Conditionals and the Selection Task

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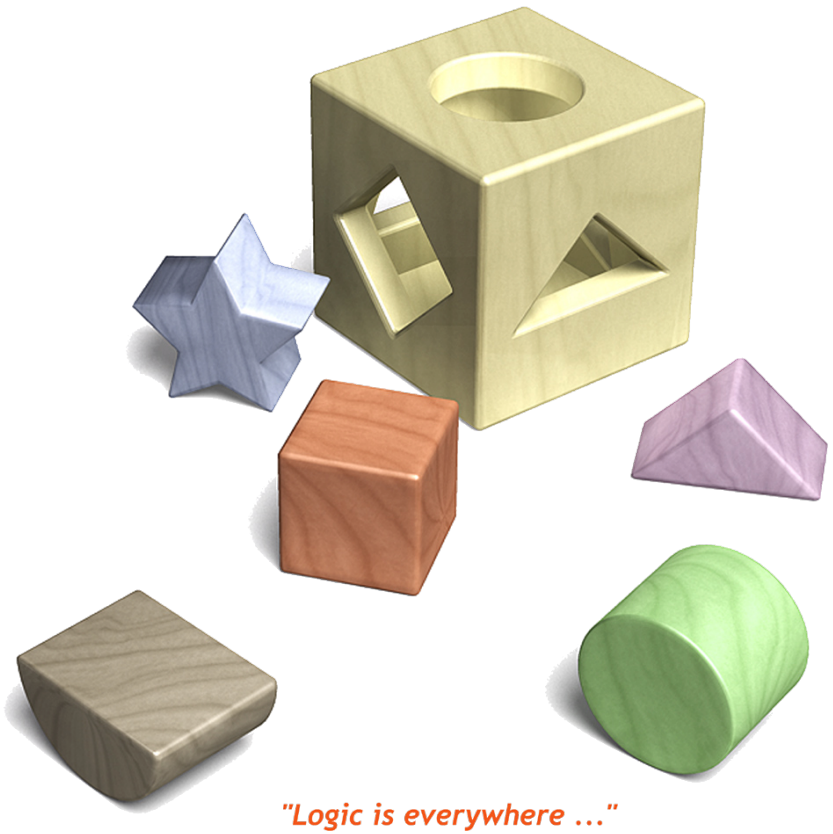
- **Obligation and Factual Conditionals**
- **The Selection Task**
- **Bounded Skeptical Abduction**



"Logic is everywhere ..."

Obligation Conditional

- Dietz Saldanha, H., Lourêdo Rocha: Obligation versus Factual Conditionals under the Weak Completion Semantics. CEUR Workshop Proc. 1837, 55-64: 2017
- **If it rains then the roofs are wet**
  - **Its consequence is obligatory**
  - **We cannot easily imagine a case, where the antecedent is true and the consequence is not**
- **Necessary antecedent**
  - **The consequence cannot be true unless the antecedent is true**

Factual Conditional

- **If it rains then she takes her umbrella**
  - **We can easily imagine a situation, where the antecedent is true and the consequence is not**
  - **Its consequence is not obligatory**
- **Sufficient antecedent**
  - **The antecedent does not appear to be necessary**

Encoding Obligation and Factual Conditionals

- **Program**

$$
\begin{aligned}
w &\leftarrow r \wedge \neg ab_4 \\
ab_4 &\leftarrow \bot \\
u &\leftarrow r \wedge \neg ab_5 \\
ab_5 &\leftarrow \bot
\end{aligned}
$$

- **Weakly completed program & least model**

$$
\begin{aligned}
w &\leftrightarrow r \wedge \neg ab_4 \\
ab_4 &\leftrightarrow \bot \\
u &\leftrightarrow r \wedge \neg ab_5 \\
ab_5 &\leftrightarrow \bot
\end{aligned}
$$

true	*false*
	ab_4
	ab_5

- **Abducibles**

$r \leftarrow \top$ $\quad$ $r \leftarrow \bot$ $\quad$ $ab_5 \leftarrow \top$ $\quad$ $u \leftarrow \top$

The Evaluation of Indicative Conditionals

- **Indicative conditional**
  - ***If X then Y***
- **Background knowledge**
  - **Weakly completed program *wc*$\mathcal{P}$ with least model $\mathcal{M}$**
  - **Set of abducibles $\mathcal{A}$**
- **Evaluation**
  - **If $\mathcal{M}(X)$ is *true*, then the conditional is evaluated to $\mathcal{M}(Y)$**
  - **If $\mathcal{M}(X)$ is *false*, then the conditional is evaluated to *true***
  - **If $\mathcal{M}(X)$ is *unknown*, then the conditional is evaluated with respect to the skeptical consequences of *wc*$\mathcal{P}$ given $\mathcal{A}$ considering *X* as an observation**

If the roofs are not wet then it did not rain

- *If* $\neg w$ *then* $\neg r$
- Explaining $\neg w$ by $r \leftarrow \bot$ we obtain
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \bot\}$
- It's least model is

true	*false*
	ab_4
	ab_5
	r
	w
	u

- The conditional is *true*

If she did not take her umbrella then it did not rain

- *If* $\neg u$ *then* $\neg r$
- **The observation** $\neg u$ **can be explained by** $r \leftarrow \bot$ **and** $ab_5 \leftarrow \top$**, and we obtain**
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \bot\}$
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot \vee \top\}$
- **Their least models are**

true	*false*
	ab_4
	ab_5
	r
	w
	u

true	*false*
ab_5	ab_4
	u

- **Reasoning skeptically, the conditional is** ***unknown***

If the roofs are wet then it rained

- *If w then r*
- Explaining w by $r \leftarrow \top$ we obtain
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \top\}$
- Its least model is

true	*false*
r	ab_4
	ab_5
w	
u	

- The conditional is *true*

If she took her umbrella then it rained

- *If u then r*
- The observation u can be explained by $r \leftarrow \top$ or $u \leftarrow \top$, and we obtain
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow r \wedge \neg ab_5,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot,\ r \leftrightarrow \top\}$
  - $\{w \leftrightarrow r \wedge \neg ab_4,\ u \leftrightarrow (r \wedge \neg ab_5) \vee \top,\ ab_4 \leftrightarrow \bot,\ ab_5 \leftrightarrow \bot\}$
- Their least models are

true	*false*
r	ab_4
	ab_5
w	
u	

true	*false*
u	ab_4
	ab_5

- Reasoning skeptically, the conditional is *unknown*

The Selection Task

- **If there is the letter d on one side of the card, then there is the number 3 on the other side**
  - **Factual conditional with necessary antecedent**
- **If a person is drinking beer, then the person must be over 19 years of age**
  - **Obligation conditional with sufficient antecedent**
- **Reasoning skeptically yields the adequate answers**
  - Dietz Saldanha, H., Lourêdo Rocha: Obligation versus Factual Conditionals under the Weak Completion Semantics. CEUR Workshop Proc. 1837, 55-64: 2017

The Abstract Case

- **Program**

$$3 \leftarrow d \wedge \neg ab_6$$
$$ab_6 \leftarrow \bot$$

- **Abducibles**

$$d \leftarrow \top \qquad d \leftarrow \bot \qquad ab_6 \leftarrow \top$$

- **Observations & least models**

$\boxed{\text{d}}$

true	*false*
d	ab_6
3	

turn
89%

$\boxed{\text{a}}\ \neg d$

true	*false*
	d
	ab_6
	3

no turn
16%

$\boxed{3}$

true	*false*
d	ab_6
3	

turn
62%

$\boxed{7}\ \neg 3$

true	*false*
	d
	ab_6
	3

true	*false*
ab_6	
	3

no turn
25%

The Social Case

- **Program**

$$o \leftarrow b \wedge \neg ab_7$$
$$ab_7 \leftarrow \bot$$

- **Abducibles**

$$b \leftarrow \top \qquad b \leftarrow \bot \qquad o \leftarrow \top$$

- **Observations & least models**

$\boxed{\text{b}}$

true	*false*
b	ab_7
o	

turn
95%

$\boxed{\text{m}}\ \neg b$

true	*false*
	b
	ab_7
	o

no turn
0,025%

$\boxed{\text{o}}$

true	*false*
o	ab_7

true	*false*
b	ab_7
o	

no turn
0,025%

$\boxed{\text{y}}\ \neg o$

true	*false*
	b
	ab_7
	o

turn
80%

Syllogistic Reasoning

- Oliviera da Costa, Dietz Saldanha, H., Ragni: A Computational Logic Approach to Human Syllogistic Reasoning. In: Proc. 39th Annual Conference of the Cognitive Science Society, 883-888: 2017

  Oliviera da Costa, Dietz Saldanha, H.: Monadic Reasoning using Weak Completion Semantics. In: CEUR Workshop Proc. 1837, 45-54: 2017

- **The Weak Completion Semantics achieves 89% when reasoning skeptically**
  - **It is better than 12 established cognitive theories**

The Complexity of Skeptical Abduction

- H., Philipp, Wernhard: An Abductive Model for Human Reasoning
  In: Logical Formalizations of Commonsense Reasoning
  Papers from the AAAI 2011 Spring Symposium
  AAAI Spring Symposium Series Technical Reports, AAAI Press, 135-138: 2011

  Dietz Saldanha, H., Philipp: Contextual Abduction and Its Complexity Issues
  In: Proc. 4th Int. Workshop on Defeasible and Ampliative Reasoning
  CEUR Workshop Proc. 1827, 58-70: 2017

- **Skeptical Reasoning is DP-complete**
  - **L is in the class DP iff there are languages $L_1 \in$ NP and $L_2 \in$ coNP such that $L = L_1 \cap L_2$**
  - **Deciding whether there exists an explanation is NP-complete**
  - **Deciding whether a formula follows from all explanations is coNP-complete**

Bounded Skeptical Abduction Hypotheses

- **Humans generate some, but usually not all possible explanations**
- **Humans reason skeptically with respect to them**

Which Explanations are Generated?

- Are short explanations preferred?
  - We have specified a connectionist system implementing the Weak Completion Semantics
  - Our system generates singleton sets first
- Are minimal explanations preferred?
  - The Weak Completion Semantics utilizes minimal explanations
- Are supersets of known explanations generated?
  - It is unnecessary to investgate explanations containing $A \leftarrow \top$ and $A \leftarrow \bot$
  - In the presented system, if $\mathcal{E}$ is an explanation, then so is each $\mathcal{E}' \supseteq \mathcal{E}$
  - However, this does not hold anymore in extensions of the presented system
- Are more explanations generated if more time is available?
- Is the generation of explanations biased and, if so, how is it biased?
- Does attention play a role?