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Navigating and Querying Answer Sets: How Hard Is It Really and Why?

joint work with: Dominik Rusovac, Markus Hecher, Martin Gebser, and Johannes K Fichte

Hanoi, 7th November 2024

Answer Set Programming (ASP)

knowledge representation

logic programming (and monotonic) reasoning **ASP**

Declarative problem solving

- planning
- product configuration
- diagnosis

logic program answer sets solution **modeling solving interpreting**

problem

⋮

ASP Modelling and Solving

Navigating ASP Solution Spaces

```
#const n=14.
{q(I, 1..n)} = 1:- I = 1..n.
{q(1..n, J)} = 1:- J = 1...n.
:- \{q(D-J,J)\} >=2, D=2..2*n.
:- {q(D+J,J)} > = 2, D=1-n...n-1.
```


Diverse Solutions: Solution: 1

 $q(1,12)$ $q(2,8)$ $q(3,6)$ $q(4,14)$ $q(5,9)$ $q(6,2)$ $q(7,5)$ q(14,1) q(9,11) q(10,7) q(11,10) q(12,4) q(13,13) q(8,3) Solution: 2 q(1,1) $q(2,10)$ $q(3,5)$ $q(4,7)$ $q(5,12)$ $q(6,3)$ $q(7,11)$ q(8,2) q(9,14) q(10,9) q(11,4) q(12,13) q(13.8) q(14.6) Solution: 3 q(1,11) $q(2,2)$ $q(3,10)$ $q(4,6)$ $q(5,3)$ $q(6,1)$ $q(7,13)$ q(8,7) q(9,12) q(10,14) q(11,8) q(12,5) q(13,9) q(14,4)

* zoom out

* ...

Visual Approach: * zoom in

* zoom out

Weighted Faceted Answer Set Navigation

[1] Johannes Klaus Fichte, Sarah Alice Gaggl, Dominik Rusovac. **Rushing and Strolling among Answer Sets - Navigation Made Easy** Proceedings of the 36th AAAI Conference on Artificial Intelligence (AAAI 2022), 2022.

	$\overline{2}$		5		$\mathbf 1$		$\mathbf{9}$	
8			\overline{c}		3			$\boldsymbol{6}$
	3			6			$\overline{7}$	
		$\mathbf{1}$						
5	$\overline{4}$						$\mathbf{1}$	9
						$\overline{7}$		
	9			3			8	
\overline{c}			8		4			$\overline{7}$
	$\mathbf{1}$		$\overline{9}$		7		6	

How to solve this Sudoku as quick as possible?

Which moves (queens) have the least/most impact?

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How to solve this Sudoku as quick as possible?

Which moves (queens) have the least/most impact?

How can we find answers without going through all solutions?

Outline

- Preliminaries
- Weighted Faceted Navigation
- Complexity Results for Facet Reasoning
- Querying Answer Sets
- Conclusion

Preliminaries

Definition (logic program)

A (normal disjunctive) logic program Π over a set of atoms $\{\alpha_0, \ldots, \alpha_n\}$ is a finite set of rules r of the form:

$$
\alpha_0 \,|\, \ldots \,|\, \alpha_k \leftarrow \alpha_{k+1}, \ldots, \alpha_m, \mathop{\sim} \alpha_{m+1}, \ldots, \mathop{\sim} \alpha_n. \text{ where } 0 \leq k \leq m \leq n
$$

Remark: We focus on ground programs without extended rules.

AS(Π) *. . .* **answer sets** (solutions) $2^{\mathcal{AS}(\Pi)}$ \dots solution space $BC(\Pi) = |$ $\bigcup AS(\Pi) \dots$ brave consequences *α* ∈ BC(Π) *. . .* **partial solution** $\mathcal{CC}(\Pi) \coloneqq \bigcap \mathcal{AS}(\Pi) \dots$ cautious consequences

Part 1 [Weighted Faceted Navigation](#page-8-0)

 Π : a | b. c | d ← b. e.

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Facets: $\mathcal{F}(\Pi) = \{a, b, c, d, \overline{a}, \overline{b}, \overline{c}, \overline{d}\}\$

ANSWER 1: a, e ANSWER 2: b, c, e ANSWER 3: b, d, e

 Π : a | b. c | d ← b. e. **Facets**: $\mathcal{F}(\Pi) = \{a, b, c, d, \overline{a}, \overline{b}, \overline{c}, \overline{d}\}\$ **Routes**: Δ^{Π} := $\{(f_0, \ldots, f_n) | f_i \in \mathcal{F}(\Pi), 0 \leq i \leq n\} \cup \{\epsilon\}$ ANSWER 1: a, e ANSWER 2: b, c, e ANSWER 3: b, d, e

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slide 9 of 24

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Facets: $\mathcal{F}(\Pi) = \{a, b, c, d, \overline{a}, \overline{b}, \overline{c}, \overline{d}\}\$

Routes: Δ^{Π} $\coloneqq \{ \langle f_0, \ldots, f_n \rangle \mid f_i \in \mathcal{F}(\Pi), 0 \leq i \leq n \} \cup \{ \epsilon \}$

ANSWER 1: a, e ANSWER 2: b, c, e ANSWER 3: b, d, e

 Π : a | b. c | d ← b. e.

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Routes: Δ^{Π} := $\{(f_0, \ldots, f_n) | f_i \in \mathcal{F}(\Pi), 0 \leq i \leq n\} \cup \{\epsilon\}$

ANSWER 1: a, e $\overline{ANSWER-2}$ \leftarrow $\overline{b}, \overline{c}, \overline{e}$ $\overbrace{\text{ANSWEB-3:}\to 0,0}$

 Π : a | b. c | d ← b. e.

Facets: $\mathcal{F}(\Pi) = \{a, b, c, d, \overline{a}, \overline{b}, \overline{c}, \overline{d}\}\$

Routes: Δ^{Π} = $\{(f_0, \ldots, f_n) | f_i \in \mathcal{F}(\Pi), 0 \leq i \leq n\} \cup \{\epsilon\}$

ANSWER 1: a, e ANSWER 2: b, c, e ANSWER $3: b, d, e$

 Π : a | b. c | d ← b. e.

Facets: $\mathcal{F}(\Pi) = \{a, b, c, d, \overline{a}, \overline{b}, \overline{c}, \overline{d}\}\$

Routes: Δ^{Π} $\coloneqq \{ \langle f_0, \ldots, f_n \rangle \mid f_i \in \mathcal{F}(\Pi), 0 \leq i \leq n \} \cup \{ \epsilon \}$

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slide 9 of 24

What is the effect of taking a certain navigation step?

Can we somehow characterize sub-spaces beforehand?

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What is the effect of taking a certain navigation step?

Can we somehow characterize sub-spaces beforehand?

Let's do some counting!

Quantifying effects of navigation steps

The Weight of a Facet

Definition (weighting function)

We call $\#\colon \{\Pi^\delta\mid \delta\in\Delta^\Pi\}\to\mathbb{N}$ *weighting function*, whenever $\#(\Pi^\delta) > 0$, if $|\mathcal{AS}(\Pi)|\geq 2.$

Definition (weight)

Let $\delta \in \Delta^{\Pi}$, $f \in \mathcal{F}(\Pi)$ and δ' be a redirection of δ w.r.t. f. The *weight* of f w.r.t. #, Π^{δ} and δ' is defined as:

$$
\omega_{\#}(f,\Pi^{\delta},\delta')\coloneqq \begin{cases} \#(\Pi^{\delta})-\#(\Pi^{\delta'}),& \text{ if } \langle \delta,f\rangle \notin \Delta_{\mathrm{s}}^{\Pi} \text{ and } \delta'\neq \epsilon; \\ \#(\Pi^{\delta})-\#(\Pi^{\langle \delta,f\rangle}),& \text{ otherwise.}\end{cases}
$$

Facet Counting Weight

Count Facets with $ω_{\#π}$

- $\omega_{\#_{\mathcal{F}}}$ provides information on the similarity/diversity of solutions
- $-$ Outputting the facet-counting weight $\omega_{\#_{\mathcal{F}}}$ for a given program Π and route δ is in $\Delta^\text{P}_3[1]$
- Precise computational complexity of facet problems remained widely open

```
AS(\Pi) = \{\{a, e\}, \{b, c, e\}, \{b, d, e\}\}\\mathcal{AS}(\Pi^{\{b\}}) = \{\{b,c,e\},\{b,d,e\}\}\\omega_{\# \mathcal{F}}(\mathbf{b}, \Pi, \epsilon) = 4
```

```
AS(\Pi) = \{\{a, e\}, \{b, c, e\}, \{b, d, e\}\}\\mathcal{AS}(\Pi^{\langle \overline{c} \rangle}) = \{\{\text{a},\text{e}\},\{\text{b},\text{d},\text{e}\}\}\omega_{\# \mathcal{F}}(\overline{\text{c}}, \Pi, \epsilon) = 2
```
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Rushing and Strolling among Answer Sets

Definition (strictly goal-oriented navigation mode)

The *stricly goal-oriented* navigation mode $\nu_{\rm sgo}^{\#}$: $\Delta_{\rm s}^{\Pi} \times \mathcal{F}(\Pi) \to 2^{\mathcal{AS}(\Pi)}$ is defined by:

 $\nu_{\rm sgo}^{\#}(\delta, f) \coloneqq \Bigg\{$ $\mathcal{AS}(\Pi^{\langle\delta,\mathrm{f}\rangle}), \text{ if } \mathrm{f} \in \max_{\omega_{\#}}(\Pi^{\delta});$ $\mathcal{AS}(\Pi^{\delta}),$ otherwise.

Definition (explore navigation mode)

The *explore* navigation mode $\nu^\#_{\rm expl}$: $\Delta^{\Pi}_{\rm s} \times \mathcal{F}(\Pi) \to 2^{\mathcal{AS}(\Pi)}$ is defined by:

$$
\nu_{\text{expl}}^{\#}(\delta,f)\coloneqq \begin{cases} \mathcal{AS}(\Pi^{\{\delta,f\}}), & \text{if } f\in\min_{\omega_{\#}}(\Pi^{\delta}); \\ \mathcal{AS}(\Pi^{\delta}), & \text{otherwise}. \end{cases}
$$

Part 2 [Complexity Results for Facet Reasoning](#page-23-0)

Complexity Results

Part 3 [Querying Answer Sets](#page-25-0)

Querying Solution Spaces

- Propositional queries to select answer sets matching specific conditions
- Program Π, prop. formula F, answer sets of Π that satisfy F $\sigma_F(\Pi) := \{ M \in \mathcal{AS}(\Pi) \mid M \in F \}$
- Express propositional query itself in ASP

Example

Π¹ = {p ← ∼q; q ← ∼p; r ∨ s ← q; t ← }.

The answer sets of Π_1 are $\mathcal{AS}(\Pi_1) = \{\{p, t\}, \{q, r, t\}, \{q, s, t\}\}\$, and $\mathcal{F}(\Pi_1) = \{p, q, r, s\}$.

Querying Solution Spaces

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Example

 $\Pi_1 = \{p \leftarrow \sim q; q \leftarrow \sim p; r \vee s \leftarrow q; t \leftarrow \}.$

The answer sets of Π_1 are $\mathcal{AS}(\Pi_1) = \{\{p, t\}, \{q, r, t\}, \{q, s, t\}\}\$, and $\mathcal{F}(\Pi_1) = \{p, q, r, s\}$.

Query for $F = (-r \wedge s) \vee (p \wedge \neg q)$?

Querying Solution Spaces

- Propositional queries to select answer sets matching specific conditions
- Program Π, prop. formula F, answer sets of Π that satisfy F $\sigma_F(\Pi)$:= {M \in *AS*(Π) | M \in F}
- Express propositional query itself in ASP

Example

 $\Pi_1 = \{p \leftarrow \sim q; q \leftarrow \sim p; r \vee s \leftarrow q; t \leftarrow \}.$

The answer sets of Π_1 are $\mathcal{AS}(\Pi_1) = \{\{p, t\}, \{q, r, t\}, \{q, s, t\}\}\$, and $\mathcal{F}(\Pi_1) = \{p, q, r, s\}$.

$$
Query for F = (\neg r \land s) \lor (p \land \neg q)?
$$

$$
\sigma_{(\neg r \land s) \lor (p \land \neg q)}(\Pi_1) = \{\{p, t\}, \{q, s, t\}\}\
$$

Matching all Elements

Terms

$\sigma_{\Lambda_{\ell-1},\ell}(\Pi) = \mathcal{AS}(\Pi \cup \{\leftarrow \sim \ell \mid \ell \in \mathbb{L}\})$

Π program, L set of literals

Matching all Elements

Terms

$$
\sigma_{\bigwedge_{\ell\in\mathbf{L}}\ell}(\Pi)=\mathcal{AS}\big(\Pi\cup\{\leftarrow\sim\ell\mid\ell\in\mathbf{L}\}\big)
$$

Π program, L set of literals

Example

$$
\Pi_1=\{p\leftarrow \sim\!\! q;\; q\leftarrow \sim\!\! p;\; r\vee s\leftarrow q;\; t\leftarrow \},\; \mathcal{AS}(\Pi_1)=\{\{p,t\},\;\{q,r,t\},\;\{q,s,t\}\}
$$

 $\sigma_{\text{d} \wedge \text{d} \alpha}(\Pi_1) = \mathcal{AS}(\Pi_1 \cup \{\leftarrow \neg p; \leftarrow q\}) = \{\{p, t\}\}\$

Matching at least one Element

Clauses

 $\sigma_{\bigvee_{\ell \in \mathbb{L}} \ell}(\Pi) = \mathcal{AS}(\Pi \cup \{\leftarrow \sim \mathbb{L}\})$

Π program, L set of literals, ∼L ∶= {∼*`* ∣ *`* ∈ L}

Matching at least one Element

Clauses

$$
\sigma_{\bigvee_{\ell\in\mathcal{L}}\ell}(\Pi)=\mathcal{AS}(\Pi\cup\{\leftarrow\sim\!\mathcal{L}\})
$$

 Π program, L set of literals, $∼L = \{ ∞ \ell \mid \ell ∈ L\}$

Example

 $\Pi_1 = \{p \leftarrow \sim q; q \leftarrow \sim p; r \vee s \leftarrow q; t \leftarrow \}$, $\mathcal{AS}(\Pi_1) = \{\{p, t\}, \{q, r, t\}, \{q, s, t\}\}\$

 $\sigma_{s\vee \neg \alpha}(\Pi_1) = \mathcal{AS}(\Pi_1 \cup \{\leftarrow \neg s, q\}) = \{\{p, t\}, \{q, s, t\}\}\$

CNFs

$\sigma_{\rm F}(\Pi) = \mathcal{AS}(\Pi \cup \{\leftarrow \sim L_{\rm i} \mid L_{\rm i} \in \mathrm{F}\})$

Π program, L set of literals, F a simple formula in CNF, F = {L1, *. . .* , Lm} set of clauses

Matching CNFs

CNFs

$$
\sigma_F\big(\Pi\big)=\mathcal{AS}\big(\Pi\cup\{\leftarrow\sim\!L_i\bigm|L_i\in F\big\}\big)
$$

 Π program, L set of literals, F a simple formula in CNF, $F = \{L_1, \ldots, L_m\}$ set of clauses

Example

$$
\Pi_1 = \{p \leftarrow \sim q;\ q \leftarrow \sim p;\ r \vee s \leftarrow q;\ t \leftarrow \},\ \mathcal{AS}(\Pi_1) = \{\{p,t\},\ \{q,r,t\},\ \{q,s,t\}\}\
$$

 $\sigma_{(p \vee \neg q) \wedge (\neg s \vee r)}(\Pi_1) = \mathcal{AS}(\Pi_1 \cup \{\leftarrow \neg p, q; \leftarrow s, \neg r\}) = \{\{p, t\}, \{q, r, t\}\}\$

Matching DNFs

DNFs

 $\sigma_F(\Pi) = \{M \setminus \{a_1, \ldots, a_m\} \mid M \in \mathcal{AS}(\Pi \cup \{a_i, \leftarrow \sim \ell \mid 1 \leq i \leq m, \ell \in L_i\} \cup \{\leftarrow a_1, \ldots, a_m\})\}\$

where a_1, \ldots, a_m are fresh atoms

 Π program, L set of literals, F a simple formula in DNF, $F = \{L_1, \ldots, L_m\}$ set of terms

Matching DNFs

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Example

$$
\Pi_1=\{p\leftarrow \sim\!\! q;\; q\leftarrow \sim\!\! p;\; r\vee s\leftarrow q;\; t\leftarrow \},\; \mathcal{AS}(\Pi_1)=\{\{p,t\},\;\{q,r,t\},\;\{q,s,t\}\}
$$

 σ _{(¬r}∧s) \vee (p∧¬q)(Π_1) = {M \ {a₁, a₂} | M \in AS($\Pi_1 \cup$ {a₁ ← r; a₁ ← ~s; a₂ ← ~p; a₂ ← q; ← a₁, a₂}}} $= \{ \{p, t\}, \{q, s, t\} \}$

fasb – **F**aceted **A**nswer **S**et **B**rowser

REPL on top of clingo solver implementing: $\nu_{\rm go}$, $\nu_{\rm sgo}^{\#}$, $\nu_{\rm exp1}^{\#}$ for $\#\in\{\#_{\mathcal{AS}},\#\mathcal{F}\}$ <https://github.com/drwadu/fasb>

<https://drwadu.github.io/web-fasb.github.io/>

Summary & Future Work

Summary:

- Weighted faceted navigation allows to quantitatively explore the solution space
- Complexity results for facet reasoning
- Empirical evaluation of facet reasoning (in paper)
- Extend facet reasoning to queries on ASP solution spaces

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Future Work:

- We expect facet reasoning to be of interest for various formalisms in KR and AI (QBFs, planning, argumentation, DL, epistemic logic programming, constraint programming and paraconsistent reasoning
- Characterise practical applications for facet reasoning, while approximate or exact solution counting would be required otherwise
- Investigate the complexity of facets in the presence of preferences

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