(X,Y).
$$

a) Give the Herbrand universe $HU_F$ and the Herbrand base $HB_{\Pi,F}$ determined by $P$.

b) Give two models of $P$.

c) Consider the following interpretations $I_1$ and $I_2$. For each case specify whether the given interpretation satisfies $P$ or not. Justify your answer.

- $I_1 : D_{I_1} = \mathbb{N}, 0_{I_1} = 1, s(t)_{I_1} = 2 \times t_{I_1},\ double_{I_1} = \{(a, a^2) | a \geq 1\}$

- $I_2 : D_{I_2} = \mathbb{N}, 0_{I_2} = 0, s(t)_{I_2} = (2 \times t_{I_2}) + 1,
  double_{I_2} = \{(0,0)\} \cup \{(a, a^2 - a + 1 | a \geq 1)\}$

Exercise 3.2:
Consider the following program which specifies the descendant relation which is the relation of being a child of, or a child of a child of, or a child of a child of a child of, …

$$
descend(X,Y) :- child(X,Y).
descend(X,Y) :- child(X,Z), descend(Z,Y).
$$

With the input database

$$
\text{child}(anne, bridget).
\text{child}(bridget, caroline).
\text{child}(caroline, donna).
\text{child}(donna, emily).
$$

Give the search tree for the query: $?= \text{descend}(anne, donna)$. 
Exercise 3.3:
Consider the program from Exercise 3.2. What happens if we change the order of the rules and goals. What is the result of the queries ?- descend(X,Y), $?- descend(anne,emily), and $?- descend(anne,bridget).

a) descend(X,Y) :- child(X,Z), descend(Z,Y).
   descend(X,Y) :- child(X,Y).

b) descend(X,Y) :- descend(Z,Y), child(X,Z).
   descend(X,Y) :- child(X,Y).

c) descend(X,Y) :- child(X,Y).
   descend(X,Y) :- descend(Z,Y), child(X,Z).

Exercise 3.4:
Consider the following program for addition.

   add(0,Y,Y).
   add(s(X),Y,s(Z)) :- add(X,Y,Z).

Give the search tree and instantiations for the query:
?- add(s(s(s(0))), s(s(0)), R).