

## Introduction to Formal Concept Analysis

### Exercise Sheet 7, Winter Semester 2017/18

**Exercise 1** (frequent concept intents and closure systems)

**Definition** (frequent concept intent). Let  $\mathbb{K} = (G, M, I)$  be a formal context.

(a) The support of a set  $B \subseteq M$  of attributes in  $\mathbb{K}$  is given by

$$\text{supp}(B) := \frac{|B'|}{|G|}.$$

(b) For a given minimal support  $\text{minsupp}$  the set of frequent concept intents is given by

$$\{B \subseteq M \mid \exists A \subseteq G : (A, B) \in \mathfrak{B}(G, M, I) \wedge \text{supp}(B) \geq \text{minsupp}\}.$$

Show that the set of frequent concept intents together with the set  $M$  forms a closure system.

**Solution:**

**Proof:** We have to show that the intersection of frequent concept intents is again a frequent concept intent. We already know that the intersection of intents produces an intent. It remains to show that it is frequent. So let  $\mathfrak{J}$  be a set of frequent intents. We pick one  $B \in \mathfrak{J}$  and observe  $\text{supp}(B) = \frac{|B'|}{|G|} \geq \text{minsupp}$ . Moreover, we have  $\bigcap \mathfrak{J} \subseteq B$  and consequently  $B' \subseteq (\bigcap \mathfrak{J})'$ . Therefore  $\text{supp}(\bigcap \mathfrak{J}) = \frac{|(\bigcap \mathfrak{J})'|}{|G|} \geq \frac{|B'|}{|G|} \geq \text{minsupp}$ , i.e.,  $\bigcap \mathfrak{J}$  is frequent.

**Exercise 2** (support)

Show the validity of the properties of the support function that are employed by the TITANIC algorithm:

Let  $(G, M, I)$  be a formal context  $X, Y \subseteq M$ . Then it holds:

- 1)  $X \subseteq Y \implies \text{supp}(X) \geq \text{supp}(Y)$
- 2)  $X'' = Y'' \implies \text{supp}(X) = \text{supp}(Y)$
- 3)  $X \subseteq Y \wedge \text{supp}(X) = \text{supp}(Y) \implies X'' = Y''$

**Solution:**

1. Let  $X \subseteq Y$ , then  $Y' \subseteq X'$  holds as we saw in Exercise Sheet 1. This implies,  $\text{supp}(Y) = \frac{|Y'|}{|G|} \leq \frac{|X'|}{|G|} = \text{supp}(X)$
2.  $X'' = Y'' \implies \text{supp}(X) = \text{supp}(Y)$   
 $X'' = Y'' \iff X''' = Y''' \iff X' = Y' \implies \text{supp}(X) = \frac{|X'|}{|G|} = \frac{|Y'|}{|G|} = \text{supp}(Y)$ .
3.  $X \subseteq Y \wedge \text{supp}(X) = \text{supp}(Y) \implies X'' = Y''$   
 $\text{supp}(X) = \text{supp}(Y) \implies |X'| = |Y'|$  and  $X \subseteq Y \implies X' \supseteq Y'$ . Hence  $X' = Y'$ , since  $X'$  and  $Y'$  are finite. It follows,  $X'' = Y''$ .

**Exercise 3** (computing concept intents with TITANIC)

The following context contains transactions in a supermarket. Compute the closure system of all concept intents using the TITANIC algorithm. (hint: use the table structure from the example computation in the lecture slides)

	apples (a)	beer (b)	chips (c)	tv magazine (d)	toothpaste (e)
$t_1$	×	×	×		
$t_2$			×	×	
$t_3$		×	×	×	
$t_4$	×	×			×
$t_5$			×		×
$t_6$		×	×	×	
$t_7$	×	×			
$t_8$			×	×	

**Solution:**

In the first pass, the algorithm deals with the empty set and singletons, all 1-sets. It returns the results for  $k=0$  and  $k=1$ :

$k = 0$ :

step 1	step 2
x	x.s $x \in K_k?$
$\emptyset$	1 yes

$k = 1$ :

steps 4+5	step 7	step 9	
$X$	$X.p_s$	$X.s$	$X \in K_k?$
{a}	1	3/8	yes
{b}	1	5/8	yes
{c}	1	6/8	yes
{d}	1	4/8	yes
{e}	1	2/8	yes

Step 8 returns:  $\emptyset.closure \leftarrow \emptyset$

$k = 2$ :

steps 12	step 7	step 9	
$X$	$X.p_s$	$X.s$	$X \in K_k?$
{a,b}	3/8	3/8	no
{a,c}	3/8	1/8	yes
{a,d}	3/8	0	yes
{a,e}	2/8	1/8	yes
{b,c}	5/8	3/8	yes
{b,d}	4/8	2/8	yes
{b,e}	2/8	1/8	yes
{c,d}	4/8	4/8	no
{c,e}	2/8	1/8	yes
{d,e}	2/8	0	yes

Step 8 returns:  
 $\{a\}.closure \leftarrow \{a, b\}$   
 $\{b\}.closure \leftarrow \{b\}$   
 $\{c\}.closure \leftarrow \{c\}$   
 $\{d\}.closure \leftarrow \{c, d\}$   
 $\{e\}.closure \leftarrow \{e\}$

Step 8 returns:

$\{a, c\}.closure \leftarrow \{a, b, c\}$   
 $\{a, d\}.closure \leftarrow \{a, b, c, d, e\}$   
 $\{a, e\}.closure \leftarrow \{a, b, e\}$   
 $\{b, c\}.closure \leftarrow \{b, c\}$   
 $\{b, d\}.closure \leftarrow \{b, c, d\}$   
 $\{b, e\}.closure \leftarrow \{a, b, c, e\}$   
 $\{c, e\}.closure \leftarrow \{c, e\}$   
 $\{d, e\}.closure \leftarrow \{a, b, c, d, e\}$

steps 12		step 7	step 9
$X$	$X.p\_s$	$X.s$	$X \in K_k?$
$\{a, c, e\}$	$1/8$	0	yes
$\{a, d, e\}$	0	0	no
$\{b, c, e\}$	$1/8$	0	yes
$\{b, d, e\}$	0	0	no

k = 3:

k= 4:

Step 12: returns the empty set. Hence there is nothing to **WEIGH** in Step 7. Step 9 sets  $k_4 = \emptyset$ ; and in step 10, the loop is exited.

Step 8 returns:

$\{a, c, e\}.closure \leftarrow \{a, b, c, d, e\}$   
 $\{b, c, e\}.closure \leftarrow \{a, b, c, d, e\}$

Step 14: Collects all concept intents: