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Complexity Theory

Exercise 6: Space Complexity

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Exercise 6.1. Show that the word problem of deterministic finite automata

$$\mathsf{A}_{\mathsf{DFA}} = \{ \langle \mathcal{A}, w \rangle \mid \mathcal{A} \text{ a DFA accepting } w \}$$

can be decided in logarithmic space.

Exercise 6.2. Show that the composition of logspace reductions again yields a logspace reduction.

Exercise 6.3. Show that the word problem $\mathsf{A}_{\mathsf{NFA}}$ of non-deterministic finite automata is NL-complete.

Exercise 6.4. Show that

$$\mathsf{BIPARTITE} = \{ \langle G \rangle \mid G \text{ a finite bipartite graph} \}$$

is in NL. For this show that $\overline{\mathsf{BIPARTITE}} \in \mathsf{NL}$ and use $\mathsf{NL} = \mathsf{coNL}$.

Hint:
Show that a graph G is bipartite if and only if it does not contain a cycle of odd length.

Exercise 6.5. Find the fault in the following proof of $\mathsf{P} \neq \mathsf{NP}$.

Assume that $\mathsf{P} = \mathsf{NP}$. Then $\mathsf{SAT} \in \mathsf{P}$ and thus there exists a $k \in \mathbb{N}$ such that $\mathsf{SAT} \in \mathsf{DTime}(n^k)$. Because every language in NP is polynomial-time reducible to SAT we have $\mathsf{NP} \subseteq \mathsf{DTime}(n^k)$. It follows that $\mathsf{P} \subseteq \mathsf{DTime}(n^k)$. But by the Time Hierarchy Theorem there exist languages in $\mathsf{DTime}(n^{k+1})$ that are not in $\mathsf{DTime}(n^k)$, contradicting $\mathsf{P} \subseteq \mathsf{DTime}(n^k)$. Therefore, $\mathsf{P} \neq \mathsf{NP}$.