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Complexity Theory

Exercise 8: Polynomial Hierarchy

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Exercise 8.1. Show that Cook-reducibility is transitive. In other words, show that if **A** is Cook-reducible to **B** and **B** is Cook-reducible to **C**, then **A** is Cook-reducible to **C**.

Exercise 8.2. Show that there exists an oracle **C** such that $\mathrm{NP}^{\mathbf{C}} \neq \mathrm{coNP}^{\mathbf{C}}$.

Hint:
What kind of Turing machines exist for languages in coNP? Use the answer to adapt the proof of the Baker-Gill-Solovay Theorem for coNP instead of P.

Exercise 8.3. Show $\mathsf{NP}^{\mathsf{SAT}} \subseteq \Sigma_2\mathsf{P}$.

Exercise 8.4. Show the following result: *If there is any k such that $\Sigma_k^{\mathrm{P}} = \Sigma_{k+1}^{\mathrm{P}}$ then $\Sigma_j^{\mathrm{P}} = \Pi_j^{\mathrm{P}} = \Sigma_k^{\mathrm{P}}$ for all $j > k$, and therefore $\mathrm{PH} = \Sigma_k^{\mathrm{P}}$.*

Exercise 8.5. Show that $\mathrm{PH} \subseteq \mathrm{PSPACE}$.

Exercise 8.6. Let **A** be a language and let **F** be a finite set with $\mathbf{A} \cap \mathbf{F} = \emptyset$.
Show that $\mathrm{P}^{\mathbf{A}} = \mathrm{P}^{\mathbf{A} \cup \mathbf{F}}$ and $\mathrm{NP}^{\mathbf{A}} = \mathrm{NP}^{\mathbf{A} \cup \mathbf{F}}$.