Exercise 8.1. Transform the following concepts into negation normal form:

(a) \( \neg (A \cap \forall r. B) \)

(b) \( \neg \forall r. \exists s. (\neg B \sqcup \exists r. A) \)

(c) \( \neg ( (\neg A \cap \exists r. T) \sqcup \geq 3 s. (A \cup \neg B) ) \)

Exercise 8.2. Apply the tableau algorithm in order to check if the axiom \( A \sqsubseteq B \) is a logical consequence of the TBox \( \{ \neg C \sqsubseteq B, A \sqcap C \sqsubseteq \bot \} \).

Exercise 8.3. Apply the tableau algorithm in order to check satisfiability of the concept \( A \sqcap \forall r. B \) w.r.t. the TBox \( \{ A \sqsubseteq \exists r. A, B \sqsubseteq \exists r. C, C \sqsubseteq \forall r. \forall r. B \} \).

Exercise 8.4. Markus wants to apply the tableau algorithm for checking the satisfiability of the concept \( B \sqcap \exists r. A \) w.r.t. the TBox \( \{ A \sqsubseteq \exists r. A \sqcap \exists r. B, \top \sqsubseteq 1 r \} \). He arrives at the situation depicted below and concludes that no further rules are applicable, since \( v_2 \) is blocked by \( v_1 \). What is Markus’ error? Continue the algorithm until its termination. (You don’t have to illustrate all intermediate steps, just provide the final state.)

\[
\begin{array}{c}
v_0 \downarrow \\
\quad r^- \\
v_1 \downarrow \\
\quad r^- \\
v_2 \downarrow \\
\end{array}
\]

\[
L(v_0) = \{ B \sqcap \exists r^- . A, B, \exists r^- . A, C_T, \neg A, \leq 1 r \}
\]

\[
L(v_1) = \{ A, C_T, \exists r^- . A, \exists r. B, \leq 1 r \}
\]

\[
L(v_2) = \{ A, C_T, \exists r^- . A, \exists r. B, \leq 1 r \}
\]

Exercise 8.5. Extend the \( \leq 1 \) rule in a way that also qualified functionality axioms of the form \( \top \sqsubseteq 1 r. A \) can be treated correctly, where \( A \) is an atomic concept. Can you also treat arbitrary axioms of the form \( C \sqsubseteq 1 r. D \)?