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Reliance-based Static Analysis of Existential Rules

Diploma Thesis (defense) // Dresden, September 18, 2025

ρ_1 : $\mathit{student}(x) \longrightarrow \exists y.\ \mathit{author}(x, y) , \mathit{thesis}(y)$

Reliances [Krötzsch, KR'22]

- When applying ρ_1, a new unsatisfied match for ρ_2 may occur
- When applying ρ_2 before ρ_1, redundant nulls may be introduced

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$\rho_3 : \mathit{thesis}(y) \longrightarrow \exists z_2.\, \mathit{examiner}(y, z_2),\ \mathit{professor}(z_2)$

$\rho_4 : \mathit{professor}(z_2) \longrightarrow \exists z_1.\, \mathit{course}(z_1, z_2)$

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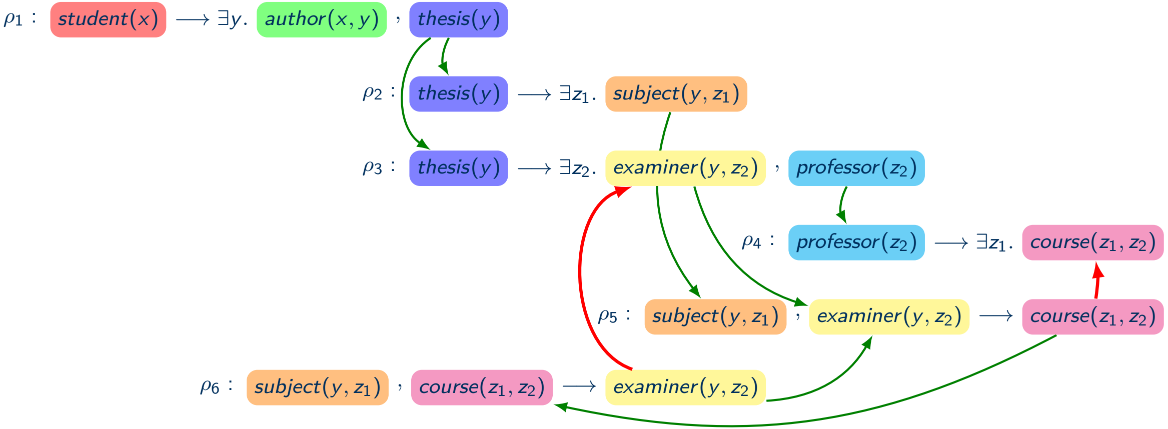

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- When applying ρ_2 before ρ_1, redundant nulls may be introduced $\Rightarrow$ **restraint reliance** $\rho_1 \overset{\square}{\prec} \rho_2$

Reliance Graph $\mathcal{G}(R)$ [Krötzsch, KR'22]

- vertices $\triangleq$ rules
- labelled edges $\triangleq$ reliance relations, with $+$ for **positive reliances** and $\square$ for **restraints**

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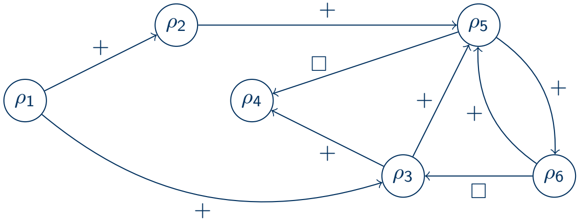

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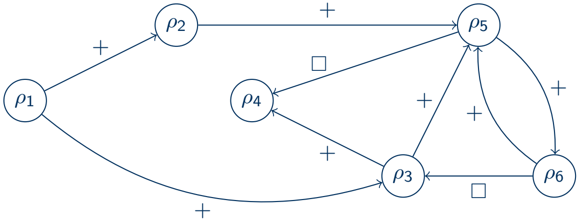

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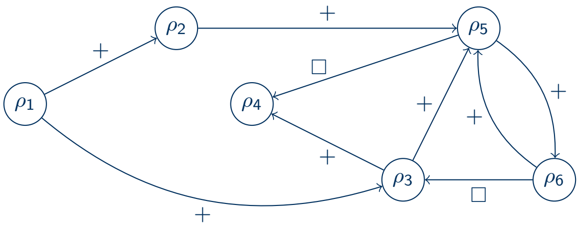

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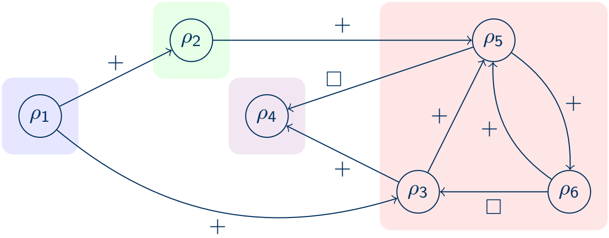

- There is an SCC $\{\rho_3, \rho_5, \rho_6\}$ with restraint edge in the reliance graph.

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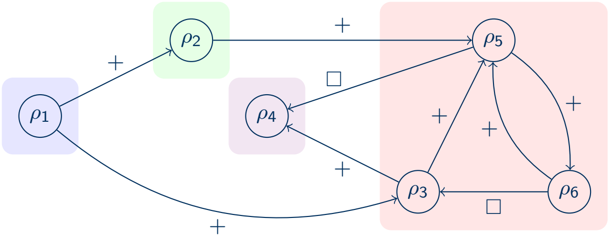

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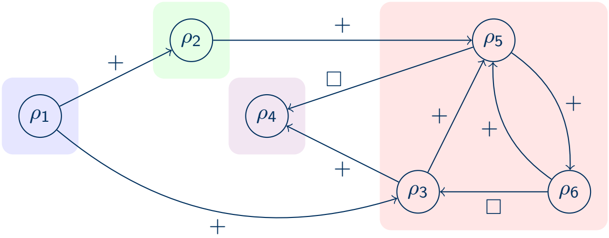

- There is an SCC $\{\rho_3, \rho_5, \rho_6\}$ with restraint edge in the reliance graph.
- The ruleset is not **core-stratified** ! $\rightarrow$ But a core can still be computed with the restricted chase !

Contractions

Definition

Let $\rho_1 : \phi_1 \to \psi_1$ and $\rho_2 : \phi_2 \to \psi_2$ be two rules with distinct variables.

$$\rho_3 : \mathit{thesis}(y) \longrightarrow \exists z_2.\ \mathit{examiner}(y, z_2), \mathit{professor}(z_2)$$

$$\rho_5 : \mathit{subject}(y, z_1), \mathit{examiner}(y, z_2) \longrightarrow \mathit{course}(z_1, z_2)$$

Contractions

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Let $\rho_1 : \phi_1 \to \psi_1$ and $\rho_2 : \phi_2 \to \psi_2$ be two rules with distinct variables.

The *contracted rule* of ρ_1 and ρ_2 for a variable substitution $\eta : \boldsymbol{V} \to (\boldsymbol{V} \cup \boldsymbol{C})$ is

$$\rho^{\oplus}_{\rho_1,\rho_2,\eta} : \underbrace{\phi_1\eta \cup (\phi_2\eta \setminus \psi_1\eta)}_{=\phi^{\oplus}} \longrightarrow \underbrace{\psi_1\eta \cup \psi_2\eta \setminus \phi^{\oplus}}_{=\psi^{\oplus}}$$

$$\rho_3 : \mathit{thesis}(y) \longrightarrow \exists z_2.\ \mathit{examiner}(y, z_2),\ \mathit{professor}(z_2)$$

$\oplus$ $\qquad\qquad \eta = \mathrm{id}$

$$\rho_5 : \mathit{subject}(y, z_1),\ \mathit{examiner}(y, z_2) \longrightarrow \mathit{course}(z_1, z_2)$$

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Contractions

Definition

We call $\rho_1 \oplus \rho_2$ their *contractions* with

$$\rho_1 \oplus \rho_2 := \{\, \rho^{\oplus}_{\rho_1,\rho_2,\eta} \mid \chi \subseteq \phi_2,\ \mu : \chi \to \psi_1,\ \eta : \boldsymbol{V} \to (\boldsymbol{V} \cup \boldsymbol{C}) \text{ leftmost MGU of } \chi\mu \text{ and } \chi,\ \mathit{var}_{\exists}(\psi_1\eta) \cap \mathit{var}(\phi_2\eta \setminus \psi_1\eta) = \varnothing \,\}$$

$$\rho_3 : \mathit{thesis}(y) \longrightarrow \exists z_2.\ \mathit{examiner}(y, z_2),\ \mathit{professor}(z_2)$$

$$\oplus$$

$$\rho_5 : \mathit{subject}(y, z_1),\ \mathit{examiner}(y, z_2) \longrightarrow \mathit{course}(z_1, z_2)$$

$$\rho_3 \oplus \rho_5 = \{\ \mathit{thesis}(y),\ \mathit{subject}(y, z_1) \longrightarrow \exists z_2.\ \mathit{examiner}(y, z_2),\ \mathit{professor}(z_2),\ \mathit{course}(z_1, z_2)\ \}$$

Enriching the ruleset with contractions

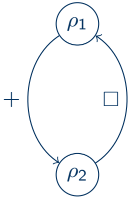
Definition

Let the contracted rules completion Comp(R) (for a ruleset R) be the smallest rulesets, s.t.

1. $R \subseteq \text{Comp}(R)$, and
2. for any pair $\langle \rho_1, \rho_2 \rangle \in \overset{+}{\prec} \cap \text{Comp}(R) \times R$ with minimal renaming ζ, s.t. $\mathit{var}(\rho_1) \cap \mathit{var}(\rho_2\zeta) = \emptyset$, their contractions $\rho_1 \oplus \rho_2\zeta \subset \text{Comp}(R)$.

Symmetry

$$\rho_1 : \quad a(x) \longrightarrow \exists v.r(x, v), b(v)$$
$$\rho_2 : \quad r(x, y) \longrightarrow r(y, x)$$



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$$\rho_1 : \quad a(x) \longrightarrow \exists v.r(x, v), b(v)$$
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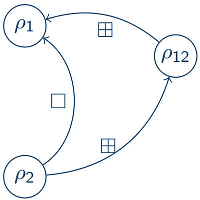
$$\rho_{12} : \quad a(x) \longrightarrow \exists v.r(x, v), b(v), r(v, x)$$



contracted reliance graph $\mathcal{G}^{\oplus}(R)$

- There is no restraint cycle in $\mathcal{G}^{\oplus}(R)$. $\rightarrow$ We call R **contraction core-stratified** !

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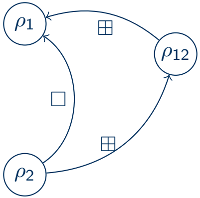

contracted reliance graph $\mathcal{G}^{\oplus}(R)$

- There is no restraint cycle in $\mathcal{G}^{\oplus}(R)$. $\to$ We call R **contraction core-stratified** !
- Here, $\mathrm{Comp}(R)$ is sure to be *finite*, as there is no positive reliance cycle in $\mathcal{G}(R)$.
  $\to$ But this is not generally the case. . .

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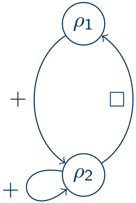

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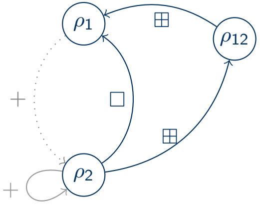

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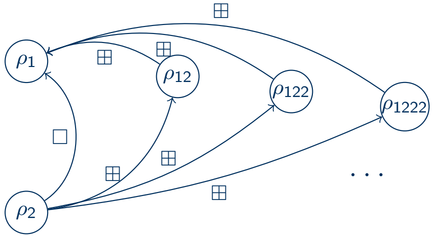
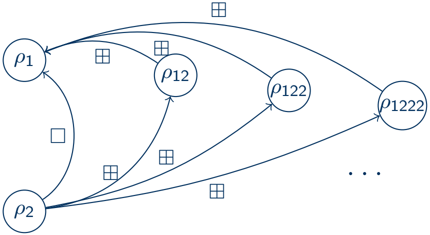


- There is no restraint cycle in $\mathcal{G}^{\oplus}(R)$. $\rightarrow$ We call R **contraction core-stratified** !
- In this case, there is a positive reliance cycle in $\mathcal{G}(R)$ and $\mathrm{Comp}(R)$ is *infinite*.

Deciding acyclicity of $\mathcal{G}^{\oplus}(R)$

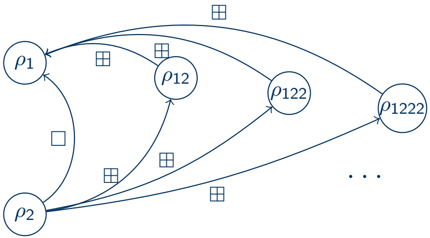
(I)



- Structure of $\mathcal{G}^{\oplus}(R)$ seams regular

$\rightarrow$ Is it sufficient to consider a finite subgraph?

Deciding acyclicity of $\mathcal{G}^{\oplus}(R)$



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→ Is it sufficient to consider a finite subgraph?

Finite contracted reliance graph $\mathcal{G}^{[\oplus]}(R)$

- Assign each (contracted) rule a set orig($\rho^{\oplus}$) of sequences over alphabet $\boldsymbol{\Sigma}$ to track information regarding their formation
- **Idea:** Stop "lengthening" $\rho^{\oplus}$ when all words in orig($\rho^{\oplus}$) are of the form $uwqwv$

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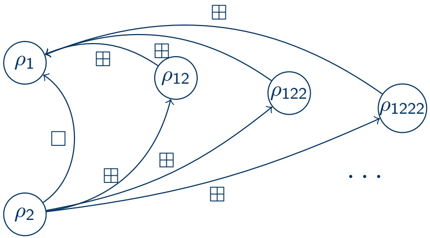

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What do we need to show?

- If there is a restraint cycle in $\mathcal{G}^{\oplus}(R)$, then there is a restraint cycle in $\mathcal{G}^{[\oplus]}(R)$
- Rules labeled *uwqw* and *uw* can be continued in the same way
- Restraints depend only on first/last letter of labels

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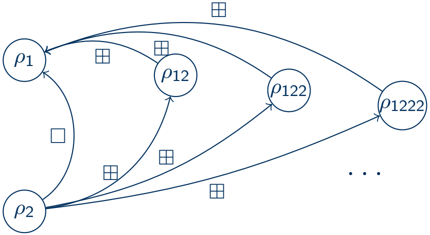

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- **Question:** Which information must be tracked in letters of $\boldsymbol{\Sigma}$?

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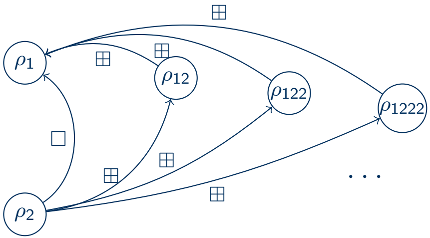

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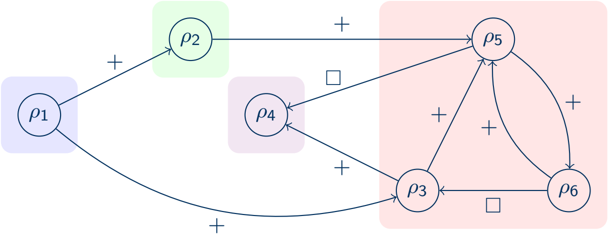
Approaches?

- $\boldsymbol{\Sigma} = R$ does not work, as the unifier can change internal structure in contractions
- Additionally tracking change of internal structure of component rules is not enough, as "surrounding" atoms in the contraction may prevent positive reliance to other rules
- $\boldsymbol{\Sigma} = 2^{\mathcal{H}} \times 2^{\mathcal{H}}$

Finite contracted reliance graph $\mathcal{G}^{[\oplus]}(R)$

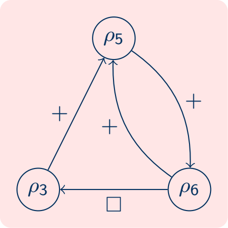
- Assign each (contracted) rule a set $\mathrm{orig}(\rho^{\oplus})$ of sequences over alphabet $\boldsymbol{\Sigma}$ to track information regarding their formation
- **Idea:** Stop "lengthening" $\rho^{\oplus}$ when all words in $\mathrm{orig}(\rho^{\oplus})$ are of the form $uwqwv$
- **Question:** Which information must be tracked in letters of $\boldsymbol{\Sigma}$?

Deciding acyclicity of $\mathcal{G}^{\oplus}(R)$ (III)



Deciding acyclicity of $\mathcal{G}^{\oplus}(R)$ (III)

- The head pieces $\tilde{\mathcal{H}}$ are
  - $\exists z_2.\ \mathit{examiner}(y, z_2)\ ,\ \mathit{professor}(z_2)$
  - $\mathit{course}(z_1, z_2)$
  - $\mathit{examiner}(y, z_2)$



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  - $\exists z_2.$ $examiner(y, z_2)$, $professor(z_2)$
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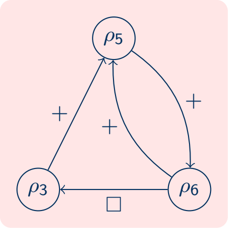
  - $examiner(y, z_2)$
- Variables are $\{y, z_1, z_2\}$
  - Replace the existentials in $\tilde{\mathcal{H}}$ with fresh variables
  - Consider all mappings of frontier variables in $\tilde{\mathcal{H}}$ to other variables



Deciding acyclicity of $\mathcal{G}^{\oplus}(R)$ (III)

- The head pieces $\tilde{\mathcal{H}}$ are
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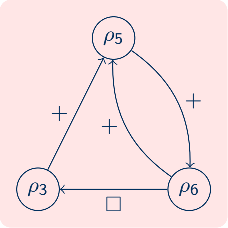
  - $course(z_1, z_2)$
  - $examiner(y, z_2)$
- Variables are $\{y, z_1, z_2\}$
  - Replace the existentials in $\tilde{\mathcal{H}}$ with fresh variables
  - Consider all mappings of frontier variables in $\tilde{\mathcal{H}}$ to other variables
- The head images $\mathcal{H}$ are
  - $examiner(y, \tilde{z}_2), professor(\tilde{z}_2)$
  - $examiner(z_1, \tilde{z}_2), professor(\tilde{z}_2)$
  - $examiner(z_2, \tilde{z}_2), professor(\tilde{z}_2)$
  - 9 options for $course(z_1, z_2)$
  - 9 options for $examiner(y, z_2)$



Deciding acyclicity of $\mathcal{G}^{\oplus}(R)$ (III)

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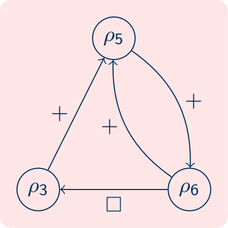

$\rightarrow$ The alphabet $\Sigma = 2^{\mathcal{H}} \times 2^{\mathcal{H}}$ tracks

. . . the head images contributed to the head of the contracted rule by the second rule

. . . the head images that are necessarily satisfied

Deciding acyclicity of $\mathcal{G}^{\oplus}(R)$ (III)

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  - $\mathit{examiner}(y, z_2)$

Repetition

$$\mathrm{orig}(\rho_{3565}) = \{\langle\{\{\exists\tilde{z}_2.\mathit{examiner}(y,\tilde{z}_2), \mathit{professor}(\tilde{z}_2)\}\}, \{\{\exists\tilde{z}_2.\mathit{examiner}(y,\tilde{z}_2), \mathit{professor}(\tilde{z}_2)\}, \{\mathit{examiner}(y, z_2)\}\}\rangle$$
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$\rightarrow$ The alphabet $\boldsymbol{\Sigma} = 2^{\mathcal{H}} \times 2^{\mathcal{H}}$ tracks

. . . the head images contributed to the head of the contracted rule by the second rule
. . . the head images that are necessarily satisfied

The selection strategy

Assumptions:

R is ...

- universally terminating and
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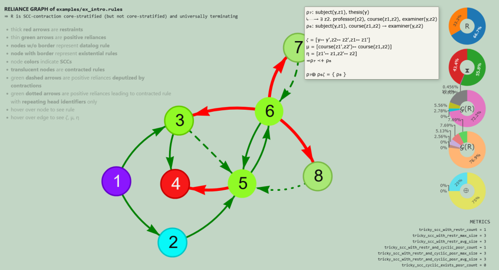
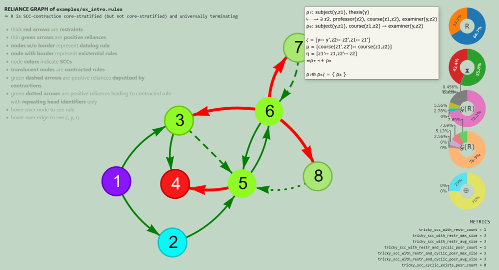
3. Otherwise, select only the triggers of R_i that are leader of contracted triggers whose rules are not restrained by other contracted triggers
   $\rightarrow$ Applying such a trigger will not lead to future appearance of new triggers that restrain the selected trigger!
   - As the restraint relation of $\mathcal{G}^{\oplus}(R)$ is acyclic by assumption, at least one selection is always possible!

Prototype implementation



- 1,913 lines of Python code
- Prototypical
  (Not fully optimized for efficiency yet)
- Interactive visualization of the **finite contracted reliance graph**



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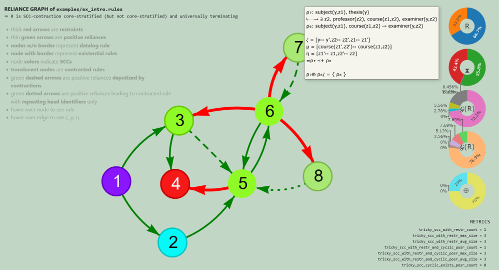

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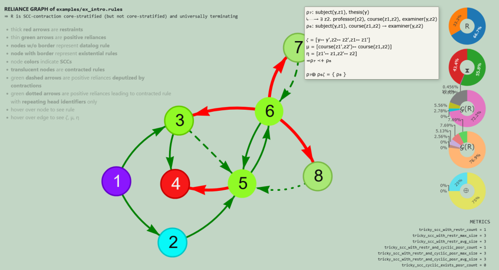

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3. Add contractions for. . . restrained rules → **contraction core-stratified** ?
   . . . existential cyclic positive reliances
   → detect absence of **unbounded contractions** ⇒ universal termination

Evaluation

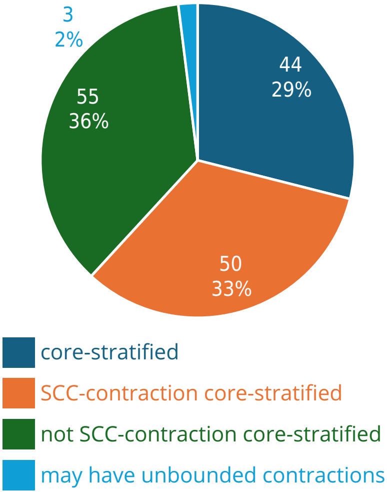
Evaluation

- *Prototypical* Python implementation to perform static analysis
  1) Build the initial reliance graph and decompose it into SCCs
  2) For each SCC with restraint, add appropriate contracted rules and check for restraint cycles
  3) For each positive-reliance-SCC with existentials, add contractions until either we are sure that the completion is finite or a full “cycle” was traversed

- Dataset: *Oxford Ontology Repository*
  - `https://www.cs.ox.ac.uk/isg/ontologies/`
  - same rulesets that were previously used to evaluate core-stratification [Ivliev, 21 & González et al., ISWC’22]
  - 201 of 787 ontologies were translated into existential rules

- Experiments:
  - on Wille server
  - with 25min timeout
    - 49 timeouts (during reliance graph computation)
    - mostly for rulesets with over 12,000 rules

Evaluation: prevalence of contraction core-stratification (II)



- Roughly 1/3 are core-stratified.
  - This is consistent with prior observations [Ivliev, 21 & González et al., ISWC'22]
- Roughly an additional 1/3 are SCC-contraction core-stratified.
  - New static analysis covers a significant portion of the evaluated real-world rulesets!
- Of the 55 not SCC-contraction core-stratified rulesets,
  - 87% have restraint cycles and
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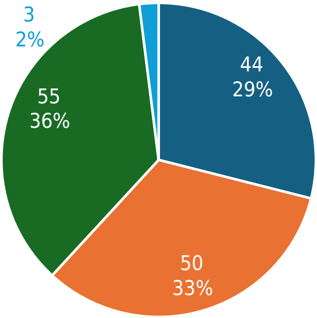

- core-stratified
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- not SCC-contraction core-stratified
- may have unbounded contractions

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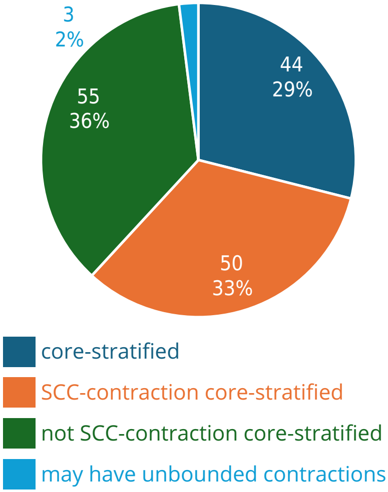

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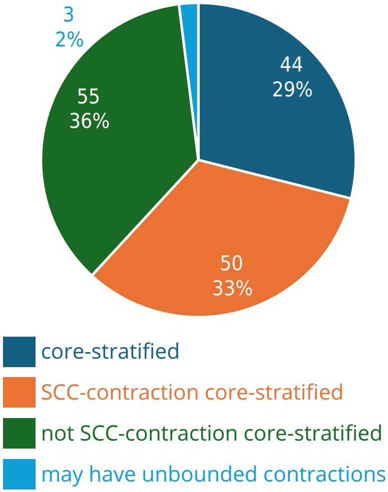

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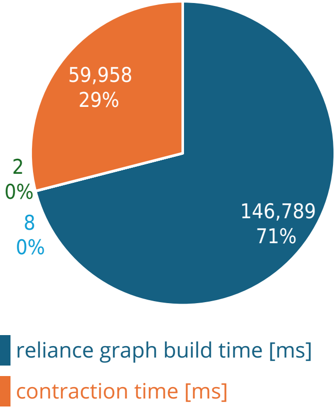

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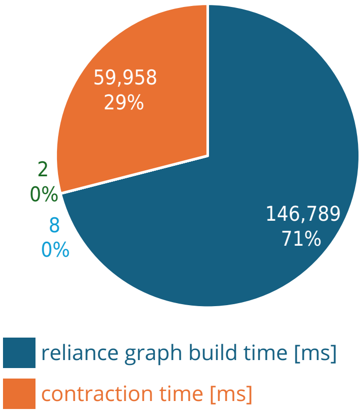

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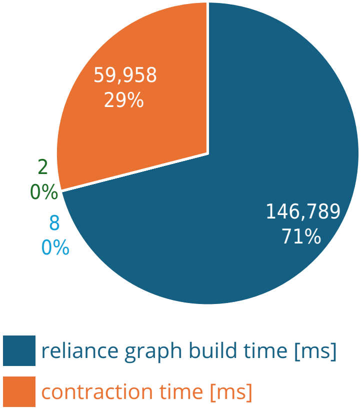

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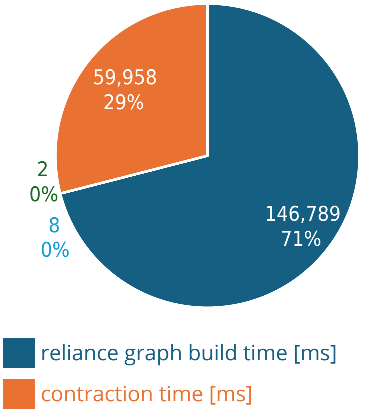

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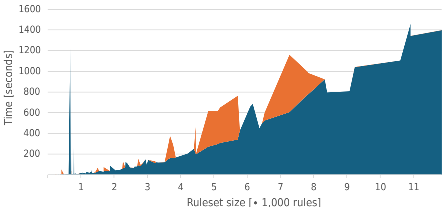

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- Contractions deputize the combined effect of applying positively relying rules
- Look ahead by adding contractions $\Rightarrow$ absence of restraint cycles

Future work

- Continue working out proof details for $\mathcal{G}^{[\oplus]}(R)$
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- *Evaluation:*
  - conduct more targeted experiments to clarify runtime behavior
- *Implementation:*
  - Improve efficiency
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  - not restricted to special cases
+ Covers 2/3 of evaluated rulesets

  $\rightarrow$ find core with restricted chase

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  - worst-case: *2Exp*

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Thank you for your attention!

Errata

- There is a mistake in the implementation of the $\textsc{check}^{\square}_{self}$ function, that causes certain self-restraints of rules with multiple pieces to be missed
  - like: $a(x) \longrightarrow \exists v.b(x), r(x, v), c(v)$