Conditionals and the Selection Task

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- Obligation and Factual Conditionals
- The Selection Task
- Bounded Skeptical Abduction

"Logic is everywhere ..."
Obligation Conditional

- Dietz Saldanha, H., Lourêdo Rocha: Obligation versus Factual Conditionals under the Weak Completion Semantics. CEUR Workshop Proc. 1837, 55-64: 2017

- If it rains then the roofs are wet
  - Its consequence is obligatory
  - We cannot easily imagine a case, where the antecedent is true and the consequence is not

- Necessary antecedent
  - The consequence cannot be true unless the antecedent is true
Factual Conditional

- If it rains then she takes her umbrella
  - We can easily imagine a situation, where the antecedent is true and the consequence is not
  - Its consequence is not obligatory

- Sufficient antecedent
  - The antecedent does not appear to be necessary
Encoding Obligation and Factual Conditionals

▶ Program

\[
\begin{align*}
w & \leftarrow r \land \neg ab_4 \\
ab_4 & \leftarrow \bot \\
u & \leftarrow r \land \neg ab_5 \\
ab_5 & \leftarrow \bot
\end{align*}
\]

▶ Weakly completed program & least model

\[
\begin{align*}
w & \leftrightarrow r \land \neg ab_4 & \text{true} & \text{false} \\
ab_4 & \leftrightarrow \bot & ab_4 & \\
u & \leftrightarrow r \land \neg ab_5 & ab_5 & \\
ab_5 & \leftrightarrow \bot &
\end{align*}
\]

▶ Abducibles

\[
\begin{align*}
r & \leftarrow \top \\
r & \leftarrow \bot \\
ab_5 & \leftarrow \top \\
u & \leftarrow \top
\end{align*}
\]
The Evaluation of Indicative Conditionals

- Indicative conditional
  - If X then Y

- Background knowledge
  - Weakly completed program \( wcP \) with least model \( M \)
  - Set of abducibles \( A \)

- Evaluation
  - If \( M(X) \) is \textit{true}, then the conditional is evaluated to \( M(Y) \)
  - If \( M(X) \) is \textit{false}, then the conditional is evaluated to \textit{true}
  - If \( M(X) \) is \textit{unknown}, then the conditional is evaluated with respect to the skeptical consequences of \( wcP \) given \( A \) considering \( X \) as an observation
If the roofs are not wet then it did not rain

- **If** $\neg w$ **then** $\neg r$

- Explaining $\neg w$ by $r \leftarrow \bot$ we obtain

  \[
  \{ w \leftrightarrow r \land \neg ab_4, \ u \leftrightarrow r \land \neg ab_5, \ ab_4 \leftrightarrow \bot, \ ab_5 \leftrightarrow \bot, \ r \leftrightarrow \bot \}
  \]

- It’s least model is

<table>
<thead>
<tr>
<th></th>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ab_4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ab_5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$w$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- The conditional is **true**
If she did not take her umbrella then it did not rain

> **If** \( \neg u \) **then** \( \neg r \)

> The observation \( \neg u \) can be explained by \( r \leftarrow \bot \) and \( ab_5 \leftarrow \top \), and we obtain

> \( \{ w \leftrightarrow r \land \neg ab_4, \ u \leftrightarrow r \land \neg ab_5, \ ab_4 \leftrightarrow \bot, \ ab_5 \leftrightarrow \bot, \ r \leftrightarrow \bot \} \)

> \( \{ w \leftrightarrow r \land \neg ab_4, \ u \leftrightarrow r \land \neg ab_5, \ ab_4 \leftrightarrow \bot, \ ab_5 \leftrightarrow \bot \lor \top \} \)

> Their least models are

<table>
<thead>
<tr>
<th>true</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( ab_4 )</td>
<td>( \top )</td>
</tr>
<tr>
<td>( ab_5 )</td>
<td>( \bot )</td>
</tr>
<tr>
<td>( r )</td>
<td>( \bot )</td>
</tr>
<tr>
<td>( w )</td>
<td>( \bot )</td>
</tr>
<tr>
<td>( u )</td>
<td>( \bot )</td>
</tr>
</tbody>
</table>

> Reasoning skeptically, the conditional is **unknown**
If the roofs are wet then it rained

- \textit{If \(w\) then \(r\)}

- Explaining \(w\) by \(r \leftarrow \top\) we obtain

\[
\{w \leftrightarrow r \land \neg ab_4, \; u \leftrightarrow r \land \neg ab_5, \; ab_4 \leftrightarrow \bot, \; ab_5 \leftrightarrow \bot, \; r \leftrightarrow \top\}
\]

- Its least model is

<table>
<thead>
<tr>
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</tr>
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<tbody>
<tr>
<td>(r)</td>
<td></td>
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</tr>
<tr>
<td>(ab_5)</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
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<tr>
<td>(u)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- The conditional is \textit{true}
If she took her umbrella then it rained

If \( u \) then \( r \)

The observation \( u \) can be explained by \( r \leftarrow \top \) or \( u \leftarrow \top \), and we obtain

\[
\{w \leftrightarrow r \land \neg ab_4, \ u \leftrightarrow r \land \neg ab_5, \ ab_4 \leftrightarrow \bot, \ ab_5 \leftrightarrow \bot, \ r \leftrightarrow \top\}
\]

\[
\{w \leftrightarrow r \land \neg ab_4, \ u \leftrightarrow (r \land \neg ab_5) \lor \top, \ ab_4 \leftrightarrow \bot, \ ab_5 \leftrightarrow \bot\}
\]

Their least models are

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<td></td>
</tr>
<tr>
<td>( w )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( u )</td>
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<td></td>
</tr>
</tbody>
</table>

Reasoning skeptically, the conditional is *unknown*
The Selection Task

- If there is the letter d on one side of the card, then there is the number 3 on the other side
  ▶ Factual conditional with necessary antecedent

- If a person is drinking beer, then the person must be over 19 years of age
  ▶ Obligation conditional with sufficient antecedent

- Reasoning skeptically yields the adequate answers
  ▶ Dietz Saldanha, H., Lourêdo Rocha: Obligation versus Factual Conditionals under the Weak Completion Semantics. CEUR Workshop Proc. 1837, 55-64: 2017
The Abstract Case

► Program

\[ 3 \leftarrow d \land \neg ab_6 \]
\[ ab_6 \leftarrow \bot \]

► Abducibles

\[ d \leftarrow \top \]
\[ d \leftarrow \bot \]
\[ ab_6 \leftarrow \top \]

► Observations & least models

\[
\begin{array}{ccc}
\text{true} & \text{false} \\
\hline
\text{d} & \text{ab}_6 \\
3 & \\
\hline
\text{a} & \neg d & 3 \\
\hline
\text{true} & \text{false} \\
\hline
\text{d} & \text{ab}_6 \\
3 & \\
\hline
\text{d} & \text{ab}_6 \\
3 & \\
\hline
\text{true} & \text{false} \\
\hline
\text{d} & \text{ab}_6 \\
3 & \\
\hline
\text{ab}_6 & \text{false} \\
3 & \\
\hline
\end{array}
\]

turn: 89%
nor turn: 16%
turn: 62%
nor turn: 25%
The Social Case

▶ Program

\[ o \leftarrow b \land \neg ab_7 \]
\[ ab_7 \leftarrow \bot \]

▶ Abducibles

\[ b \leftarrow \top \quad b \leftarrow \bot \quad o \leftarrow \top \]

▶ Observations & least models

<table>
<thead>
<tr>
<th></th>
<th>true</th>
<th>false</th>
<th></th>
<th>true</th>
<th>false</th>
<th></th>
<th>true</th>
<th>false</th>
<th></th>
<th>true</th>
<th>false</th>
<th></th>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>true</td>
<td>false</td>
<td>ab_7</td>
<td>true</td>
<td>false</td>
<td>ab_7</td>
<td>true</td>
<td>false</td>
<td>ab_7</td>
<td>true</td>
<td>false</td>
<td></td>
<td>true</td>
<td>false</td>
</tr>
<tr>
<td>o</td>
<td>true</td>
<td>false</td>
<td></td>
<td>true</td>
<td>false</td>
<td></td>
<td>true</td>
<td>false</td>
<td></td>
<td>true</td>
<td>false</td>
<td></td>
<td>true</td>
<td>false</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>turn</th>
<th>95%</th>
<th>no turn</th>
<th>0,025%</th>
<th>no turn</th>
<th>0,025%</th>
<th>turn</th>
<th>80%</th>
</tr>
</thead>
</table>

Steffen Hölldobler
Conditionals and the Selection Task
Syllogistic Reasoning


- The Weak Completion Semantics achieves 89% when reasoning skeptically
  - It is better than 12 established cognitive theories
The Complexity of Skeptical Abduction

- H., Philipp, Wernhard: An Abductive Model for Human Reasoning
  In: Logical Formalizations of Commonsense Reasoning
  Papers from the AAAI 2011 Spring Symposium

- Dietz Saldanha, H., Philipp: Contextual Abduction and Its Complexity Issues
  In: Proc. 4th Int. Workshop on Defeasible and Ampliative Reasoning
  CEUR Workshop Proc. 1827, 58-70: 2017

- Skeptical Reasoning is DP-complete
  - $L$ is in the class DP
    iff there are languages $L_1 \in \text{NP}$ and $L_2 \in \text{coNP}$ such that $L = L_1 \cap L_2$
  - Deciding whether there exists an explanation is NP-complete
  - Deciding whether a formula follows from all explanations is coNP-complete
Bounded Skeptical Abduction Hypotheses

- Humans generate some, but usually not all possible explanations
- Humans reason skeptically with respect to them
Which Explanations are Generated?

► Are short explanations preferred?
  ▶ We have specified a connectionist system implementing the Weak Completion Semantics
  ▶ Our system generates singleton sets first

► Are minimal explanations preferred?
  ▶ The Weak Completion Semantics utilizes minimal explanations

► Are supersets of known explanations generated?
  ▶ It is unnecessary to investigate explanations containing $A \leftarrow \top$ and $A \leftarrow \bot$
  ▶ In the presented system, if $E$ is an explanation, then so is each $E' \supseteq E$
  ▶ However, this does not hold anymore in extensions of the presented system

► Are more explanations generated if more time is available?

► Is the generation of explanations biased and, if so, how is it biased?

► Does attention play a role?