TU Dresden, Fakultät Informatik
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Complexity Theory

Exercise 3: Time Complexity

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Exercise 3.1. Use the approach presented in lecture 4 to create a quine in your favourite programming language (or just use Python). What is the equivalent of "TM concatenation" here? Also note that the function q is often more complicated than one might think, due to character escaping.

Exercise 3.2. Show that P is closed under concatenation and star.

Exercise 3.3. A language $\mathbf{L} \in \mathrm{P}$ is complete for P under polynomial-time reductions if $\mathbf{L}' \leq_p \mathbf{L}$ for every $\mathbf{L}' \in \mathrm{P}$. Show that every language in P except $\emptyset$ and Σ^* is complete for P under polynomial-time reductions.

Exercise 3.4. Show that the following problem is NP-complete:

Input:	A propositional formula φ in CNF
Question:	Does φ have at least 2 different satisfying assignments?

Exercise 3.5. We recall some definitions.

- Given some language $\mathbf{L}$, $\mathbf{L} \in \mathrm{coNP}$ if, and only if, $\overline{\mathbf{L}} \in \mathrm{NP}$.
- $\mathbf{L}$ is coNP-hard if, and only if, $\mathbf{L}' \leq_p \mathbf{L}$ for every $\mathbf{L}' \in \mathrm{coNP}$.
- $\mathbf{L}$ is coNP-complete if, and only if, $\mathbf{L} \in \mathrm{coNP}$ and $\mathbf{L}$ is coNP-hard.

Show that if any coNP-complete problem is in NP, then $\mathrm{NP} = \mathrm{coNP}$.

Exercise 3.6. If G is an undirected graph, a *vertex cover* of G is a subset of the nodes where every edge of G touches one of those nodes. The vertex cover problem asks whether a graph contains a vertex cover of a specified size.

$$\textsc{Vertex-Cover} = \{\langle G, k\rangle \mid G \text{ is an undirected graph that has a } k\text{-node vertex cover.}\}$$

Show that **Vertex-Cover** is NP-complete.

Hint:
Try to find a reduction from 3-Sat

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