Introduction to Formal Argumentation

* slides adapted from Stefan Woltran’s lecture on Abstract Argumentation

Sarah Gaggl

Dresden, 24th October 2014
Organisation

Goal:

- Get an overview of abstract argumentation and it’s most resent research topics.
- Learn to prepare a scientific talk.

Organisation:

- 3 lectures to introduce necessary background.
- In last lecture: topic selection.
- Students should read related literature and prepare a presentation (of 30 min).
- Send the slides no later than 1 Week before presentation to sarah.gaggl@tu-dresden.de
Organisation

Goal:

- Get an overview of abstract argumentation and its most recent research topics.
- Learn to prepare a scientific talk.

Organisation:

- 3 lectures to introduce necessary background.
- In last lecture: topic selection.
- Students should read related literature and prepare a presentation (of 30 min).
- Send the slides no later than 1 Week before presentation to sarah.gaggl@tu-dresden.de

Influence on evaluation:

- If I did receive the slides in time!
- Quality of the slides.
- Quality of the presentation (time limit, easy to follow, clarity, reaction to questions).
First International Competition on Computational Models of Argumentation (ICCMA’15), see http://argumentationcompetition.org

Student project for optimizing ASP encodings for abstract argumentation.

If you are interested have a look at http://www.inf.tu-dresden.de/?node_id=3657&ln=en and contact me!
Roadmap for the Lecture

- Introduction
- Abstract Argumentation Frameworks
- Implementation Techniques
- Extensions of Abstract Argumentation Frameworks
- Students’ Topics
Introduction

Arguementation:

...the study of processes “concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held”.

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]
Introduction

Argumentation:

...the study of processes “concerned with how assertions are **proposed**, **discussed**, and **resolved** in the context of issues upon which several **diverging opinions** may be held”.

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]

Formal Models of Argumentation are concerned with

- representation of an argument
- representation of the relationship between arguments
- solving conflicts between the arguments (“acceptability”)
Increasingly important area

- “Argumentation” as keyword at all major AI conferences
- dedicated conference: COMMA, TAFA workshop; and several more workshops
- specialized journal: *Argument and Computation* (Taylor & Francis)
- two text books:
Increasingly important area

- “Argumentation” as keyword at all major AI conferences
- dedicated conference: COMMA, TAFA workshop; and several more workshops
- specialized journal: *Argument and Computation* (Taylor & Francis)
- two text books:

Applications

- PARMENIDES-system for E-Democracy: facilitates structured arguments over a proposed course of action [Atkinson et al.; 2006]
- IMPACT project: argumentation toolbox for supporting open, inclusive and transparent deliberations about public policy
- Decision support systems, etc.
The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions
The Overall Process

Steps
- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example
\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]
The Overall Process

Steps
- Starting point: knowledge-base
- **Form arguments**
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]

\[ \langle \{w, w \rightarrow \neg s\}, \neg s \rangle \]

\[ \langle \{s, s \rightarrow \neg r\}, \neg r \rangle \]

\[ \langle \{r, r \rightarrow \neg w\}, \neg w \rangle \]
The Overall Process

Steps
- Starting point: knowledge-base
- Form arguments
- **Identify conflicts**
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

\[ \Delta = \{ s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s \} \]

\[
\langle \{ s, s \rightarrow \neg r \}, \neg r \rangle \\
\langle \{ w, w \rightarrow \neg s \}, \neg s \rangle \\
\langle \{ r, r \rightarrow \neg w \}, \neg w \rangle 
\]
The Overall Process

Steps
- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

\[ \Delta = \{ s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s \} \]

\[ F_\Delta : \]

\[ \alpha \rightarrow \beta \rightarrow \gamma \rightarrow \alpha \]
The Overall Process

Steps
- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

\[ \Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\} \]

\[ F_\Delta: \]

\[ \text{pref}(F_\Delta) = \{\emptyset\} \]
\[ \text{stage}(F_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\} \]
The Overall Process

Steps

- Starting point: knowledge-base
- Form arguments
- Identify conflicts
- Abstract from internal structure
- Resolve conflicts
- Draw conclusions

Example

\[ \Delta = \{ s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s \} \]

\[
\begin{align*}
\langle \{ w, w \rightarrow \neg s \}, \neg s \rangle \\
\langle \{ s, s \rightarrow \neg r \}, \neg r \rangle \\
\langle \{ r, r \rightarrow \neg w \}, \neg w \rangle
\end{align*}
\]

\[
\begin{align*}
C_{n_{\text{pref}}} (F_\Delta) &= Cn(\top) \\
C_{n_{\text{stage}}} (F_\Delta) &= Cn(\neg r \lor \neg w \lor \neg s)
\end{align*}
\]
Some Remarks

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
- Separation between logical (forming arguments) and nonmonotonic reasoning ("abstract argumentation frameworks")
- Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")
Some Remarks

- Main idea dates back to Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, etc.)
- Separation between logical (forming arguments) and nonmonotonic reasoning ("abstract argumentation frameworks")
- Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")

Main Challenge

- All Steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)
Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

• Given is a KB (a set of propositions) $\Delta$
• argument is a pair $(\Phi, \alpha)$, such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
• conflicts between arguments $(\Phi, \alpha)$ and $(\Phi', \alpha')$ arise if $\Phi$ and $\alpha'$ are contradicting.
Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) $\Delta$
- argument is a pair $(\Phi, \alpha)$, such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
- conflicts between arguments $(\Phi, \alpha)$ and $(\Phi', \alpha')$ arise if $\Phi$ and $\alpha'$ are contradicting.

Example

$\langle \{s, s \rightarrow \neg r\}, \neg r \rangle \rightarrow \langle \{r, r \rightarrow \neg w\}, \neg w \rangle$
Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) $\Delta$
- argument is a pair $(\Phi, \alpha)$, such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subset \Phi$, $\Psi \models \alpha$
- conflicts between arguments $(\Phi, \alpha)$ and $(\Phi', \alpha')$ arise if $\Phi$ and $\alpha'$ are contradicting.

Example

$\langle \{s, s \rightarrow \neg r\}, \neg r\rangle \rightarrow \langle \{r, r \rightarrow \neg w\}, \neg w\rangle$

Other Approaches

- Arguments are trees of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.
Dung’s Abstract Argumentation Frameworks

Example

\[ \alpha \xrightarrow{} \gamma \xrightarrow{} \beta \]

Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
Dung’s Abstract Argumentation Frameworks

Example

Main Properties

- Abstract from the concrete content of arguments but only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful, formalism
- Most active research area in the field of argumentation.
  - “plethora of semantics”
An argumentation framework (AF) is a pair \((A, R)\) where

- \(A\) is a set of arguments
- \(R \subseteq A \times A\) is a relation representing the conflicts ("attacks")
Dung’s Abstract Argumentation Frameworks

**Definition**

An argumentation framework (AF) is a pair \((A, R)\) where

- \(A\) is a set of arguments
- \(R \subseteq A \times A\) is a relation representing the conflicts ("attacks")

**Example**

\[
F = (\{a,b,c,d,e\}, \{(a,b),(c,b),(c,d),(d,c),(d,e),(e,e)\})
\]
Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$. 
Basic Properties

Conflict-Free Sets

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

Example

$cf(F) = \{\{a, c\}\}$,
Basic Properties

Conflict-Free Sets

Given an AF $F = (A, R)$. A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

Example

$$cf(F) = \{\{a, c\}, \{a, d\}\}.$$
Basic Properties

**Conflict-Free Sets**

Given an AF $F = (A, R)$. A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

**Example**

$$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\},$$
Basic Properties

**Conflict-Free Sets**

Given an AF $F = (A, R)$.

A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

**Example**

$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$
Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is **admissible** in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is **defended** by $S$ in $F$
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$. 

Example

<table>
<thead>
<tr>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ADM ($F$) = \{\{a, c\}\}
Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is defended by $S$ in $F$
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

$adm(F) = \{\{a, c\}\}$,
Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is defended by $S$ in $F$
- $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

$$\text{adm}(F) = \{\{a, c\}, \{a, d\}\}.$$
Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is defended by $S$ in $F$
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

$$adm(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\}$$
Admissible Sets [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is defended by $S$ in $F$
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

$adm(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$
Dung’s Fundamental Lemma

Let $S$ be admissible in an AF $F$ and $a, a'$ arguments in $F$ defended by $S$ in $F$. Then,

1. $S' = S \cup \{a\}$ is admissible in $F$

2. $a'$ is defended by $S'$ in $F$
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a \textit{naive extension} of $F$, if

- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S \not\subseteq T$
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of $F$, if

- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S \not\subset T$

**Example**

$\text{naive}(F) = \{\{a, c\}\}$,
Naive Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of $F$, if
- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S \not\subset T$

Example

$naive(F) = \{\{a, c\}, \{a, d\}\}$
Naive Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of $F$, if

1. $S$ is conflict-free in $F$
2. for each $T \subseteq A$ conflict-free in $F$, $S \not\subset T$

Example

$\text{naive}(F) = \{ \{a, c\}, \{a, d\}, \{b, d\}, \}$
Naive Extensions

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a naive extension of $F$, if
- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S \not\subset T$

Example

$\text{naive}(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$
Semantics (ctd.)

Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of $F$ is defined as the outcome $S$ of the following “algorithm”:

1. put each argument $a \in A$ which is not attacked in $F$ into $S$; if no such argument exists, return $S$;
2. remove from $F$ all (new) arguments in $S$ and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.
Semantics (ctd.)

Grounded Extension [Dung, 1995]

Given an AF $F = (A, R)$. The unique grounded extension of $F$ is defined as the outcome $S$ of the following "algorithm":

1. put each argument $a \in A$ which is not attacked in $F$ into $S$; if no such argument exists, return $S$;

2. remove from $F$ all (new) arguments in $S$ and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.

Example

$ground(F) = \{\{a\}\}$
Semantics (ctd.)

**Complete Extension [Dung, 1995]**

Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if

- \(S\) is admissible in \(F\)
- each \(a \in A\) defended by \(S\) in \(F\) is contained in \(S\)
  - Recall: \(a \in A\) is defended by \(S\) in \(F\), if for each \(b \in A\) with \((b, a) \in R\), there exists a \(c \in S\), such that \((c, b) \in R\).
Semantics (ctd.)

Complete Extension [Dung, 1995]

Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if

- \(S\) is admissible in \(F\)
- each \(a \in A\) defended by \(S\) in \(F\) is contained in \(S\)
  - Recall: \(a \in A\) is defended by \(S\) in \(F\), if for each \(b \in A\) with \((b, a) \in R\), there exists a \(c \in S\), such that \((c, b) \in R\).

Example

\[
comp(F) = \{a, c\},
\]
Complete Extension [Dung, 1995]

Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if

- \(S\) is admissible in \(F\)
- each \(a \in A\) defended by \(S\) in \(F\) is contained in \(S\)
  - Recall: \(a \in A\) is defended by \(S\) in \(F\), if for each \(b \in A\) with \((b, a) \in R\), there exists a \(c \in S\), such that \((c, b) \in R\).

Example

\[
\begin{align*}
\text{comp}(F) &= \{\{a, c\}, \{a, d\}\},
\end{align*}
\]
Semantics (ctd.)

**Complete Extension [Dung, 1995]**

Given an AF $(A, R)$. A set $S \subseteq A$ is complete in $F$, if

- $S$ is admissible in $F$
- each $a \in A$ defended by $S$ in $F$ is contained in $S$
  - Recall: $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

**Example**

\[\begin{align*}
\text{comp}(F) &= \{\{a, c\}, \{a, d\}, \{a\}\},
\end{align*}\]
Semantics (ctd.)

Complete Extension [Dung, 1995]

Given an AF \((A, R)\). A set \(S \subseteq A\) is complete in \(F\), if
- \(S\) is admissible in \(F\)
- each \(a \in A\) defended by \(S\) in \(F\) is contained in \(S\)

Recall: \(a \in A\) is defended by \(S\) in \(F\), if for each \(b \in A\) with \((b, a) \in R\), there exists a \(c \in S\), such that \((c, b) \in R\).

Example

\[
\begin{align*}
\text{comp}(F) &= \{\{a, c\}, \{a, d\}, \{a\}, \{e\}, \{d\}, \emptyset\}
\end{align*}
\]
Semantics (ctd.)

Properties of the Grounded Extension

For any AF $F$, the grounded extension of $F$ is the subset-minimal complete extension of $F$. 

Remark: Since there exists exactly one grounded extension for each AF $F$, we often write $\text{ground}(F) = S$ instead of $\text{ground}(F) = \{S\}$. 

TU Dresden, 24th October 2014 Seminar Abstract Argumentation slide 49 of 94
Properties of the Grounded Extension
For any AF $F$, the grounded extension of $F$ is the subset-minimal complete extension of $F$.

Remark
Since there exists exactly one grounded extension for each AF $F$, we often write $\text{ground}(F) = S$ instead of $\text{ground}(F) = \{S\}$. 
Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of $F$, if

- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $F$, $S \not\subset T$

Example:

```
b c d e
pref(F) = {{a, c}, {a, d}, {a}, {c}, {d}, ∅}
```
Preferred Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a preferred extension of $F$, if

- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $F$, $S \not\subseteq T$

Example

$\text{pref}(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$
Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$
Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a **stable extension** of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

**Example**

```
stable(F) = \{\{a, e\}\}
```
Semantics (ctd.)

Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

Example

\[ stable(F) = \{ \{a, e\}, \{a, d\} \}, \]
Semantics (ctd.)

Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

Example

\[
\text{stable}(F) = \{\{a, e\}, \{a, d\}, \{b, d\}\}
\]
Stable Extensions [Dung, 1995]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stable extension of $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$

Example

$\text{stable}(F) = \{\{a, e\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{e\}, \{d\}, \emptyset\}$
Semantics (ctd.)

Some Relations

For any AF $F$ the following relations hold:

1. Each stable extension of $F$ is admissible in $F$
2. Each stable extension of $F$ is also a preferred one
3. Each preferred extension of $F$ is also a complete one
Semi-Stable Extensions [Caminada, 2006]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of $F$, if

- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $F$, $S^+ \not\subseteq T^+$
- for $S \subseteq A$, define $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$
Semi-Stable Extensions [Caminada, 2006]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a semi-stable extension of $F$, if

- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $F$, $S^+ \not\subseteq T^+$
  - for $S \subseteq A$, define $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

Example

$$semi(F) = \{\{a, e\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$$
Stage Extensions [Verheij, 1996]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of $F$, if

- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S^+ \not\subset T^+$
  - recall $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$
Semantics (ctd.)

### Stage Extensions [Verheij, 1996]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of $F$, if

- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S^+ \nsubseteq T^+$
  - recall $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$

### Ideal Extension [Dung, Mancarella & Toni 2007]

Given an AF $F = (A, R)$. A set $S \subseteq A$ is an ideal extension of $F$, if

- $S$ is admissible in $F$ and contained in each preferred extension of $F$
- there is no $T \supseteq S$ admissible in $F$ and contained in each of $\text{pref}(F)$
Stage Extensions [Verheij, 1996]
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a stage extension of $F$, if
- $S$ is conflict-free in $F$
- for each $T \subseteq A$ conflict-free in $F$, $S^+ \not\subseteq T^+$
  - recall $S^+ = S \cup \{a | \exists b \in S \text{ with } (b, a) \in R\}$

Ideal Extension [Dung, Mancarella & Toni 2007]
Given an AF $F = (A, R)$. A set $S \subseteq A$ is an ideal extension of $F$, if
- $S$ is admissible in $F$ and contained in each preferred extension of $F$
- there is no $T \supset S$ admissible in $F$ and contained in each of $\text{pref}(F)$

Eager Extension [Caminada, 2007]
Given an AF $F = (A, R)$. A set $S \subseteq A$ is an eager extension of $F$, if
- $S$ is admissible in $F$ and contained in each semi-stable extension of $F$
- there is no $T \supset S$ admissible in $F$ and contained in each of $\text{semi}(F)$
Semantics (ctd.)

Properties of Ideal Extensions

For any AF $F$ the following observations hold:

1. there exists exactly one ideal extension of $F$
2. the ideal extension of $F$ is also a complete one

The same results hold for the eager extension and similar variants [Dvořák et al., 2011].
Resolution-based grounded Extensions
[Baroni, Giacomin 2008]

A resolution $\beta$ of an AF $F = (A, R)$ contains exactly one of the attacks $(a, b)$, $(b, a)$ for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of $F$, if

- there exists a resolution $\beta$ such that $\text{ground}((A, R \setminus \beta)) = S$
- and there is no resolution $\beta'$ such that $\text{ground}((A, R \setminus \beta')) \subset S$
A resolution $\beta$ of an AF $F = (A, R)$ contains exactly one of the attacks $(a, b)$, $(b, a)$ for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of $F$, if

- there exists a resolution $\beta$ such that $\text{ground}((A, R \setminus \beta)) = S$
- and there is no resolution $\beta'$ such that $\text{ground}((A, R \setminus \beta')) \subset S$

Example

$$\text{ground}^*(F) = \{\{a, c\}\}$$
Resolution-based grounded Extensions
[Baroni,Giacomin 2008]

A resolution $\beta$ of an AF $F = (A, R)$ contains exactly one of the attacks $(a, b), (b, a)$ for each pair $a, b \in A$ with $\{(a, b), (b, a)\} \subseteq R$.

A set $S \subseteq A$ is a resolution-based grounded extension of $F$, if
- there exists a resolution $\beta$ such that $\text{ground}((A, R \setminus \beta)) = S$
- and there is no resolution $\beta'$ such that $\text{ground}((A, R \setminus \beta')) \subset S$

Example

$$\text{ground}^*(F) = \{\{a, c\}, \{a, d\}\}$$
Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from $b$ to $a$. We define $[[F]] = \bigcup_{C \in SCCs(F)} F|_C$ and call $[[F]]$ the separation of $F$.

Example
Definition (Separation)

An AF $F = (A, R)$ is called separated if for each $(a, b) \in R$, there exists a path from $b$ to $a$. We define $[[F]] = \bigcup_{C \in \text{SCCs}(F)} F|_C$ and call $[[F]]$ the separation of $F$.

Example
Definition (Reachability)

Let $F = (A, R)$ be an AF, $B$ a set of arguments, and $a, b \in A$. We say that $b$ is reachable in $F$ from $a$ modulo $B$, in symbols $a \Rightarrow^{F}_B b$, if there exists a path from $a$ to $b$ in $F|_B$. 
Definition (Reachability)

Let $F = (A, R)$ be an AF, $B$ a set of arguments, and $a, b \in A$. We say that $b$ is reachable in $F$ from $a$ modulo $B$, in symbols $a \Rightarrow^B_F b$, if there exists a path from $a$ to $b$ in $F|_B$.

Definition ($\Delta_{F,S}$)

For an AF $F = (A, R)$, $D \subseteq A$, and a set $S$ of arguments,

$$\Delta_{F,S}(D) = \{ a \in A \mid \exists b \in S : b \neq a, (b, a) \in R, a \not\Rightarrow^{A\setminus D}_F b \}.$$  

By $\Delta_{F,S}$, we denote the lfp of $\Delta_{F,S}(\emptyset)$.
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([F - \Delta_{F,S}])$. 

**Example** $S = \{c, f, h\}$, $S \in cf2(F)$. 

**cf2 Semantics (ctd.)**
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([[F - \Delta_F, S]])$.

**Example**

$S = \{c, f, h\}$, $S \in cf(F)$. 

TU Dresden, 24th October 2014  Seminar Abstract Argumentation  slide 73 of 94
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in naive([F - \Delta_{F,S}])$.

Example

$S = \{c, f, h\}$, $S \in cf(F)$, $\Delta_{F,S}(\emptyset) = \{d, e\}$. 

TU Dresden, 24th October 2014

Seminar Abstract Argumentation
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if

- $S$ is conflict-free in $F$
- and $S \in \text{naive}([F - \Delta_{F,S}])$.

Example

$S = \{c, f, h\}$, $S \in cf(F)$, $\Delta_{F,S}(\{d, e\}) = \{d, e\}$.
Given an AF $F = (A, R)$. A set $S \subseteq A$ is a cf2-extension of $F$, if
- $S$ is conflict-free in $F$
- and $S \in \text{naive}([[F - \Delta_{F,S}]]).$

Example

$S = \{c, f, h\}, \ S \in cf(F), \ \Delta_{F,S} = \{d, e\}, \ S \in \text{naive}([[F - \Delta_{F,S}]]).$
Relations between Semantics

Figure: An arrow from semantics $\sigma$ to semantics $\tau$ encodes that each $\sigma$-extension is also a $\tau$-extension.
Characteristics of Argumentation Semantics

**Example**

\[
pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}
\]
\[
naive(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b, e\}\}
\]

**Natural Questions**

- How to change the AF if we want \{a, b, e\} instead of \{a, b\} in \(pref(F)\)?
- How to change the AF if we want \{a, b, d\} instead of \{a, b\} in \(pref(F)\)?
- Can we have equivalent AFs without argument \(f\)?

→ Realizability
Some Properties . . .

Theorem

For any AFs $F$ and $G$, we have

- $\text{adm}(F) = \text{adm}(G) \implies \sigma(F) = \sigma(G)$, for $\sigma \in \{\text{pref}, \text{ideal}\}$;
- $\text{comp}(F) = \text{comp}(G) \implies \vartheta(F) = \vartheta(G)$, for $\vartheta \in \{\text{pref}, \text{ideal}, \text{ground}\}$;
- no other such relation between the different semantics ($\text{adm}$, $\text{pref}$, $\text{ideal}$, $\text{semi}$, $\text{eager}$, $\text{ground}$, $\text{comp}$, $\text{stable}$) in terms of standard equivalence holds.
Strong Equivalence [Oikarinen & Woltran 2011, G & Woltran 2011]

Definition

Two AFs $F$ and $G$ are strongly equivalent wrt. a semantics $\sigma \in \{\text{stable, adm, pref, ideal, semi, comp, ground, stage}\}$, in symbols $F \equiv^\sigma_s G$, iff $\sigma(F \cup H) = \sigma(G \cup H)$, for each AF $H$.

- Idea: Find “$\sigma$-kernels” of AFs, such that the $\sigma$-kernels of $F$ and $G$ coincide iff $F \equiv^\sigma_s G$.
- Verification of strong equivalence then reduces to checking syntactical equivalence.
Kernel for stable semantics

For AF $F = (A, R)$, we define stable-kernel of $F$ as $F^\kappa = (A, R^\kappa)$ with

$$R^\kappa = R \setminus \{(a, b) \mid a \neq b, (a, a) \in R\}.$$ 

Theorem

For any AFs $F$ and $G$: $F^\kappa = G^\kappa$ iff $F \equiv_s^{\text{stable}} G$ iff $F \equiv_s^{\text{stage}} G$. 
Decision Problems on AFs

<table>
<thead>
<tr>
<th>Credulous Acceptance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cred$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is $a$ contained in at least one $\sigma$-extension of $F$?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Skeptical Acceptance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skept$_\sigma$: Given AF $F = (A, R)$ and $a \in A$; is $a$ contained in every $\sigma$-extension of $F$?</td>
</tr>
</tbody>
</table>

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted\(^1\).

---

\(^1\)This is only relevant for stable semantics.

TU Dresden, 24th October 2014   Seminar Abstract Argumentation   slide 82 of 94
Credulous Acceptance

Cred_σ: Given AF \( F = (A, R) \) and \( a \in A \); is \( a \) contained in at least one \( \sigma \)-extension of \( F \)?

Skeptical Acceptance

Skept_σ: Given AF \( F = (A, R) \) and \( a \in A \); is \( a \) contained in every \( \sigma \)-extension of \( F \)?

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted\(^1\).

Hence we are also interested in the following problem:

Skeptically and Credulously accepted

Skept'_σ: Given AF \( F = (A, R) \) and \( a \in A \); is \( a \) contained in every and at least one \( \sigma \)-extension of \( F \)?

\(^1\) This is only relevant for stable semantics.
Further Decision Problems

Verifying an extension

\[ \text{Ver}_\sigma : \text{Given } AF \, F = (A, R) \text{ and } S \subseteq A; \text{ is } S \text{ a } \sigma\text{-extension of } F? \]
### Verifying an extension

\[ \text{Ver}_\sigma : \text{Given } AF \ F = (A, R) \text{ and } S \subseteq A; \text{ is } S \text{ a } \sigma\text{-extension of } F? \]

### Does there exist an extension?

\[ \text{Exists}_\sigma : \text{Given } AF \ F = (A, R); \text{ Does there exist a } \sigma\text{-extension for } F? \]
### Verifying an extension

Ver$_\sigma$: Given AF $F = (A, R)$ and $S \subseteq A$; is $S$ a $\sigma$-extension of $F$?

### Does there exist an extension?

Exists$_\sigma$: Given AF $F = (A, R)$; Does there exist a $\sigma$-extension for $F$?

### Does there exist a nonempty extension?

Exists$\neg \emptyset$$_\sigma$: Does there exist a non-empty $\sigma$-extension for $F$?
Complexity Results (Summary)

Complexity for decision problems in AFs.

<table>
<thead>
<tr>
<th>σ</th>
<th>Cred&lt;sub&gt;σ&lt;/sub&gt;</th>
<th>Skept&lt;sub&gt;σ&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>ground</td>
<td>P-c</td>
<td>P-c</td>
</tr>
<tr>
<td>naive</td>
<td>in L</td>
<td>in L</td>
</tr>
<tr>
<td>stable</td>
<td>NP-c</td>
<td>co-NP-c</td>
</tr>
<tr>
<td>adm</td>
<td>NP-c</td>
<td>trivial</td>
</tr>
<tr>
<td>comp</td>
<td>NP-c</td>
<td>P-c</td>
</tr>
<tr>
<td>pref</td>
<td>NP-c</td>
<td>Π&lt;sub&gt;2&lt;/sub&gt;-c</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>σ</th>
<th>Cred&lt;sub&gt;σ&lt;/sub&gt;</th>
<th>Skept&lt;sub&gt;σ&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>semi</td>
<td>Σ&lt;sub&gt;2&lt;/sub&gt;-c</td>
<td>Π&lt;sub&gt;2&lt;/sub&gt;-c</td>
</tr>
<tr>
<td>stage</td>
<td>Σ&lt;sub&gt;2&lt;/sub&gt;-c</td>
<td>Π&lt;sub&gt;2&lt;/sub&gt;-c</td>
</tr>
<tr>
<td>ideal</td>
<td>in Θ&lt;sub&gt;p&lt;/sub&gt;&lt;sub&gt;2&lt;/sub&gt;</td>
<td>in Θ&lt;sub&gt;p&lt;/sub&gt;&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td>eager</td>
<td>Π&lt;sub&gt;2&lt;/sub&gt;-c</td>
<td>Π&lt;sub&gt;2&lt;/sub&gt;-c</td>
</tr>
<tr>
<td>ground*</td>
<td>NP-c</td>
<td>co-NP-c</td>
</tr>
<tr>
<td>cf2</td>
<td>NP-c</td>
<td>co-NP-c</td>
</tr>
</tbody>
</table>

Intractable problems in Abstract Argumentation

Most problems in Abstract Argumentation are computationally intractable, i.e. at least NP-hard. To show intractability for a specific reasoning problem we follow the schema given below:

**Goal:** Show that a reasoning problem is NP-hard.

**Method:** Reducing the NP-hard SAT problem to the reasoning problem.

- Consider an arbitrary CNF formula $\varphi$
- Give a reduction that maps $\varphi$ to an Argumentation Framework $F_\varphi$ containing an argument $\varphi$.
- Show that $\varphi$ is satisfiable iff the argument $\varphi$ is accepted.
Definition

For $\varphi = \bigwedge_{i=1}^{m} l_{i1} \lor l_{i2} \lor l_{i3}$ over atoms $Z$, build $F_{\varphi} = (A_{\varphi}, R_{\varphi})$ with

$$
A_{\varphi} = Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\varphi\}
$$

$$
R_{\varphi} = \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \varphi) \mid i \in \{1, \ldots, m\}\} \cup
\{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup
\{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\}
$$
**Canonical Reduction**

**Definition**

For $\varphi = \bigwedge_{i=1}^{m} l_{i1} \lor l_{i2} \lor l_{i3}$ over atoms $Z$, build $F_{\varphi} = (A_{\varphi}, R_{\varphi})$ with

$$A_{\varphi} = Z \cup \bar{Z} \cup \{C_1, \ldots, C_m\} \cup \{\varphi\}$$

$$R_{\varphi} = \{(z, \bar{z}), (\bar{z}, z) \mid z \in Z\} \cup \{(C_i, \varphi) \mid i \in \{1, \ldots, m\}\} \cup \{(z, C_i) \mid i \in \{1, \ldots, m\}, z \in \{l_{i1}, l_{i2}, l_{i3}\}\} \cup \{(\bar{z}, C_i) \mid i \in \{1, \ldots, m\}, \neg z \in \{l_{i1}, l_{i2}, l_{i3}\}\}$$

**Example**

Let $\Phi = (z_1 \lor z_2 \lor z_3) \land (\neg z_2 \lor \neg z_3 \lor \neg z_4) \land (\neg z_1 \lor z_2 \lor z_4)$. 
Theorem

The following statements are equivalent:

1. $\phi$ is satisfiable
2. $F_\phi$ has an admissible set containing $\phi$
3. $F_\phi$ has a complete extension containing $\phi$
4. $F_\phi$ has a preferred extension containing $\phi$
5. $F_\phi$ has a stable extension containing $\phi$
Complexity Results

Theorem

1. $\text{Cred}_{\text{stable}}$ is NP-complete
2. $\text{Cred}_{\text{adm}}$ is NP-complete
3. $\text{Cred}_{\text{comp}}$ is NP-complete
4. $\text{Cred}_{\text{pref}}$ is NP-complete

Proof.

(1) The hardness is immediate by the last theorem. For the NP-membership we use the following guess & check algorithm:

- Guess a set $E \subseteq A$
- verify that $E$ is stable
  - for each $a, b \in E$ check $(a, b) \not\in R$
  - for each $a \in A \setminus E$ check if there exists $b \in E$ with $(b, a) \in R$

As this algorithm is in polynomial time we obtain NP-membership.

□
P. Baroni, P. E. Dunne, and M. Giacomin.
On the resolution-based family of abstract argumentation semantics and its grounded instance.

P. Baroni and M. Giacomin.
Semantics of abstract argument systems.


T.J.M. Bench-Capon and P.E.Dunne.
Argumentation in AI,
AIJ 171:619-641, 2007

M. Caminada.
Semi-stable semantics.

M. Caminada.
Comparing two unique extension semantics for formal argumentation: ideal and eager

S. Coste-Marquis, C. Devred, and P. Marquis.
Symmetric argumentation frameworks.

Y. Dimopoulos and A. Torres.
Graph theoretical structures in logic programs and default theories.

P. M. Dung.
On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games.

P. M. Dung, P. Mancarella, and F. Toni.
Computing ideal sceptical argumentation.

P. E. Dunne.
Computational properties of argument systems satisfying graph-theoretic constraints.

P. E. Dunne.
The computational complexity of ideal semantics I: Abstract argumentation frameworks.

P. E. Dunne and T. J. M. Bench-Capon.
Coherence in finite argument systems.

P. E. Dunne and T. J. M. Bench-Capon.
Complexity in value-based argument systems.

W. Dvořák, P. Dunne, and S. Woltran.
Parametric properties of ideal semantics.

W. Dvořák and S. Woltran
On the intertranslatability of argumentation semantics

S. Gaggl and S. Woltran.
cf2 semantics revisited.

S. Gaggl and S. Woltran.
Strong equivalence for argumentation semantics based on conflict-free sets.

E. Oikarinen and S. Woltran.
Characterizing strong equivalence for argumentation frameworks.

B. Verheij.
Two approaches to dialectical argumentation: admissible sets and argumentation stages.