TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

AGM Revision in Description Logics under Fixed-Domain Semantics

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Revision Problem

Belief revision: incorporating new information into knowledge base consistently with minimal change.

- $\mathcal{K} = \{Professor \sqsubseteq Lecturer, Professor(sebastian)\}$
- $\mathcal{K}' = \{Professor \sqsubseteq \top, Lecturer \sqsubseteq \top, Professor(markus), \neg Lecturer(sebastian)\}$

How to add $\mathcal{K}'$ to $\mathcal{K}$ so that the revision result ($\mathcal{K} \circ \mathcal{K}'$) is still consistent?

$\circ : \mathcal{P}_{fin}(\mathcal{L}) \times \mathcal{P}_{fin}(\mathcal{L}) \to \mathcal{P}_{fin}(\mathcal{L})$ is a change operator, where $\mathcal{L}$ is the set of all sentences in DL

AGM Postulates [AGM85]

(G1) $\mathcal{K} \circ \mathcal{K}' \models \mathcal{K}'$.

(G2) If $\mathcal{K} \cup \mathcal{K}'$ is consistent, then $\mathcal{K} \circ \mathcal{K}' \equiv \mathcal{K} \cup \mathcal{K}'$.

(G3) If $\mathcal{K}'$ is consistent then $\mathcal{K} \circ \mathcal{K}'$ is consistent.

(G4) If $\mathcal{K}_1 \equiv \mathcal{K}_2$ and $\mathcal{K}_1 \equiv \mathcal{K}_2$ then $\mathcal{K}_1 \circ \mathcal{K}_1 \equiv \mathcal{K}_2 \circ \mathcal{K}_2$.

(G5) $(\mathcal{K} \circ \mathcal{K}_1) \cup \mathcal{K}_2 \models \mathcal{K} \circ (\mathcal{K}_1 \cup \mathcal{K}_2)$.

(G6) If $(\mathcal{K} \circ \mathcal{K}_1) \cup \mathcal{K}_2$ is consistent, then $\mathcal{K} \circ (\mathcal{K}_1 \cup \mathcal{K}_2) \models (\mathcal{K} \circ \mathcal{K}_1) \cup \mathcal{K}_2$.

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Postulates $\Longleftarrow$ Operators $\Longrightarrow$ Construction

Construction:

- Syntax-based approach
- Semantic-based approach/model-based approach

Approaches for DL Revision (1/2)

Syntax-based approaches: modify or remove the axioms of the KB.

$$\mathcal{K} = \{Professor \sqsubseteq Lecturer, Professor(sebastian)\}$$
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e.g. Justification-based revision [HWKP06]:
$\mathcal{K} \circ \mathcal{K}' = \{Professor \sqsubseteq Lecturer, Professor \sqsubseteq \top, Lecturer \sqsubseteq \top, Professor(markus), \neg Lecturer(sebastian)\}$

OR

$\mathcal{K} \circ \mathcal{K}' = \{Professor(sebastian), Professor \sqsubseteq \top, Lecturer \sqsubseteq \top, Professor(markus), \neg Lecturer(sebastian)\}$

Issues: considered only semi-revision [HWKP06] and could not satisfy all AGM postulates [QLB06]

Approaches for DL Revision (2/2)

Semantic-based approaches:

- investigate the models of the KB,
- search for the most plausible set of models to be the revision result, and
- generate a KB which corresponds to the produced model set.

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$[\![\mathcal{K}]\!]$?

- $\mathcal{I}_1 : \Delta = \{a\}, Professor^{\mathcal{I}_1} = \{a\}, Lecturer^{\mathcal{I}_1} = \{a\}, sebastian^{\mathcal{I}_1} = a$,
- $\ldots$

Issues in standard semantics: infinitely many models and some models might not be expressible [Liu+06]

Fixed-Domain Semantics for DL [GRS16]

Given a non-empty finite set $\Delta \subseteq N_I$ called fixed domain, an interpretation $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$ is said to be Δ-fixed if $\Delta^{\mathcal{I}} = \Delta$ and $a^{\mathcal{I}} = a$ for all $a \in \Delta$.

- Restrict the models to have the domain that is fixed upfront.
- Reasoner: `Wolpertinger`[1], allows satisfiability checking and model enumeration.

> How do we define (concrete) AGM revision operators in DL under fixed-domain semantics?

Answer:

1) Model-based approach using distance between models.

2) Individual-based approach by axioms weakening.

[1] https://github.com/wolpertinger-reasoner

Semantic Characterization (Adapted from [KM91])

- Assignment $\preceq_{(.)}: \mathcal{P}_{fin}(\mathcal{L}) \rightarrow \mathcal{P}(\Omega \times \Omega)$, maps $\mathcal{K}$ to $\preceq_{\mathcal{K}}$, where $\preceq_{\mathcal{K}}$ is a total relation over the set of all interpretations Ω.
  Idea: for each KB, assign an order to measure "how close" the interpretations to the KB models.

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- faithfulness-conditions for $\preceq_{(.)}$:
  - (F1) If $\mathcal{I}, \mathcal{I}' \models \mathcal{K}$, then $\mathcal{I} \prec_{\mathcal{K}} \mathcal{I}'$ does not hold.
  - (F2) If $\mathcal{I} \models \mathcal{K}$ and $\mathcal{I}' \not\models \mathcal{K}$, then $\mathcal{I} \prec_{\mathcal{K}} \mathcal{I}'$.
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Representation Theorem

In $\mathcal{SROIQ}$ under fixed-domain semantics, an operator $\circ$ satisfies (G1)–(G6) if and only if $\circ$ is compatible with some **faithful preorder** assignment.

Model-based Approach

- Idea: find the minimal models of $\mathcal{K}'$ by calculating distance between models (inspired by [Dal88])
- Let $Gr(\mathcal{I})$ be the set of all ground facts (incl. individual equality) of $\mathcal{I}$
- $dist(\mathcal{I}, \mathcal{I}') = |(Gr(\mathcal{I}) \cup Gr(\mathcal{I}')) \setminus (Gr(\mathcal{I}) \cap Gr(\mathcal{I}'))|$
- $dist([\![\mathcal{K}]\!]_\Delta, \mathcal{I}') = \min_{\mathcal{I} \in [\![\mathcal{K}]\!]_\Delta} dist(\mathcal{I}, \mathcal{I}')$

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Model-based revision operator

We define $\mathcal{I}_1 \preceq^\Delta_\mathcal{K} \mathcal{I}_2$ if and only if $dist([\![\mathcal{K}]\!]_\Delta, \mathcal{I}_1) \leq dist([\![\mathcal{K}]\!]_\Delta, \mathcal{I}_2)$ for all interpretations $\mathcal{I}_1$ and $\mathcal{I}_2$. We define the **model-based revision operator** $\circ_\Delta$ as follows:

$$[\![\mathcal{K} \circ_\Delta \mathcal{K}']\!]_\Delta = \min([\![\mathcal{K}']\!]_\Delta, \preceq^\Delta_\mathcal{K})$$

From models to KB

Let $\mathcal{I}_i$ be a Δ-interpretation, we define

$$\tau(\mathcal{I}_i) = \Bigg(\bigsqcap_{a \in N_I(\mathcal{K}) \setminus \Delta, d \in \Delta \text{ and } a^{\mathcal{I}_i} = d} \exists u.(\{a\} \sqcap \{d\}) \Bigg) \sqcap \Bigg(\bigsqcap_{C \in N_C} \bigsqcap_{d \in \Delta \text{ and } d \in C^{\mathcal{I}_i}} \exists u.(\{d\} \sqcap C) \Bigg) \sqcap \Bigg(\bigsqcap_{C \in N_C} \bigsqcap_{d \in \Delta \text{ and } d \notin C^{\mathcal{I}_i}} \exists u.(\{d\} \sqcap \neg C) \Bigg) \sqcap$$

$$\Bigg(\bigsqcap_{r \in N_R} \bigsqcap_{d,e \in \Delta \text{ and } (d,e) \in r^{\mathcal{I}_i}} \exists u.(\{d\} \sqcap \exists r.\{e\}) \Bigg) \sqcap \Bigg(\bigsqcap_{r \in N_R} \bigsqcap_{d,e \in \Delta \text{ and } (d,e) \notin r^{\mathcal{I}_i}} \exists u.(\{d\} \sqcap \neg \exists r.\{e\}) \Bigg), \text{where } u \text{ is the universal role.}$$

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Given a set of Δ-interpretations $\{\mathcal{I}_1, \ldots, \mathcal{I}_n\}$, we define

$$form(\{\mathcal{I}_1, \ldots, \mathcal{I}_n\}) = \top \sqsubseteq \bigsqcup_{1 \leq i \leq n} (\tau(\mathcal{I}_i))$$

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Proposition

Given $\{\mathcal{I}_1, \ldots, \mathcal{I}_n\}$, we have $[\![form(\mathcal{I}_1, \ldots, \mathcal{I}_n)]\!]_\Delta = \{\mathcal{I}_1, \ldots, \mathcal{I}_n\}$ holds.

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Model-based revision

$\mathcal{K} \circ_\Delta \mathcal{K}' = \{form(\min([\![\mathcal{K}']\!]_\Delta, \preceq^\Delta_\mathcal{K}))\}$

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$\mathcal{K} \circ_\Delta \mathcal{K}' = \{form(\min([\![\mathcal{K}']\!]_\Delta, \preceq^\Delta_\mathcal{K}))\}$

Proposition

The model-based change operator $\circ_\Delta$ satisfies the postulates (G1)–(G6).

Individual-based Approach

- Syntax-based revision
- Idea: when inconsistent, axioms are weakened, i.e. modified by adding some exceptions from the domain elements
- Preprocessing: transform KB $\mathcal{K}$ to RBox-free KB $\mathsf{trans}_\Delta(\mathcal{K})$

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Weakened knowledge base

Given some exceptional individuals $\Delta' \subseteq \Delta$ with $\Delta' = \{a_1, ..., a_n\}$. Consider an axiom $\sigma \in \mathcal{K}$:

(1) If $\sigma = C \sqsubseteq D$, then $\sigma^{-\Delta'} = C \sqcap \neg\{a_1\} \sqcap ... \sqcap \neg\{a_n\} \sqsubseteq D$.
(2) If $\sigma = C(a_i)$, then $\sigma^{-\Delta'} = \top(a_i)$ if $a_i \in \Delta'$ and $\sigma^{-\Delta'} = C(a_i)$ otherwise.
(3) If $\sigma = r(a, b)$, then $\sigma^{-\Delta'} = u(a, b)$ if $a \in \Delta'$, and $\sigma^{-\Delta'} = r(a, b)$ otherwise.

The weakened knowledge base $\mathcal{K}^{-\Delta'}$ of $\mathcal{K}$ w.r.t. Δ' is $\mathcal{K}^{-\Delta'} = \{\sigma^{-\Delta'} \mid \sigma \in \mathcal{K}\}$

Individual-based Revision

Exceptional individual set

A set of exceptional individuals w.r.t. $\mathcal{K}$ *and* $\mathcal{K}'$ is a set $Exc \subseteq \Delta$ such that $\mathcal{K}^{-Exc} \cup \mathcal{K}'$ is consistent. We use $\mathcal{E}(\mathcal{K}, \mathcal{K}')$ to denote the set of all sets of exceptional individuals w.r.t. $\mathcal{K}$ and $\mathcal{K}'$.

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Individual-based Revision

$$\mathcal{K} \circ^{\pi}_{\Delta} \mathcal{K}' = \begin{cases} \text{trans}_{\Delta}(\mathcal{K})^{-\pi(\mathcal{E}(\mathcal{K},\mathcal{K}'))} \cup \mathcal{K}' & \text{if } \mathcal{K}' \text{ is consistent,} \\ \mathcal{K}' & \text{otherwise,} \end{cases}$$

where $\pi : \mathcal{P}(\mathcal{P}(\Delta)) \to \mathcal{P}(\Delta)$ is a selection function retrieving subset-minimal elements, i.e. $\pi(\mathcal{X}) \in \mathcal{X}$ and there is no $Y \in \mathcal{X}$ such that $Y \subset \pi(\mathcal{X})$.

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Proposition

The individual-based change operator $\circ^{\pi}_{\Delta}$ satisfies postulates (G1)-(G3), (G5), and (G6).

Example

$$\mathcal{K} = \{Professor \sqsubseteq Lecturer, Professor(sebastian)\}$$

$$\mathcal{K}' = \{Professor \sqsubseteq \top, Lecturer \sqsubseteq \top, Professor(markus), \neg Lecturer(sebastian)\}$$

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Model-based revision:

$$\mathcal{K} \circ_{\Delta} \mathcal{K}' = \{\top \sqsubseteq (\exists u.(\{markus\} \sqcap Professor)) \sqcap (\exists u.(\{sebastian\} \sqcap Professor)) \sqcap (\exists u.(\{markus\} \sqcap Lecturer)) \sqcap (\exists u.(\{sebastian\} \sqcap \neg Lecturer))\}$$

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Individual-based revision:

$$\mathcal{K} \circ^{\pi}_{\Delta} \mathcal{K}' = \{Professor \sqcap \neg\{sebastian\} \sqsubseteq Lecturer, \top(sebastian), Professor \sqsubseteq \top, Lecturer \sqsubseteq \top, Professor(markus), \neg Lecturer(sebastian)\}$$

Conclusions

- Semantic characterization of AGM revision operator in DL under fixed-domain semantics
- Concrete revision approaches for DL under fixed-domain semantics
  - Model-based approach
  - Individual-based approach by axioms weakening

Future work:

- new axiom construction for model-based revision to be more human-readable
- efficient implementation and evaluation