The Belief Bias Effect under the Weak Completion Semantics

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A Syllogistic Reasoning Task by Evans, Barston, and Pollard [1983]

Let's consider S_{dog}

PREMISE 1	*No police dogs are vicious.*
PREMISE 2	*Some highly trained dogs are vicious.*
CONCLUSION	*Therefore, some highly trained dogs are not police dogs.*

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Evans, Barston, and Pollard [1983] call this phenomenon the belief bias effect

It occurs when we think to be judging something based on our reasoning, but are actually influenced by our beliefs and our prior knowledge.

Possible Explanations

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- The atmosphere of the premises influences the acceptance for the conclusions.
- Humans heuristically accept any syllogism having a believable conclusion and only check on the logic if the conclusion contradicts their beliefs.
- If the conclusion is neutral or believable, humans attempt to construct a model that supports it; otherwise, they attempt to construct a model, which rejects it.

Adequate Framework for Human Reasoning

The belief bias can take effect on two stages:

1. Beliefs can influence our interpretation of the premises.
2. Beliefs can determine whether we search for alternative models.

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Stenning and van Lambalgen (2008) propose a two step process:
Human reasoning should be modeled by

1. Reasoning towards an appropriate representation,
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Classical logic cannot adequately represent the syllogistic reasoning task.

Syllogism S_{dog}

Premise 1	*No police dogs are vicious.*
Premise 2	*Some highly trained dogs are vicious.*
Conclusion	*Therefore, some highly trained dogs are not police dogs.*

Reasoning Towards an Appropriate Logical Form

The first premise of S_{dog} is

No police dogs are vicious.

1. $\neg \exists X(police\_dog(X) \wedge vicious(X))$

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3.	$\forall X(\neg police\_dog(X) \vee \neg vicious(X))$	because	$p \rightarrow q$	$\equiv$	$\neg p \vee q$
4.	$\forall X(police\_dog(X) \rightarrow \neg vicious(X))$				

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3.	$\forall X(\neg police\_dog(X) \vee \neg vicious(X))$	because	$p \rightarrow q$	$\equiv$	$\neg p \vee q$
4.	$\forall X(police\_dog(X) \rightarrow \neg vicious(X))$	because	$p \rightarrow q$	$\equiv$	$\neg q \rightarrow \neg p$
		and	$\neg\neg p$	$\equiv$	p
5.	$\forall X(vicious(X) \rightarrow \neg police\_dog(X))$				

Reasoning Towards an Appropriate Logical Form

The first premise of S_{dog} is

No police dogs are vicious.

If something is vicious, then it is not a police dog.

and *If something is a police dog, then it is not vicious.*

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The first premise of S_{dog} is

No police dogs are vicious.

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If something is vicious, then it is not a police dog.
If something is a police dog, then it is not vicious.

We obtain the following preliminary representation of the first premise of S_{dog}:

$$police\_dog'(X) \leftarrow vicious(X), \qquad police\_dog(X) \leftarrow \neg police\_dog'(X),$$

where $police\_dog(X)$, $police\_dog'(X)$, and $vicious(X)$ denote that X is a police dog, X is not a police dog, and X is vicious, respectively.

Reasoning Towards an Appropriate Logical Form

Stenning and van Lambalgen [2005, 2008] suggest to implement conditionals by default licenses for implications. PREMISE 1 in S_{dog} can be modeled accordingly:

If something is vicious and not abnormal (in that respect),
then it is not a police dog.
Nothing (by default) is abnormal (regarding the previous sentence).

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This information for PREMISE 1 in S_{dog} can now be encoded as:

$$\begin{array}{rcl} police\_dog'(X) & \leftarrow & vicious(X) \wedge \neg ab_{dog'}(X), \\ police\_dog(X) & \leftarrow & \neg police\_dog'(X), \\ ab_{dog'}(X) & \leftarrow & \bot. \end{array}$$

Syllogism S_{dog}

PREMISE 2 states that there are some highly trained dogs that are vicious.
That presupposes that there actually exists something, e.g. a, for which it holds that:

$$highly\_trained(a) \leftarrow \top \qquad \text{and} \qquad vicious(a) \leftarrow \top.$$

$\mathcal{P}_{dog}$ represents the first two premises of S_{dog}:

$$\begin{array}{rcl} police\_dog'(X) & \leftarrow & vicious(X) \wedge \neg ab_{dog'}(X), \\ police\_dog(X) & \leftarrow & \neg police\_dog'(X), \\ ab_{dog'}(X) & \leftarrow & \bot, \\ highly\_trained(a) & \leftarrow & \top, \\ vicious(a) & \leftarrow & \top. \end{array}$$

Reasoning with respect to Least Models

$\mathcal{P}_{dog}$ represents S_{dog}. Its weak completion, $\mathrm{wc\,g}\,\mathcal{P}_{dog}$, is:

$$
\begin{array}{rcl}
police\_dog'(a) & \leftrightarrow & vicious(a) \wedge \neg ab_{dog'}(a), \\
police\_dog(a) & \leftrightarrow & \neg police\_dog'(a), \\
ab_{dog'}(a) & \leftrightarrow & \bot, \\
highly\_trained(a) & \leftrightarrow & \top, \\
vicious(a) & \leftrightarrow & \top.
\end{array}
$$

Its least model is:

$$\langle \{highly\_trained(a), vicious(a), police\_dog'(a)\}, \{police\_dog(a), ab_{dog'}(a)\} \rangle.$$

This model entails the CONCLUSION: *Some highly trained dogs are not police dogs*.

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PREMISE 1	*No nutritional things are inexpensive.*
PREMISE 2	*Some vitamin tablets are inexpensive.*
CONCLUSION	*Therefore, some vitamin tablets are not nutritional.*

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It is commonly known that

The purpose of vitamin tablets is to aid nutrition.

This belief and the clause representing PREMISE 1 leads to

If something is a vitamin tablet, then it is abnormal (regarding PREMISE 1 of S_{vit}).

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$\mathcal{P}_{vit}$ represents PREMISE 1 and PREMISE 2 together with the background knowledge:

$$\begin{aligned}
nutritional'(X) &\leftarrow inex(X) \wedge \neg ab_{nut'}(X), \\
nutritional(X) &\leftarrow \neg nutritional'(X), \\
ab_{nut'}(X) &\leftarrow \bot, \\
ab_{nut'}(X) &\leftarrow vitamin(X), \\
vitamin(a) &\leftarrow \top, \\
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After the first step of the weak completion transformation:

$$\begin{array}{rcl} nutritional'(a) & \leftarrow & inex(a) \wedge \neg ab_{nut'}(a), \\ nutritional(a) & \leftarrow & \neg nutritional'(a), \\ ab_{nut'}(a) & \leftarrow & \bot \vee vitamin(a), \\ \\ vitamin(a) & \leftarrow & \top, \\ inex(a) & \leftarrow & \top. \end{array}$$

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After the second step of the weak completion transformation, we have $\mathrm{wc}\,\mathrm{g}\,\mathcal{P}_{vit}$:

$$\begin{array}{rcl} nutritional'(a) & \leftrightarrow & inex(a) \wedge \neg ab_{nut'}(a), \\ nutritional(a) & \leftrightarrow & \neg nutritional'(a), \\ ab_{nut'}(a) & \leftrightarrow & \bot \vee vitamin(a), \\ & & \\ vitamin(a) & \leftrightarrow & \top, \\ inex(a) & \leftrightarrow & \top. \end{array}$$

Syllogism S_{vit}

There are two kinds of participants each taking a different interpretation of S_{vit}. The group that invalidated the syllogism has been influenced by their belief. $\mathsf{lm}_{\mathsf{L}}\mathsf{wc}\,\mathcal{P}_{vit}$ is:

$$\langle\{vitamin(a), inex(a), nutritional(a), ab_{nut'}(a)\}, \{nutritional'(a)\}\rangle.$$

The CONCLUSION of S_{vit} is not entailed.

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The Conclusion of S_{vit} is not entailed.

The other group was not influenced. $\mathsf{lm}_{Ł}\mathsf{wc}\,(\mathcal{P}_{vit} \setminus \{ab_{nut'}(X) \leftarrow vitamin(X)\})$ is:

$$\langle\{vitamin(a), inex(a), nutritional'(a)\}, \{nutritional(a), ab_{nut'}(a)\}\rangle,$$

which entails the conclusion, that some vitamin tables are not nutritional.

The Second Part of Evans, Barston and Pollard Syllogistic Reasoning Task

Let's consider S_{mil}

PREMISE 1	*No millionaires are hard workers.*
PREMISE 2	*Some rich people are hard workers.*
CONCLUSION	*Therefore, some millionaires are not rich people.*

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Both Syllogisms can be modeled with abduction.

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3. $\forall X(\neg add(X) \lor \neg inex(X))$ because $A \rightarrow B \equiv \neg A \lor B$
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PREMISE 1	*No addictive things are inexpensive.*
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4.	$\forall X(add(X) \rightarrow \neg inex(X))$	because	$A \rightarrow B$	$\equiv$	$\neg B \rightarrow \neg A$
			and $\neg\neg A$	$\equiv$	A
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The first premise implies that

If something costs  or less then not addictive. (1)

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If  then it costs . (2)

And additionally it is common knowledge that

If  then addictive. (3)

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PREMISE 1	*No addictive things are inexpensive.*
PREMISE 2	*Some cigarettes are inexpensive.*
CONCLUSION	*Therefore, some addictive things are not cigarettes.*

The first premise implies that

If something costs  or less then not addictive. (1)

From the second premise, we generalize to

If  then it costs . (2)

And additionally it is common knowledge that

If  then addictive. (3)

The first part of the conclusion is about addictive things.

Explaining the Addictive Things

PREMISE 1	*No addictive things are inexpensive.*
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The first part of the conclusion is about addictive things.

1. According to (1), they cannot cost  or less.
2. As they cannot cost  or less according to (2), they cannot be .

Modeling S_{cig}

Consider S_{cig} again

PREMISE 1	*No addictive things are inexpensive.*	$add'(X) \leftarrow inex(X) \wedge \neg ab(X)$ $add(X) \leftarrow \neg add'(X)$ $ab(X) \leftarrow \bot$
PREMISE 2	*Some cigarettes are inexpensive.*	$cig(a) \leftarrow \top \quad inex(a) \leftarrow \top$

Modeling S_{cig}

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PREMISE 1	*No addictive things are inexpensive.*	$add'(X) \leftarrow inex(X) \wedge \neg ab(X)$ $add(X) \leftarrow \neg add'(X)$ $ab(X) \leftarrow \bot$
PREMISE 2	*Some cigarettes are inexpensive.*	$cig(a) \leftarrow \top \quad inex(a) \leftarrow \top$

We generalize PREMISE 2 and state that cigarettes are abnormal

Cigarettes are inexpensive.	$inex(X) \leftarrow cig(X)$
Cigarettes are abnormal.	$ab(X) \leftarrow cig(X)$

Modeling S_{cig}

Consider S_{cig} again

PREMISE 1	*No addictive things are inexpensive.*	$add'(X) \leftarrow inex(X) \wedge \neg ab(X)$ $add(X) \leftarrow \neg add'(X)$ $ab(X) \leftarrow \bot$
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Cigarettes are inexpensive.	$inex(X) \leftarrow cig(X)$
Cigarettes are abnormal.	$ab(X) \leftarrow cig(X)$

Additionally, to explain why people validate S_{cig} we assume that they use abduction.

CONCLUSION *Some addictive things are not cigarettes.*

Modeling S_{cig}

Consider S_{cig} again

PREMISE 1	*No addictive things are inexpensive.*	$add'(X) \leftarrow inex(X) \wedge \neg ab(X)$ $add(X) \leftarrow \neg add'(X)$ $ab(X) \leftarrow \bot$
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Additionally, to explain why people validate S_{cig} we assume that they use abduction.

CONCLUSION	*Some addictive things are not cigarettes.*	$add(b)$

1. We know that there are addictive things, let's say b.

   b is addictive.

Modeling S_{cig}

Consider S_{cig} again

PREMISE 1	*No addictive things are inexpensive.*	$add'(X) \leftarrow inex(X) \wedge \neg ab(X)$ $add(X) \leftarrow \neg add'(X)$ $ab(X) \leftarrow \bot$
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Cigarettes are inexpensive.	$inex(X) \leftarrow cig(X)$
Cigarettes are abnormal.	$ab(X) \leftarrow cig(X)$

Additionally, to explain why people validate S_{cig} we assume that they use abduction.

CONCLUSION *Some addictive things are not cigarettes.* $add(b) \wedge \neg inex(b)$

1. We know that there are addictive things, let's say *b*.

   b is addictive.

2. Given PREMISE 1, these addictive things cannot be inexpensive.

   b is not inexpensive.

Modeling S_{cig}

Consider S_{cig} again

PREMISE 1	*No addictive things are inexpensive.*	$add'(X) \leftarrow inex(X) \wedge \neg ab(X)$ $add(X) \leftarrow \neg add'(X)$ $ab(X) \leftarrow \bot$
PREMISE 2	*Some cigarettes are inexpensive.*	$cig(a) \leftarrow \top \quad inex(a) \leftarrow \top$

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Cigarettes are inexpensive.	$inex(X) \leftarrow cig(X)$
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Additionally, to explain why people validate S_{cig} we assume that they use abduction.

CONCLUSION	*Some addictive things are not cigarettes.*	$add(b) \wedge \neg inex(b) \wedge \neg cig(b)$

1. We know that there are addictive things, let's say *b*.

   b is addictive.

2. Given PREMISE 1, these addictive things cannot be inexpensive.

   b is not inexpensive.

3. we credulously abduce, that these cannot be the cigarettes.

Modeling S_{cig}

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PREMISE 1	*No addictive things are inexpensive.*	$add'(X) \leftarrow inex(X) \wedge \neg ab(X)$ $add(X) \leftarrow \neg add'(X)$ $ab(X) \leftarrow \bot$
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2. Given PREMISE 1, these addictive things cannot be inexpensive.

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3. we credulously abduce, that these cannot be the cigarettes.

   b is not a cigarette.

Syllogism S_{mil}

PREMISE 1	*No millionaires are hard workers.*
PREMISE 2	*Some rich people are hard workers.*
CONCLUSION	*Therefore, some millionaires are not rich people.*

Modeling S_{mil}

PREMISE 1 *No millionaires are hard workers.*	$mil'(X)$	$\leftarrow$	$hard\_worker(X) \wedge \neg ab(X)$
	$mil(X)$	$\leftarrow$	$\neg mil'(X)$
	$ab(X)$	$\leftarrow$	$\bot$
PREMISE 2 *Some rich people are hard workers.*	$rich(a)$	$\leftarrow$	$\top$, $hard\_worker(a) \leftarrow \top$

Modeling S_{mil}

PREMISE 1 *No millionaires are hard workers.*	$mil'(X) \leftarrow hard\_worker(X) \wedge \neg ab(X)$
	$mil(X) \leftarrow \neg mil'(X)$
	$ab(X) \leftarrow \bot$
PREMISE 2 *Some rich people are hard workers.*	$rich(a) \leftarrow \top$, $hard\_worker(a) \leftarrow \top$

Let us analogously to S_{cig} assume the generalization of PREMISE 2 as

Rich people are hard workers.	$hard\_worker(X) \leftarrow rich(X)$

Modeling S_{mil}

PREMISE 1 *No millionaires are hard workers.*	$mil'(X)$	$\leftarrow hard\_worker(X) \wedge \neg ab(X)$
	$mil(X)$	$\leftarrow \neg mil'(X)$
	$ab(X)$	$\leftarrow \bot$
PREMISE 2 *Some rich people are hard workers.*	$rich(a)$	$\leftarrow \top$, $hard\_worker(a) \leftarrow \top$

Let us analogously to S_{cig} assume the generalization of PREMISE 2 as

Rich people are hard workers. $hard\_worker(X) \leftarrow rich(X)$

Additionally, to explain why people validate S_{mil} we assume that they use abduction.

CONCLUSION *Some millionaires are not rich.*

Modeling S_{mil}

PREMISE 1 *No millionaires are hard workers.*	$mil'(X)$	$\leftarrow hard\_worker(X) \wedge \neg ab(X)$
	$mil(X)$	$\leftarrow \neg mil'(X)$
	$ab(X)$	$\leftarrow \bot$
PREMISE 2 *Some rich people are hard workers.*	$rich(a)$	$\leftarrow \top$, $hard\_worker(a) \leftarrow \top$

Let us analogously to S_{cig} assume the generalization of PREMISE 2 as

Rich people are hard workers. $hard\_worker(X) \leftarrow rich(X)$

Additionally, to explain why people validate S_{mil} we assume that they use abduction.

CONCLUSION *Some millionaires are not rich.* $mil(b)$

1. We know that there are millionaires, let's say b.

b is a millionaire.

Modeling S_{mil}

PREMISE 1 *No millionaires are hard workers.*	$mil'(X) \leftarrow hard\_worker(X) \wedge \neg ab(X)$
	$mil(X) \leftarrow \neg mil'(X)$
	$ab(X) \leftarrow \bot$
PREMISE 2 *Some rich people are hard workers.*	$rich(a) \leftarrow \top,\ hard\_worker(a) \leftarrow \top$

Let us analogously to S_{cig} assume the generalization of PREMISE 2 as

Rich people are hard workers.	$hard\_worker(X) \leftarrow rich(X)$

Additionally, to explain why people validate S_{mil} we assume that they use abduction.

CONCLUSION *Some millionaires are not rich.*	$mil(b) \wedge \neg hard\_worker(b)$

1. We know that there are millionaires, let's say *b*.

   b is a millionaire.

2. Given PREMISE 1, these millionaires cannot be hard workers.

   b is not a hard worker.

Modeling S_{mil}

PREMISE 1	*No millionaires are hard workers.*	$mil'(X) \leftarrow hard\_worker(X) \wedge \neg ab(X)$
		$mil(X) \leftarrow \neg mil'(X)$
		$ab(X) \leftarrow \bot$
PREMISE 2	*Some rich people are hard workers.*	$rich(a) \leftarrow \top, hard\_worker(a) \leftarrow \top$

Let us analogously to S_{cig} assume the generalization of PREMISE 2 as

Rich people are hard workers. $hard\_worker(X) \leftarrow rich(X)$

Additionally, to explain why people validate S_{mil} we assume that they use abduction.

CONCLUSION *Some millionaires are not rich.* $mil(b) \wedge \neg hard\_worker(b) \wedge \neg rich(b)$

1. We know that there are millionaires, let's say *b*.

   b is a millionaire.

2. Given PREMISE 1, these millionaires cannot be hard workers.

   b is not a hard worker.

3. By the generalization of PREMISE 2, we abduce that *b* cannot be rich.

Modeling S_{mil}

PREMISE 1	*No millionaires are hard workers.*	$mil'(X) \leftarrow hard\_worker(X) \wedge \neg ab(X)$
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Let us analogously to S_{cig} assume the generalization of PREMISE 2 as

Rich people are hard workers. $hard\_worker(X) \leftarrow rich(X)$

Additionally, to explain why people validate S_{mil} we assume that they use abduction.

CONCLUSION *Some millionaires are not rich.* $mil(b) \wedge \neg hard\_worker(b) \wedge \neg rich(b)$

1. We know that there are millionaires, let's say *b*.

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2. Given PREMISE 1, these millionaires cannot be hard workers.

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Modeling S_{mil}

PREMISE 1 *No millionaires are hard workers.*	$mil'(X) \leftarrow hard\_worker(X) \wedge \neg ab(X)$
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CONCLUSION *Some millionaires are not rich.* $mil(b) \wedge \neg hard\_worker(b) \wedge \neg rich(b)$

1. We know that there are millionaires, let's say *b*.

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2. Given PREMISE 1, these millionaires cannot be hard workers.

   b is not a hard worker.

3. By the generalization of PREMISE 2, we abduce that *b* cannot be rich.

   b is not rich.

Even though not tested yet, our hypothesis is, while checking S_{mil}, participants did not make this assumption and thus, had not been influenced by the belief-bias effect.

Conclusions

Yet another human reasoning task can be modeled
under the weak completion semantics.

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under the weak completion semantics.

Open Questions

- The weak completion semantics differs wrt other approaches, in the way that the undefined atoms in the program stay unknown, instead of becoming false.
- Syllogistic reasoning tasks never give the option I don't know to the participants.

Thank you very much for your attention!

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Logic Programs

We restrict ourselves to datalog programs. A logic program $\mathcal{P}$ is a finite set of clauses

$$A \leftarrow A_1 \wedge \ldots \wedge A_n \wedge \neg B_1 \wedge \ldots \wedge \neg B_m, \quad (1)$$

$$A \leftarrow \bot, \quad (2)$$

- where A and A_i, $0 \leq i \leq n$, are atoms and $\neg B_j$, $1 \leq j \leq m$, are negated atoms.
- If $i = 0$, then we write $A \leftarrow \top$, which is called a positive fact.
- A clause of the form (2) is called a negative fact.
- A is undefined if it is not the head of any clause.
- $\mathrm{g}\,\mathcal{P}$ denotes ground $\mathcal{P}$, that is, it contains all ground instances of its clauses.
- $\mathrm{undef}(\mathcal{P})$ is the set of all undefined atoms in $\mathrm{g}\,\mathcal{P}$.

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The following transformation is the weak completion of $\mathcal{P}$

1. Replace all clauses in $\mathrm{g}\,\mathcal{P}$ with the same head $A \leftarrow body_1, \ldots, A \leftarrow body_n$ by the single expression $A \leftarrow body_1 \vee \ldots \vee body_n$.
2. Replace all occurrences of $\leftarrow$ by $\leftrightarrow$.

Three-Valued Łukasiewicz Logic

	$\neg$
$\top$	$\bot$
$\bot$	$\top$
U	U

$\wedge$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
U	U	U	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$

$\vee$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	$\top$	U	U
$\bot$	$\top$	U	$\bot$

$\leftarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	$\top$	$\top$
U	U	$\top$	$\top$
$\bot$	$\bot$	U	$\top$

$\leftrightarrow_L$	$\top$	U	$\bot$
$\top$	$\top$	U	$\bot$
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Table : $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

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$\top$	$\top$	$\top$	$\top$
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$\bot$	$\bot$	U	$\top$

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$\bot$	$\bot$	U	$\top$

Table : $\top$, $\bot$, and U denote *true*, *false*, and *unknown*, respectively.

An interpretation I of g$\mathcal{P}$ is a mapping of the Herbrand base $\mathcal{B}_\mathcal{P}$ to $\{\top, \bot, \mathrm{U}\}$ and is represented by an unique pair, $\langle I^\top, I^\bot \rangle$, where

$$I^\top = \{A \in \mathcal{B}_\mathcal{P} \mid A \text{ is mapped to } \top\} \text{ and } I^\bot = \{A \in \mathcal{B}_\mathcal{P} \mid A \text{ is mapped to } \bot\}.$$

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- For every I it holds that $I^\top \cap I^\bot = \emptyset$.
- A model of a formula F is an interpretation I such that F is true under I.
- A model of $g\,\mathcal{P}$ is an interpretation that is a model of each clause in $g\,\mathcal{P}$.

Computing Least Models

Hölldobler and Kencana Ramli [2009] propose to compute the least model of the weak completion of $\mathcal{P}$ ($\text{lm}_{\text{Ł}}\text{wc}\,\mathcal{P}$) which is identical to the least fixed point of $\Phi_{\mathcal{P}}$, by an operator defined by Stenning and van Lambalgen [2008].

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Let I be an interpretation in $\Phi_{\mathcal{P}}(I) = \langle J^{\top}, J^{\bot} \rangle$, where

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Hölldobler and Kencana Ramli showed that the model intersection property holds for weakly completed programs. This guarantees the existence of least models for every $\mathcal{P}$.

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In (Dietz, Hölldobler, and Wernhard [2014]) we show that weak completion semantics corresponds to well-founded semantics for modified tight logic programs.

Abduction (Kakas, Kowalski, and Toni [1993])

Given an abductive framework $\langle \mathcal{P}, \mathcal{A}, \models_{Ł}^{\mathsf{lmwc}} \rangle$ where

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- $\mathcal{E}$ is an explanation and a consistent subset of $\mathcal{A}$,
- logical consequence relation $\models_{Ł}^{\mathsf{lmwc}}$, where $\mathcal{P} \models_{Ł}^{\mathsf{lmwc}} F$ iff $\mathsf{lm}_{Ł}\mathsf{wc}\,\mathcal{P}(F) = \top$, and
- $\mathcal{O}$ is an observation which is a set of (at least one) literals.

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$\mathcal{O}$ is **explained by** $\mathcal{E}$ **given** $\mathcal{P}$ iff $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{\text{lmwc}} O$, where $\mathcal{P} \not\models_{Ł}^{\text{lmwc}} \mathcal{O}$.

$\mathcal{O}$ is **explained given** $\mathcal{P}$ iff there exists an $\mathcal{E}$ such that $\mathcal{O}$ is explained by $\mathcal{E}$ given $\mathcal{P}$.

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$\mathcal{O}$ is **explained by** $\mathcal{E}$ **given** $\mathcal{P}$ iff $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} O$, where $\mathcal{P} \not\models_{Ł}^{lmwc} \mathcal{O}$.

$\mathcal{O}$ is **explained given** $\mathcal{P}$ iff there exists an $\mathcal{E}$ such that $\mathcal{O}$ is explained by $\mathcal{E}$ given $\mathcal{P}$.

F **follows skeptically from** $\mathcal{P}$**,and** $\mathcal{O}$ iff $\mathcal{O}$ can be explained given $\mathcal{P}$, and for all minimal explanations $\mathcal{E}$ we find that $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} \mathcal{O}$.

F **follows credulously from** $\mathcal{P}$**, and** $\mathcal{O}$ iff there exists a minimal explanation $\mathcal{E}$ such that $\mathcal{P} \cup \mathcal{E} \models_{Ł}^{lmwc} \mathcal{O}$.

Abducing the CONCLUSION

Given our background knowledge we know, there are addictive things, let's say about b

$$\mathcal{O}_{add(b)} = \{add(b)\}$$

We have two minimal explanations for $\mathcal{O}_{add(b)}$

$$\begin{array}{rcll} \mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{cig} \cup \mathcal{E}_{cig(b)}) & = & \langle\{add(b), cig(b), inex(b), \ldots\}, & \{\ldots\}\rangle \\ \mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{cig} \cup \mathcal{E}_{\neg cig(b)}) & = & \langle\{add(b), \ldots\}, & \{cig(b), inex(b), \ldots\}\rangle \end{array}$$

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Recall $\mathcal{P}_{cig}$. Together with $\mathcal{E}_{cig(b)}$ it contains

$$\begin{array}{rcl rcl} add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\ inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), \\ ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot, \\ cig(b) & \leftarrow & \top. & & & \end{array}$$

Abducing the CONCLUSION

Given our background knowledge we know, there are addictive things, let's say about b

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Recall $\mathcal{P}_{cig}$. Together with $\mathcal{E}_{\neg cig(b)}$ it contains

$$\begin{array}{rcl rcl} add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\ inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), \\ ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot, \\ cig(b) & \leftarrow & \bot. & & & \end{array}$$

Abducing the CONCLUSION

Given our background knowledge we know, there are addictive things, let's say about b

$$\mathcal{O}_{add(b)} = \{add(b)\}$$

We have two minimal explanations for $\mathcal{O}_{add(b)}$

$$\begin{array}{rcll}
\mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{cig} \cup \mathcal{E}_{cig(b)}) & = & \langle\{add(b), cig(b), inex(b), \ldots\}, & \{\ldots\}\rangle \\
\mathrm{Im}_{Ł}\mathrm{wc}\,(\mathcal{P}_{cig} \cup \mathcal{E}_{\neg cig(b)}) & = & \langle\{add(b), \ldots\}, & \{cig(b), inex(b), \ldots\}\rangle
\end{array}$$

Recall $\mathcal{P}_{cig}$. Together with $\mathcal{E}_{\neg cig(b)}$ it contains

$$\begin{array}{rclrcl}
add'(X) & \leftarrow & inex(X) \wedge \neg ab_{add'}(X), & add(X) & \leftarrow & \neg add'(X), \\
inex(X) & \leftarrow & cig(X) \wedge \neg ab_{inex}(X), & ab_{add'}(X) & \leftarrow & cig(X), \\
ab_{add'}(X) & \leftarrow & \bot, & ab_{inex}(X) & \leftarrow & \bot, \\
cig(b) & \leftarrow & \bot. & & &
\end{array}$$

Credulously, we validate some addictive things are not cigarettes.