TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Computer Science Institute of Theoretical Computer Science, Chair of Automata Theory

Database Theory

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Exercise Sheet 7 – CQ Optimisation and FO Expressivity

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Exercise 7.1 Apply the conjunctive query minimisation algorithm to find a core of the following CQs:

(a) $\exists x, y, z.\ R(x,y) \wedge R(x,z)$

(b) $\exists x, y, z.\ R(x,y) \wedge R(x,z) \wedge R(y,z)$

(c) $\exists x, y, z.\ R(x,y) \wedge R(x,z) \wedge R(y,z) \wedge R(x,x)$

(d) $\exists v, w.\ S(x,a,y) \wedge S(x,v,y) \wedge S(x,w,y) \wedge S(x,x,x)$

Exercise 7.2 Consider a fixed set of relation names (each with a given arity). Show that there is a Boolean CQ Q_{min} without constant symbols that is most specific in the following sense:

For every BCQ Q that does not use constants, we find that $Q_{\mathsf{min}} \sqsubseteq Q$.

Is there also a most general BCQ Q_{max} that contains all BCQs without constant names?

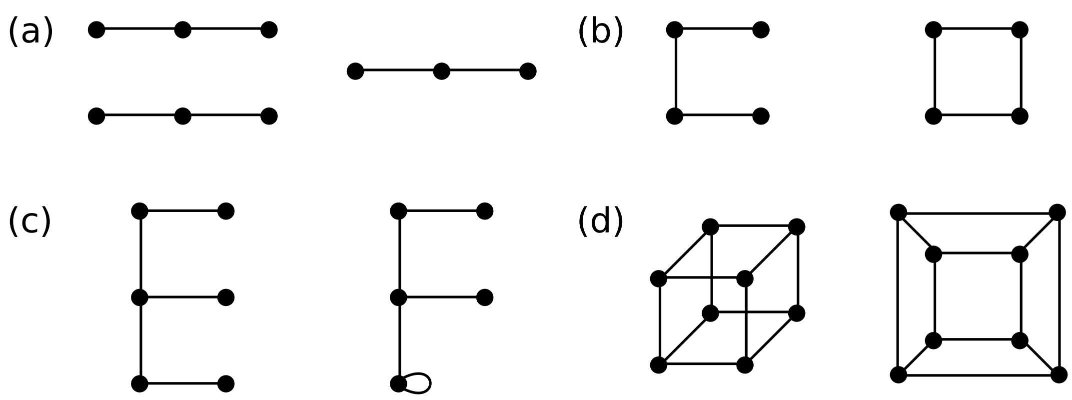
What is the answer to these questions if the considered BCQs may use constant names? What if we consider FO queries instead?

Exercise 7.3 Explain why the CQ minimisation algorithm is correct:

(a) Why is the result guaranteed to be a minimal CQ?

(b) Why is the result guaranteed to be unique up to bijective renaming of variables?

Exercise 7.4 For the following pairs of structures, find the maximal r such that $\mathcal{I} \sim_r \mathcal{J}$:



Exercise 7.5 A *linear order* is a relational structure with one binary relational symbol $\leq$ that is interpreted as a reflexive, asymmetric, transitive and total relation over the domain. Up to renaming of domain elements there is exactly one linear order for every finite domain, which can be depicted as a chain of elements. We denote the linear order of size n by $\mathcal{L}_n$. For example:

$$\mathcal{L}_6 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \qquad \text{and} \qquad \mathcal{L}_7 : 1 \leq 2 \leq 3 \leq 4 \leq 5 \leq 6 \leq 7$$

(a) For which r are $\mathcal{L}_6 \sim_r \mathcal{L}_7$?

(b) More generally, for which r are $\mathcal{L}_n \sim_r \mathcal{L}_{n+1}$? (∗)