Exercise 3.1. A language $L \in P$ is complete for $P$ under polynomial-time reductions if $L' \leq_p L$ for every $L' \in P$. Show that every language in $P$ except $\emptyset$ and $\Sigma^*$ is complete for $P$ under polynomial-time reductions.

Exercise 3.2. Let

$$A_{PNTM} = \{\langle M, p, w \rangle \mid M \text{ is a non-deterministic TM that accepts } w \text{ in time } p(|w|) \}$$

with $p$ a polynomial function.

Show that $A_{PNTM}$ is NP-complete.

Exercise 3.3. Show that the following problem is NP-complete:

Input: A propositional formula $\varphi$ in CNF
Question: Does $\varphi$ have at least 2 different satisfying assignments?

Exercise 3.4. We recall some definitions.

- Given some language $L$, $L \in \text{coNP}$ if and only if $\overline{L} \in \text{NP}$.
- $L$ is $\text{coNP}$-hard if and only if $L' \leq_p L$ for every $L' \in \text{coNP}$.
- $L$ is $\text{coNP}$-complete if and only if $L \in \text{coNP}$ and $L$ is $\text{coNP}$-hard.

Show that if any $\text{coNP}$-complete problem is in NP, then $\text{NP} = \text{coNP}$.

Exercise 3.5. If $G$ is an undirected graph, a vertex cover of $G$ is a subset of the nodes where every edge of $G$ touches one of those nodes. The vertex cover problem asks whether a graph contains a vertex cover of a specified size.

$$\text{VERTEX-COVER} = \{(G, k) \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover.}\}$$

Show that $\text{VERTEX-COVER}$ is NP-complete.

Hint: This problem is related to the 3-SAT problem.