The Sticky Path to Expressive Querying:

Decidability of Navigational Queries under Existential Rules

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[1] TU Dresden [2] University of Wrocław

Introduction

Basic Setup

In this talk, we are going to discuss:

Instances

Relational structures

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Regular path queries

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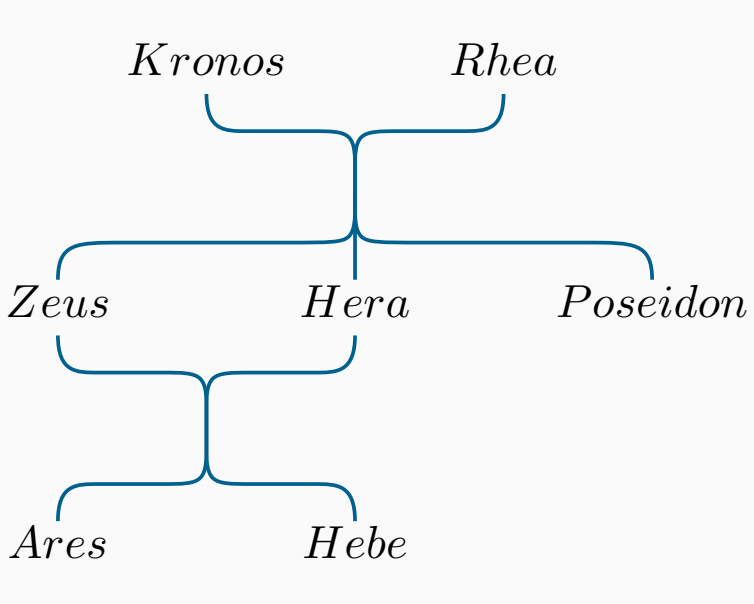
Entailment

Of Regular Path Queries under existential rules

Logic Based Knowledge Representation



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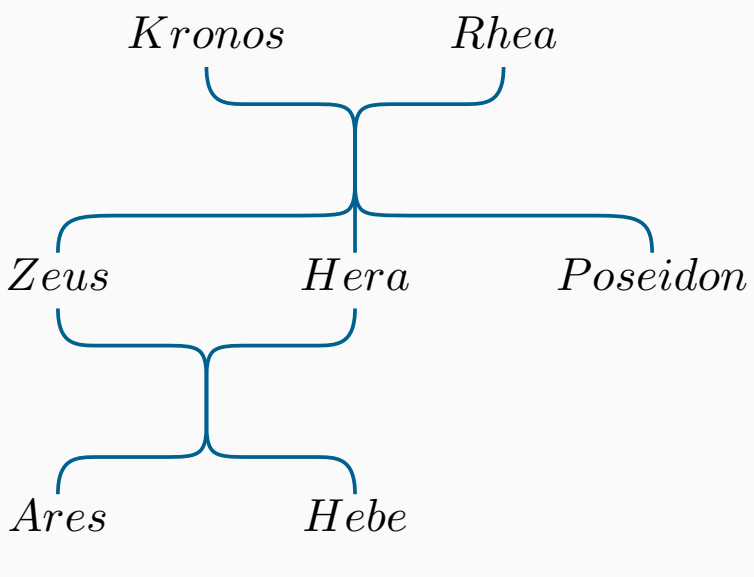

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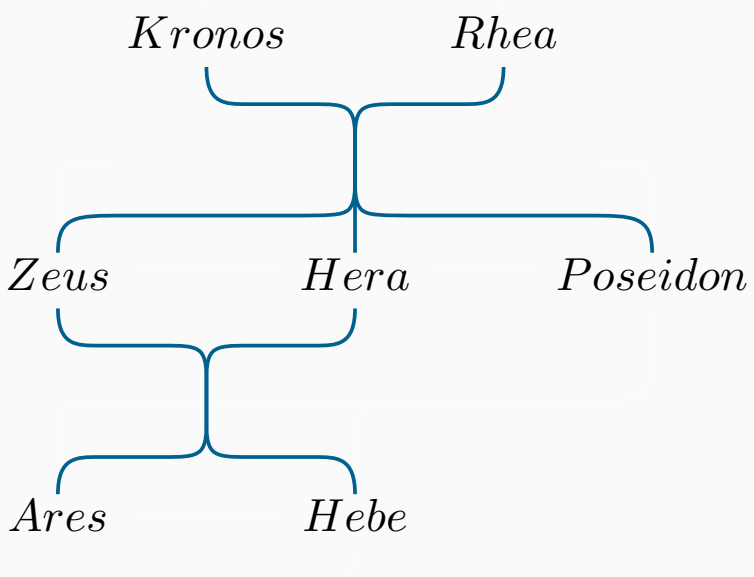

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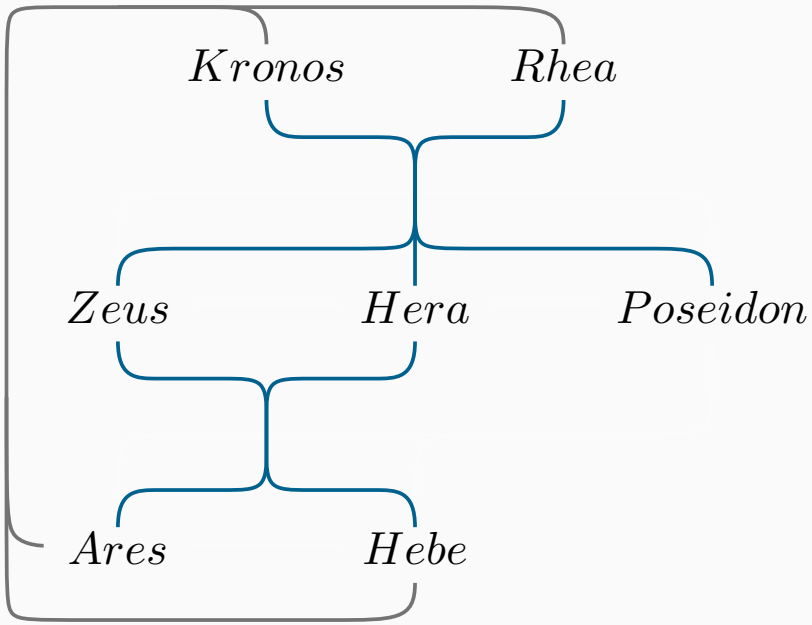

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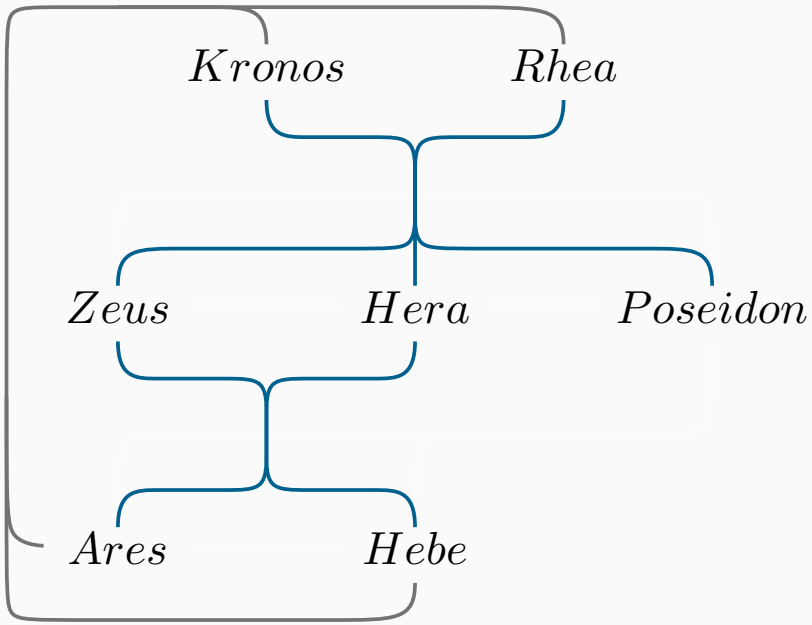

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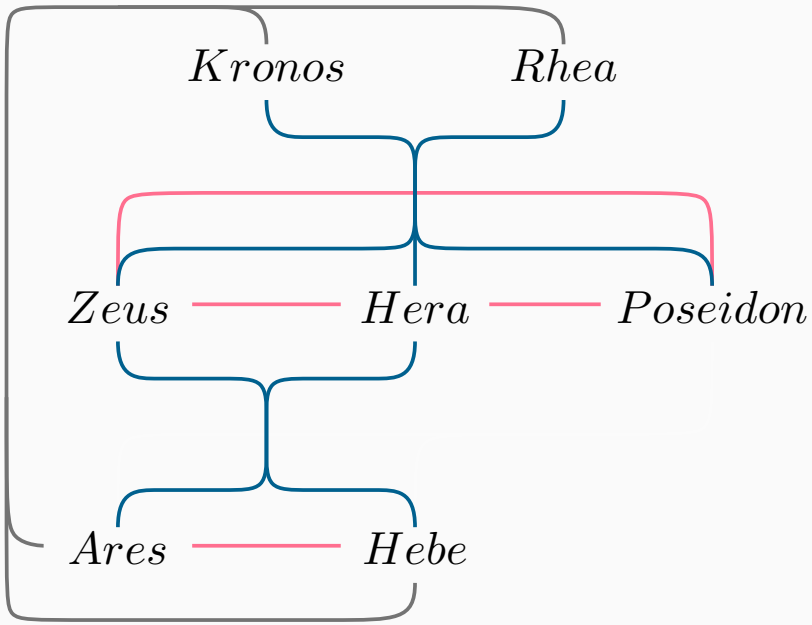

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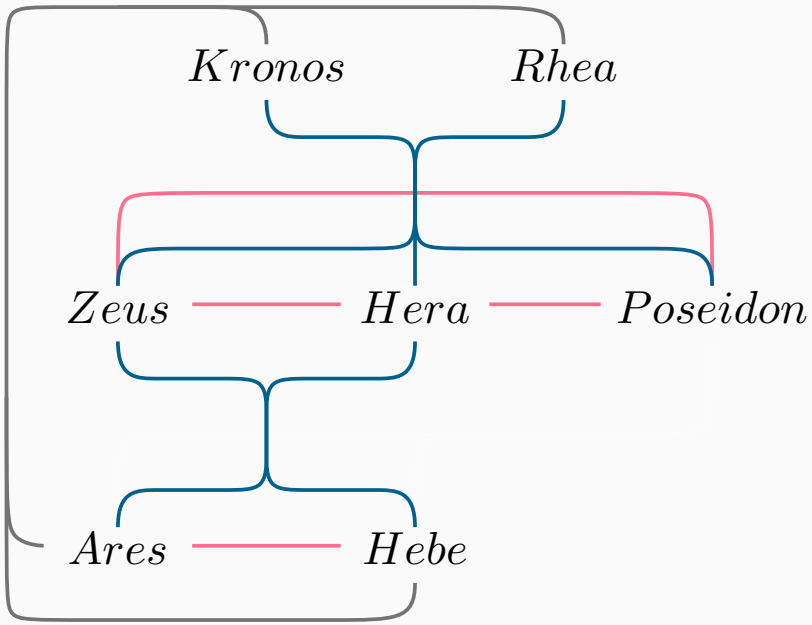

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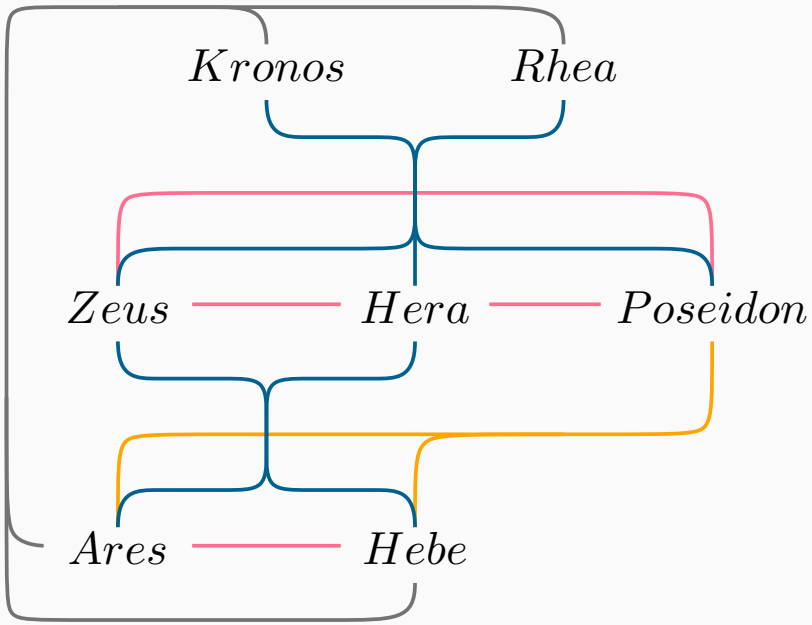

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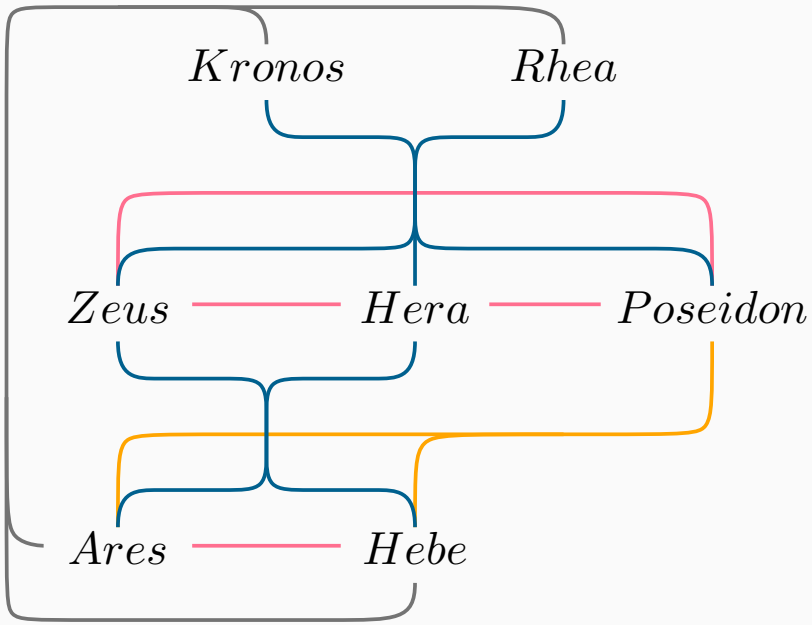

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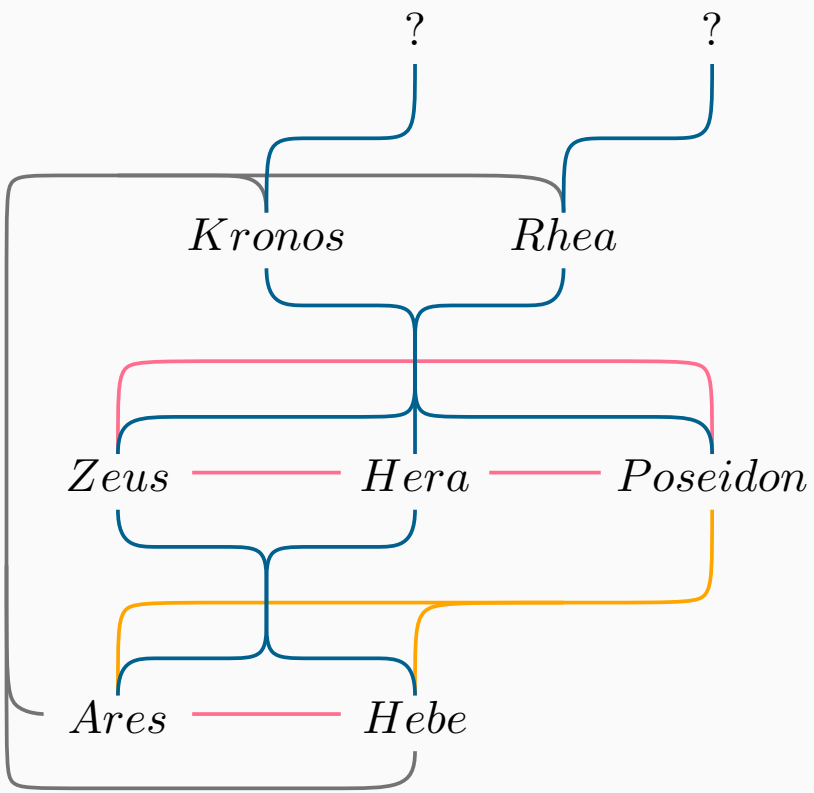

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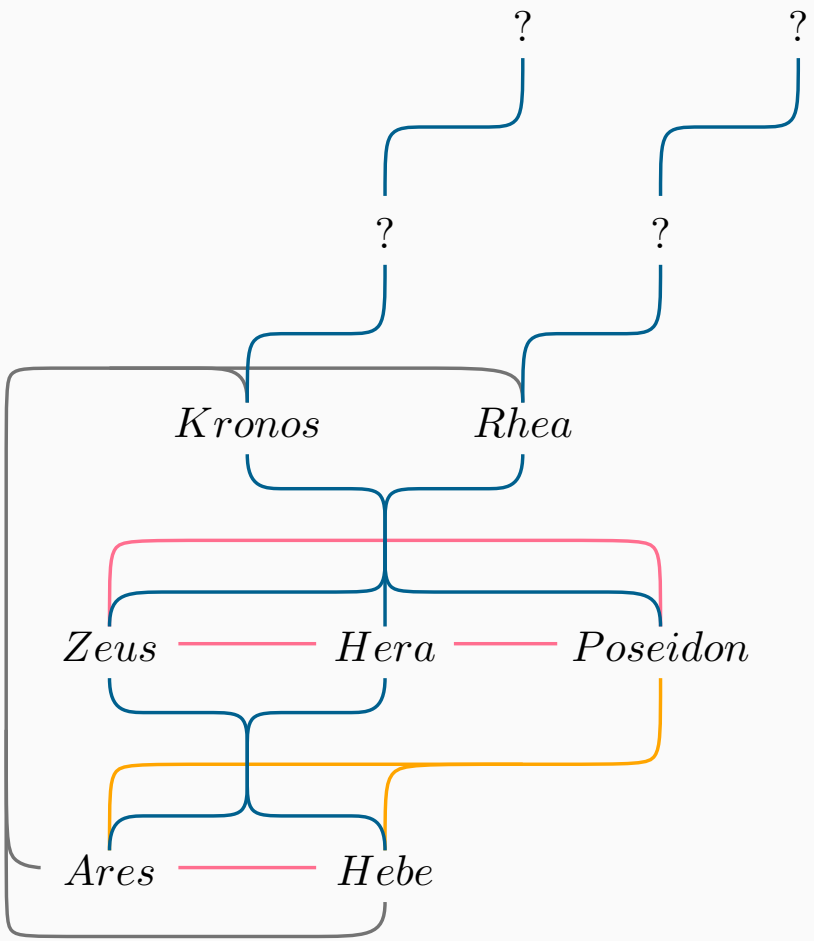

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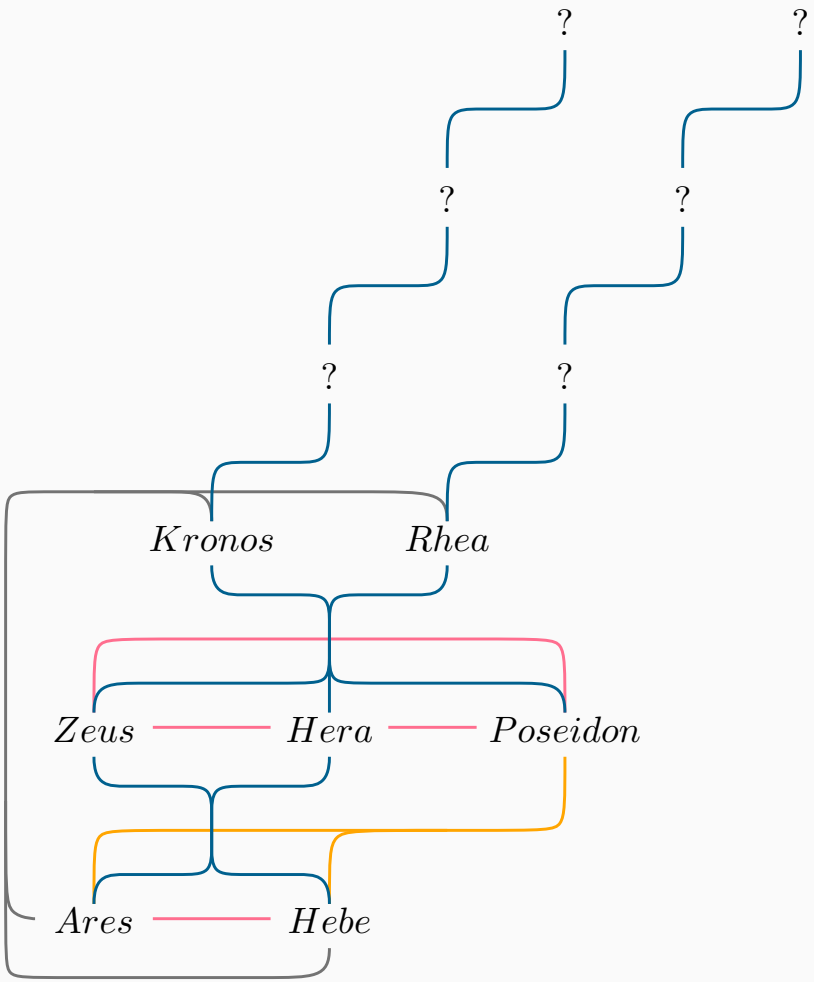

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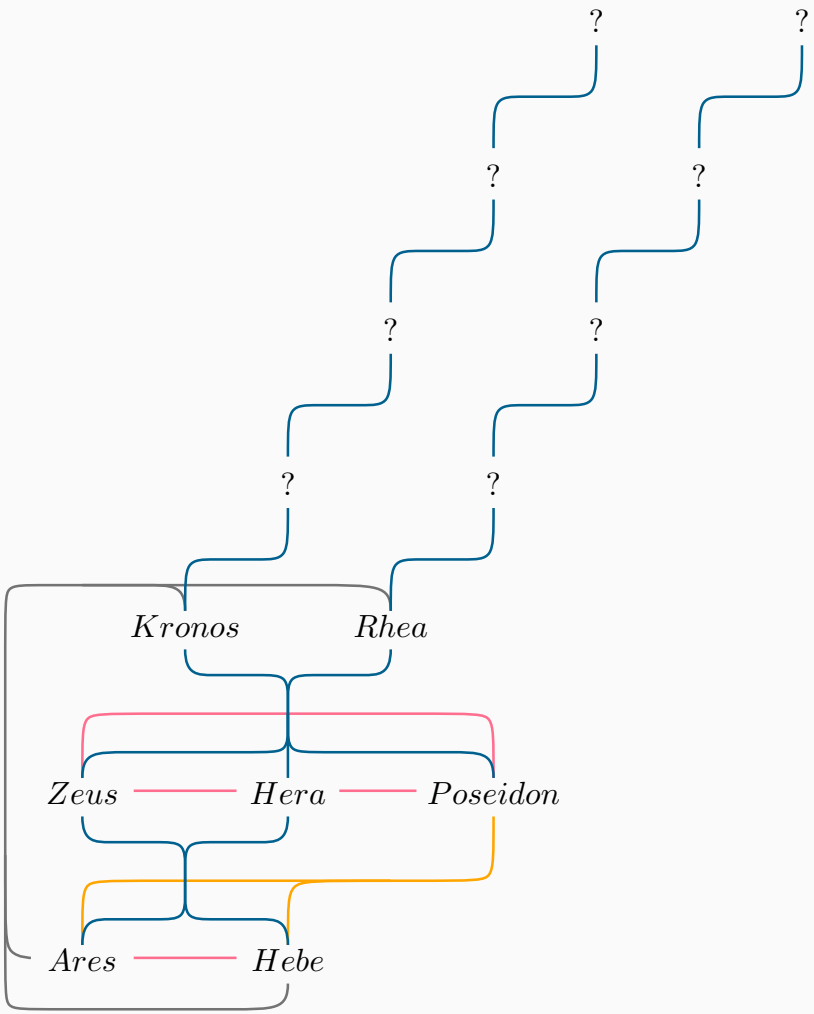

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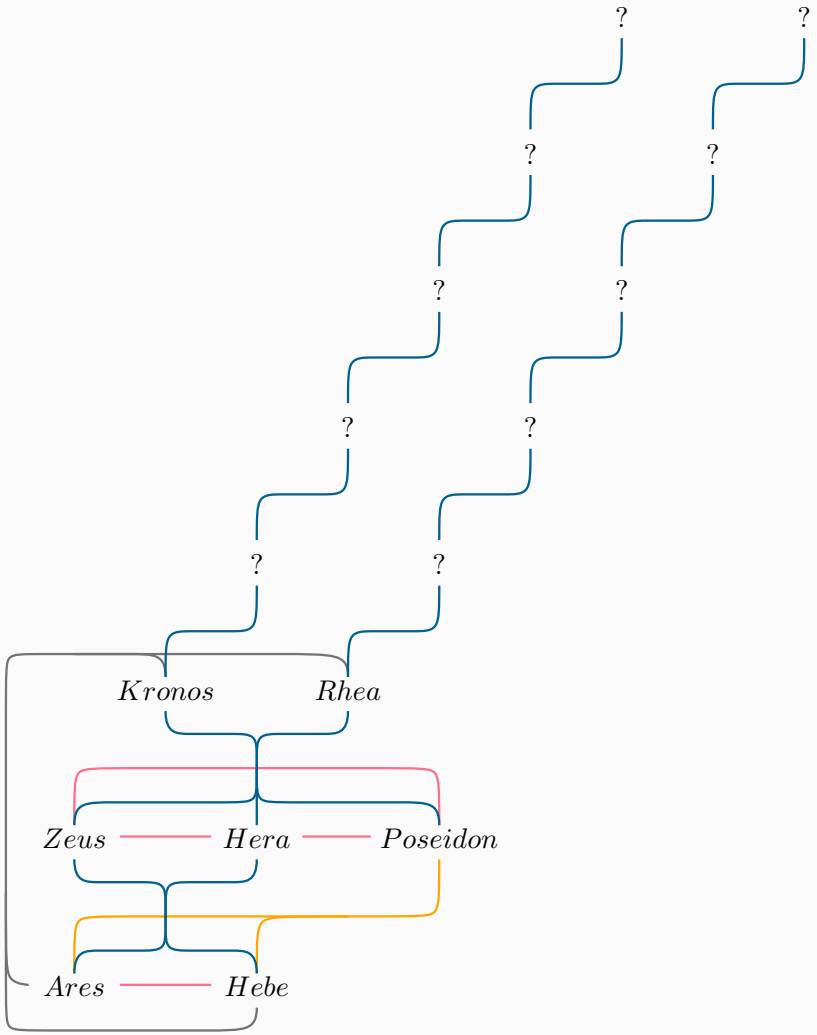

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In general, we are interested about *entailment*

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Example query languages:

- Conjunctive Queries (SQL),
- Regular Path Queries (Cypher, Gremlin and SPARQL),
- Datalog.

Introduction: Existential Rules

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Models are gridlike

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Universal models can be computed with *the chase* algorithm!

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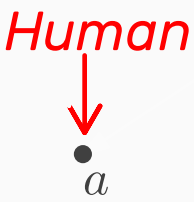

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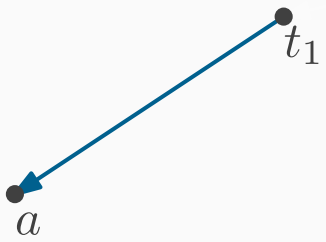

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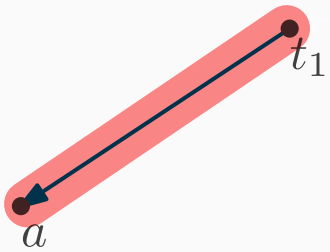

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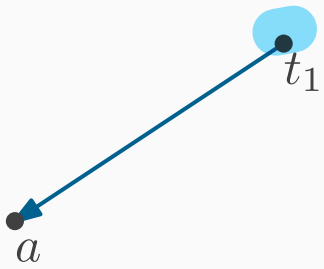

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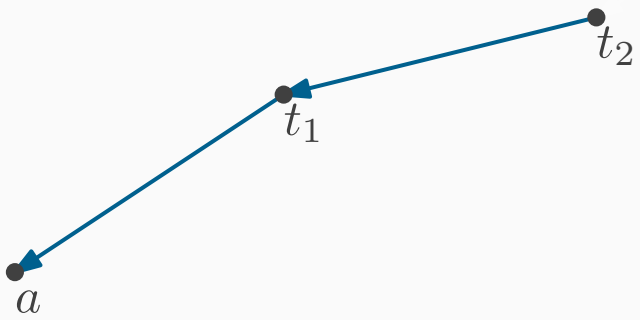

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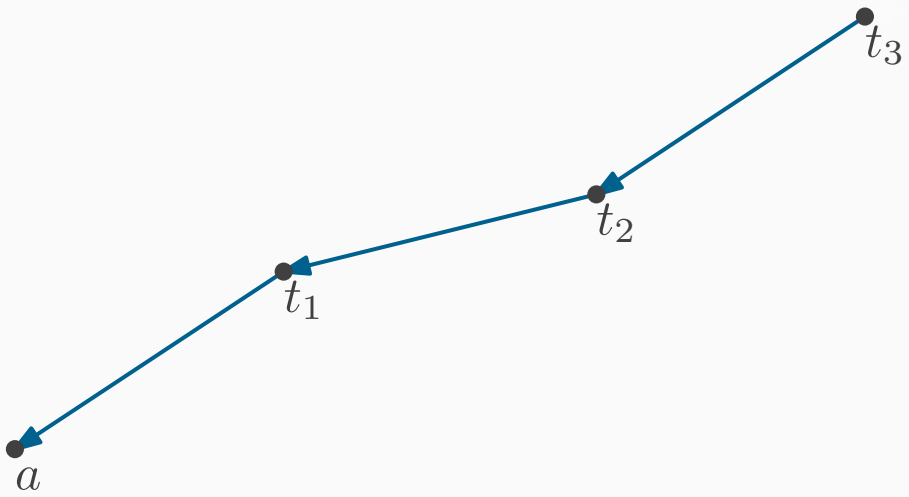

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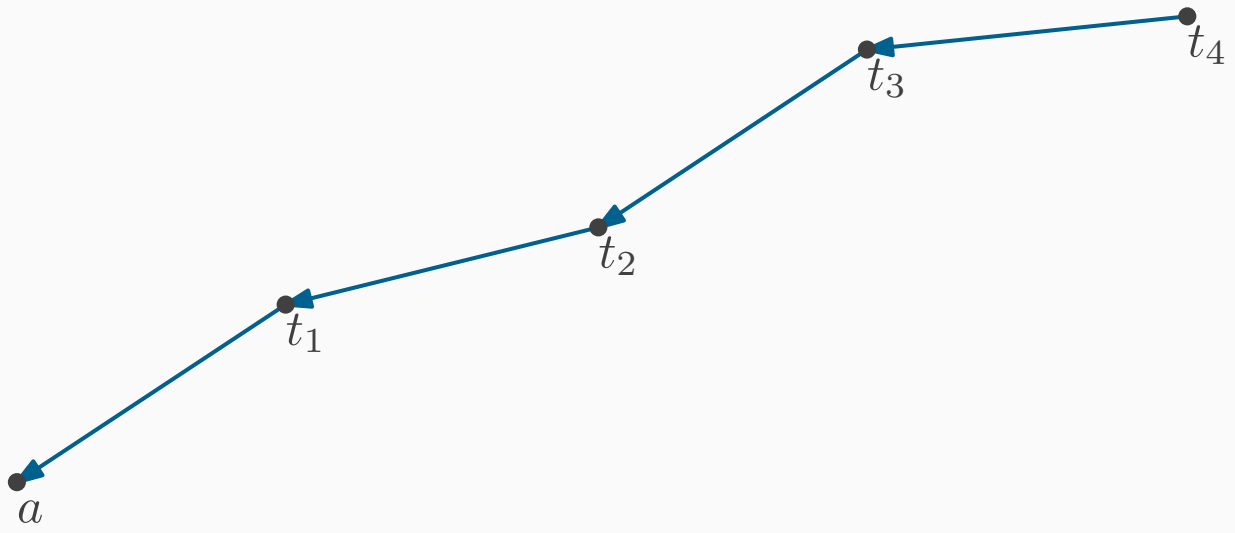

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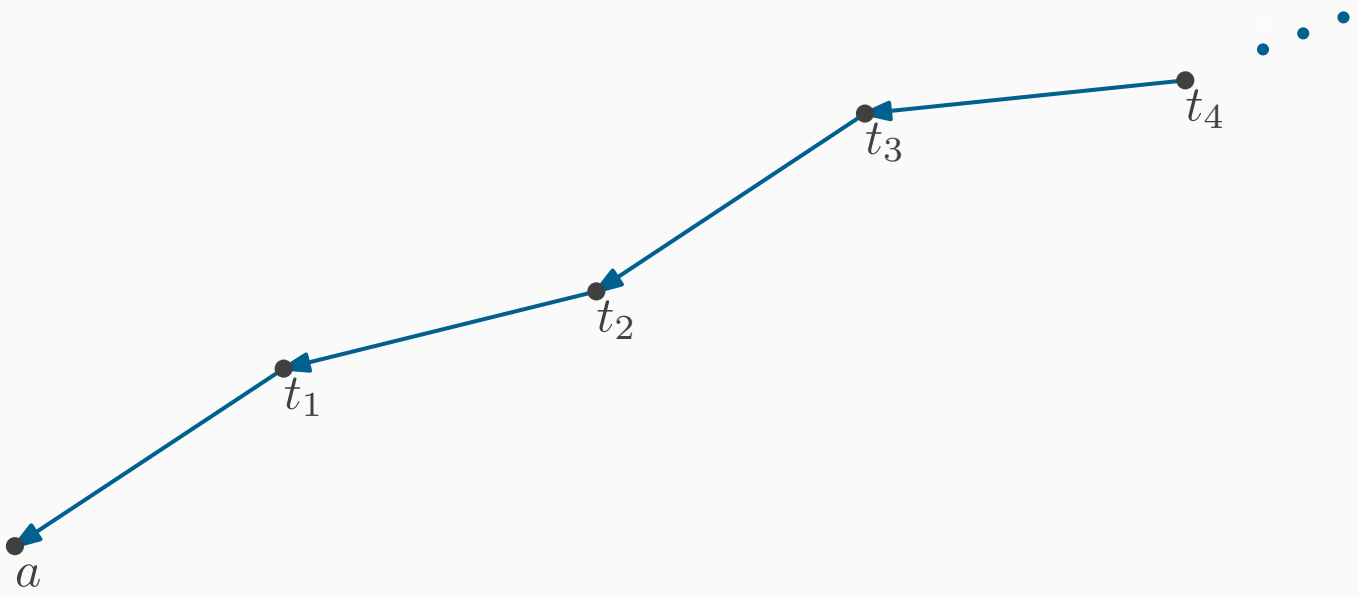

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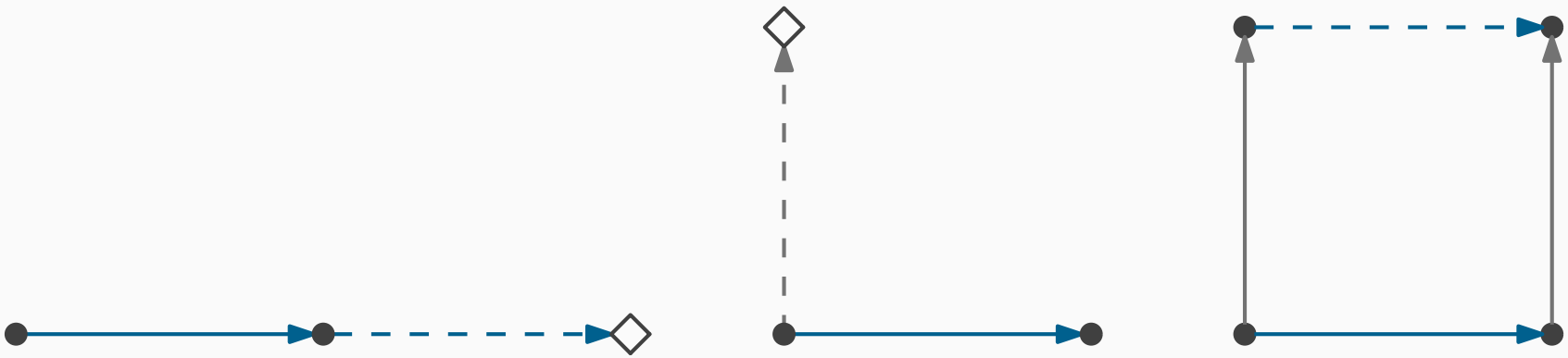

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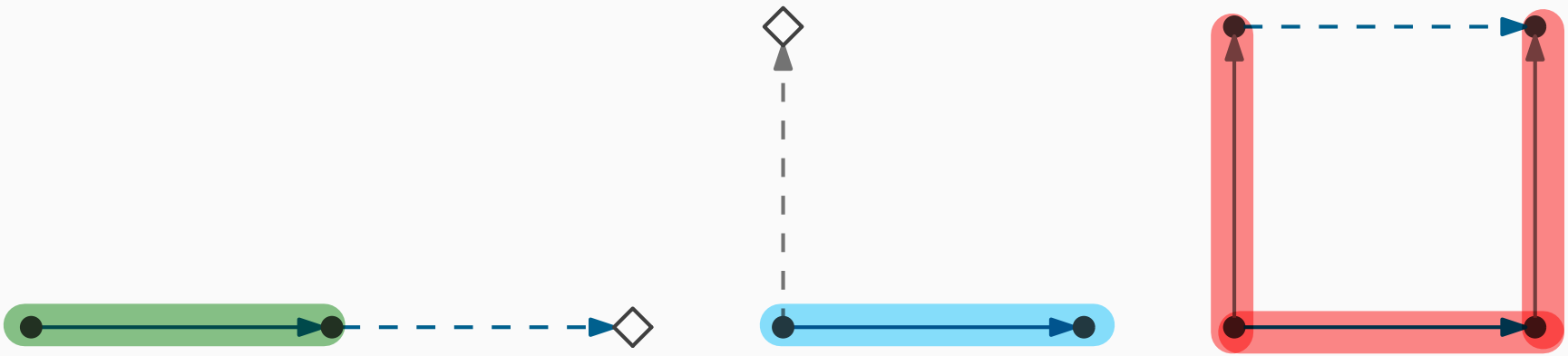

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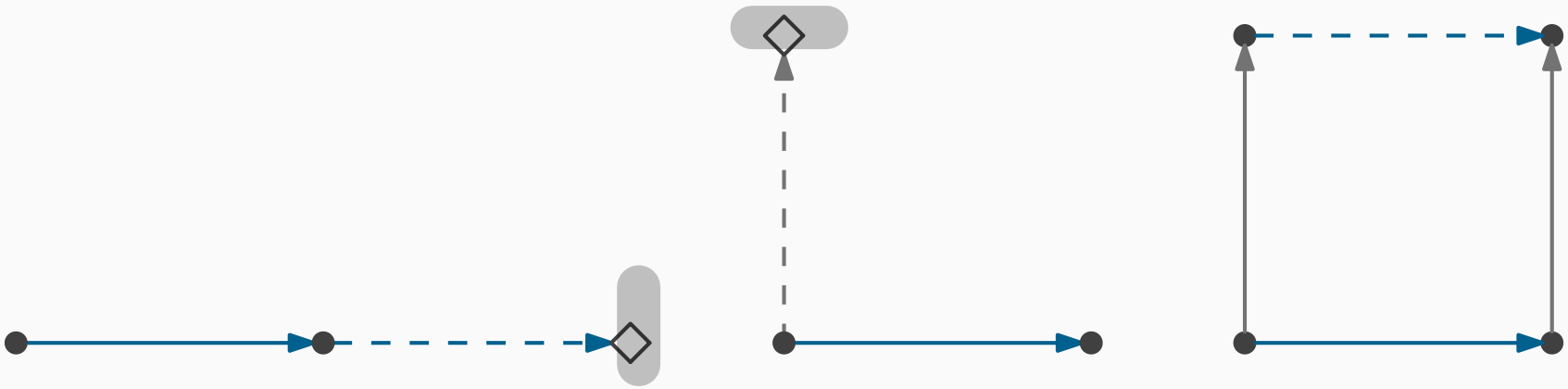

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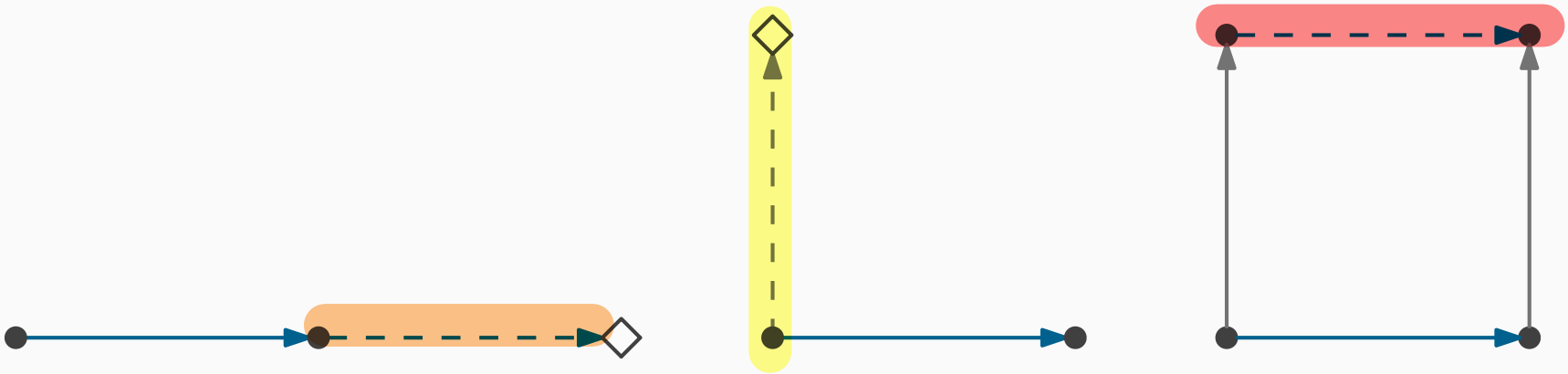

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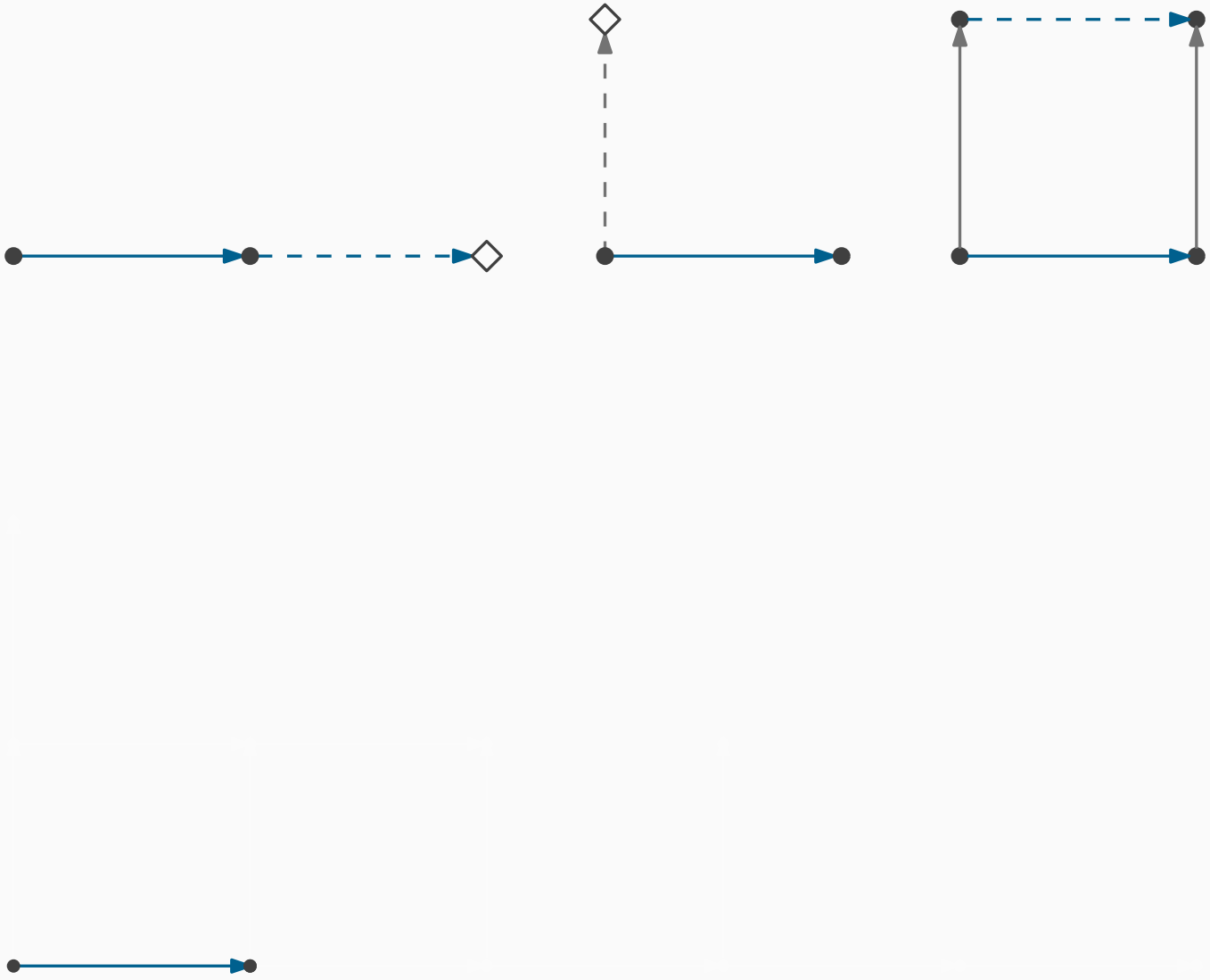

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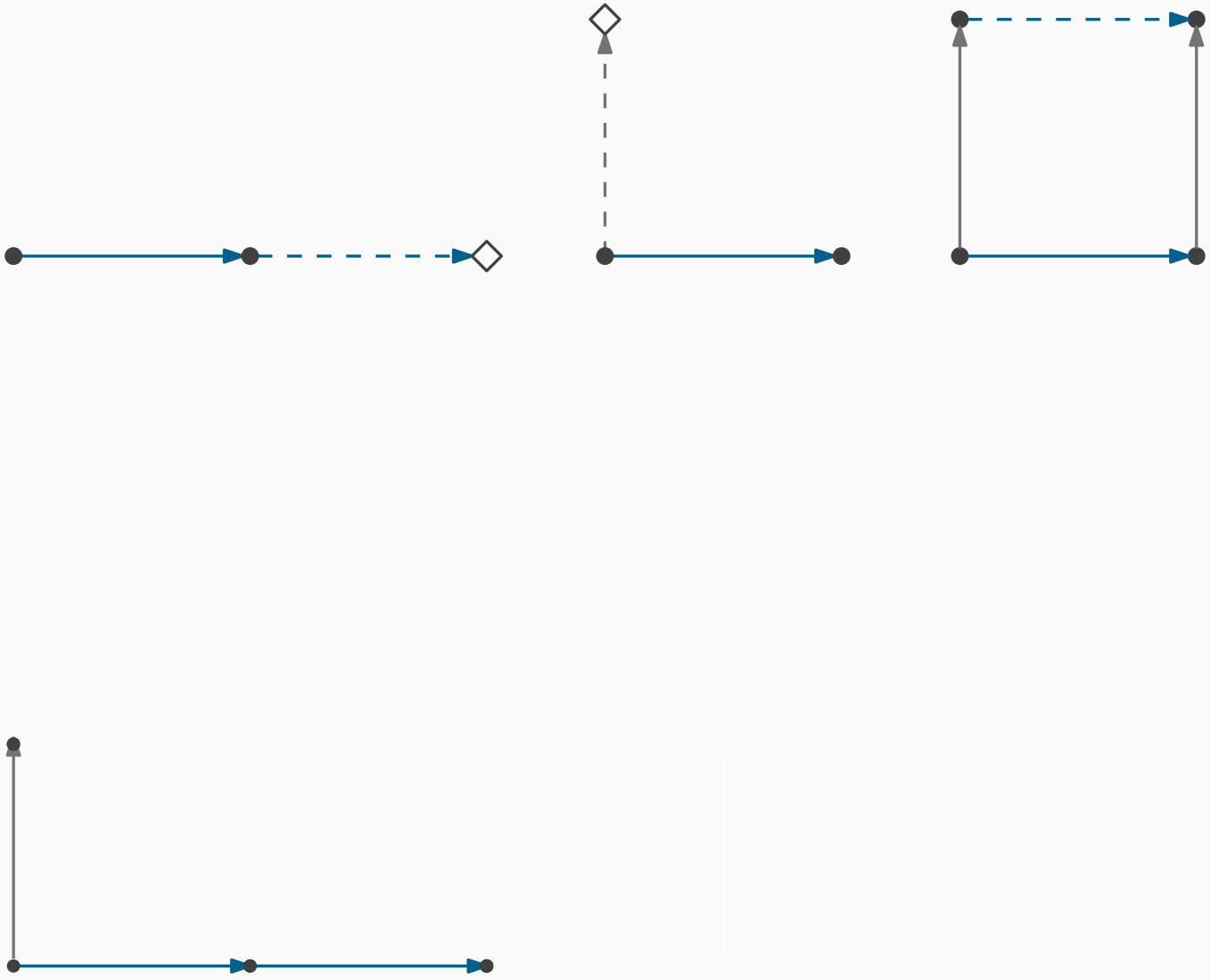

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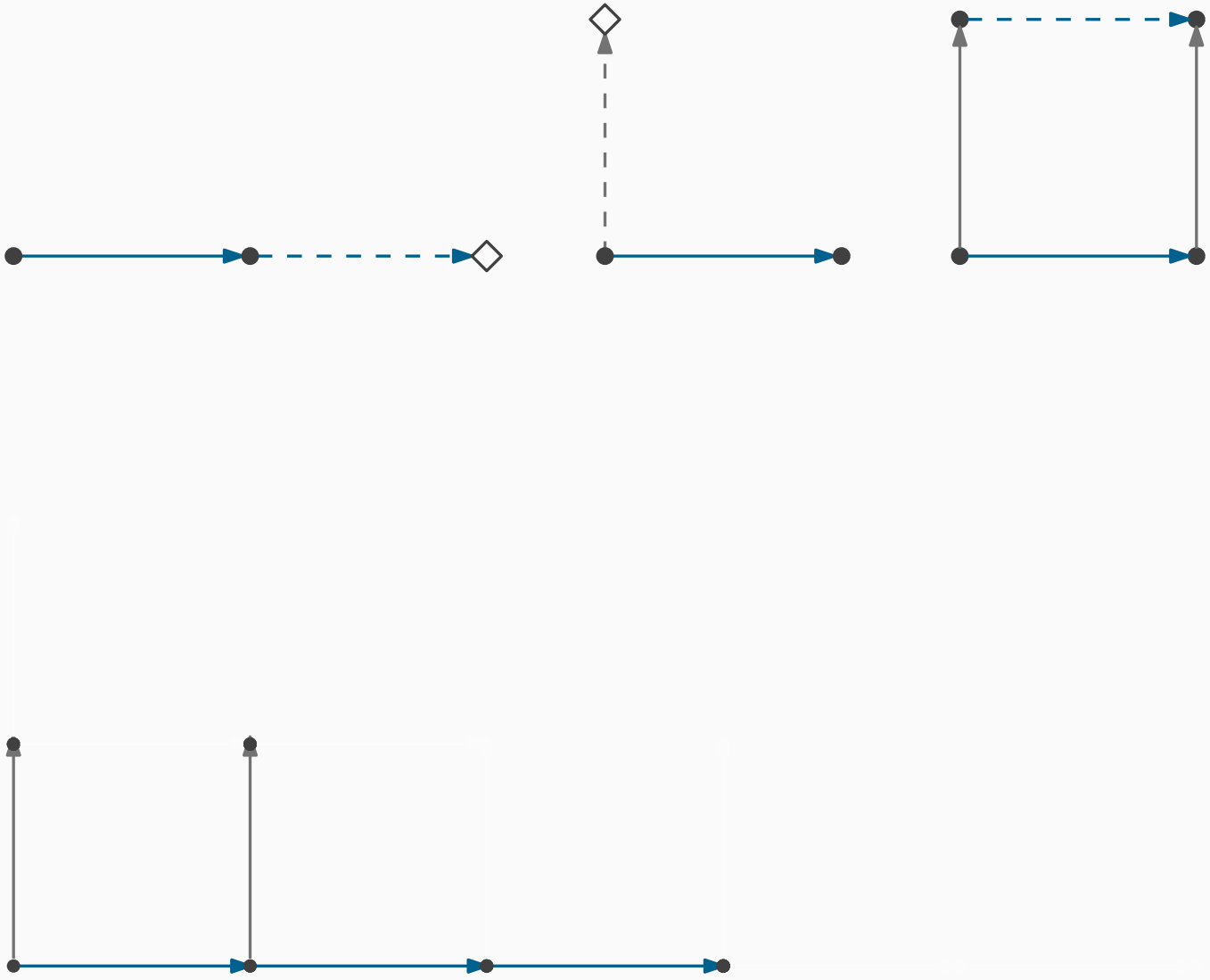

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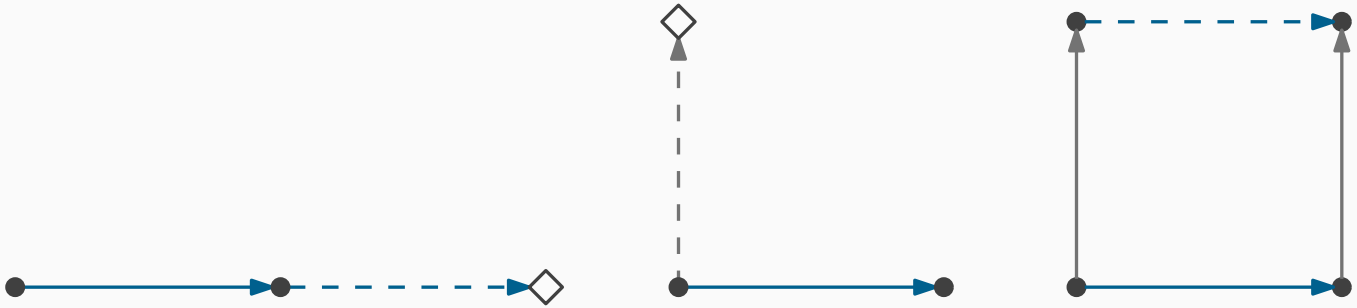

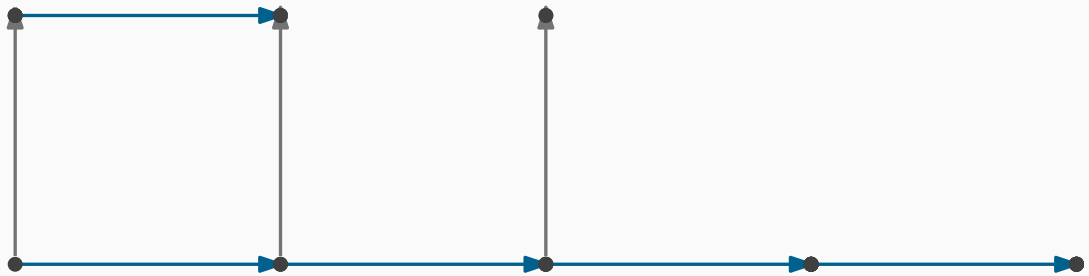

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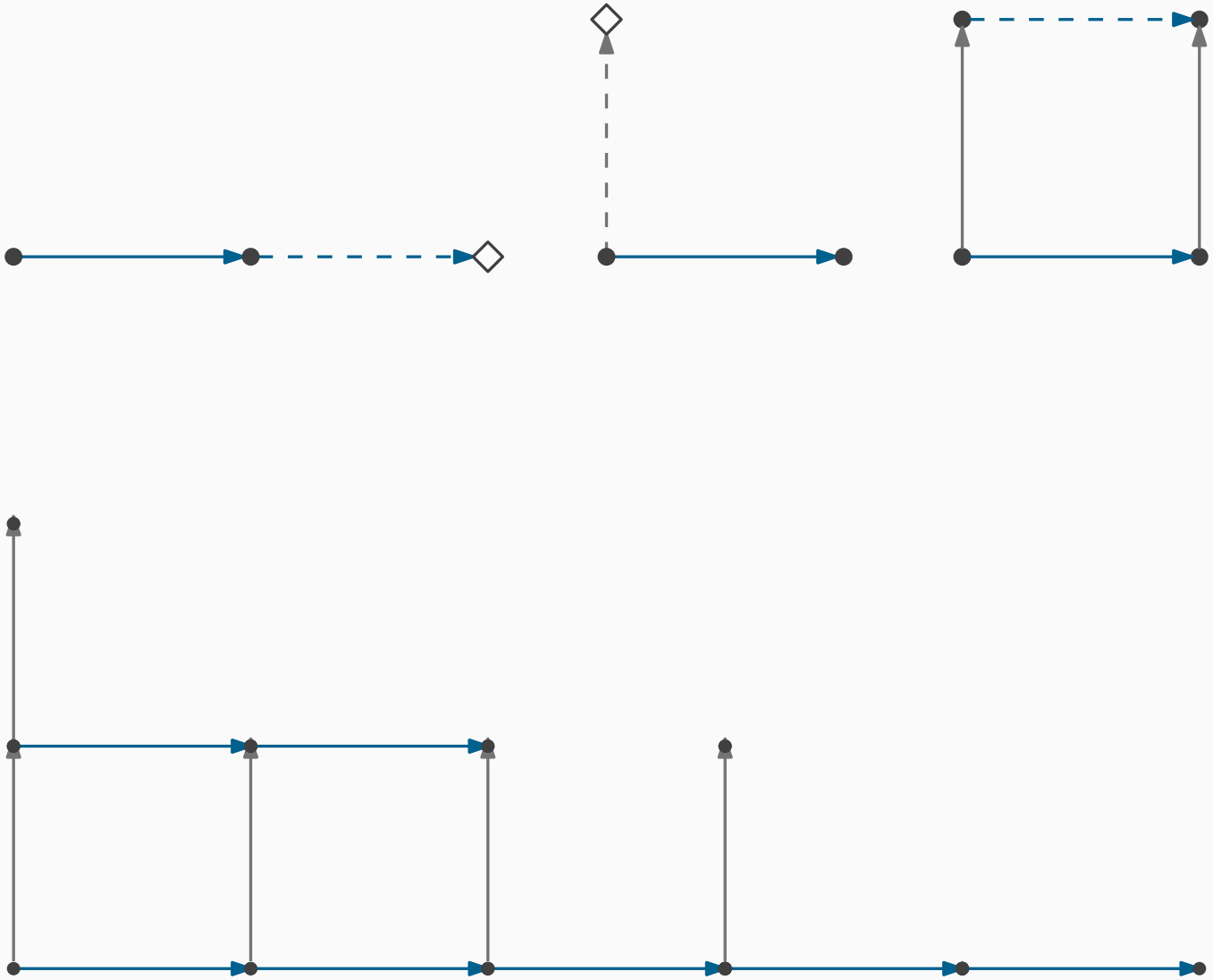
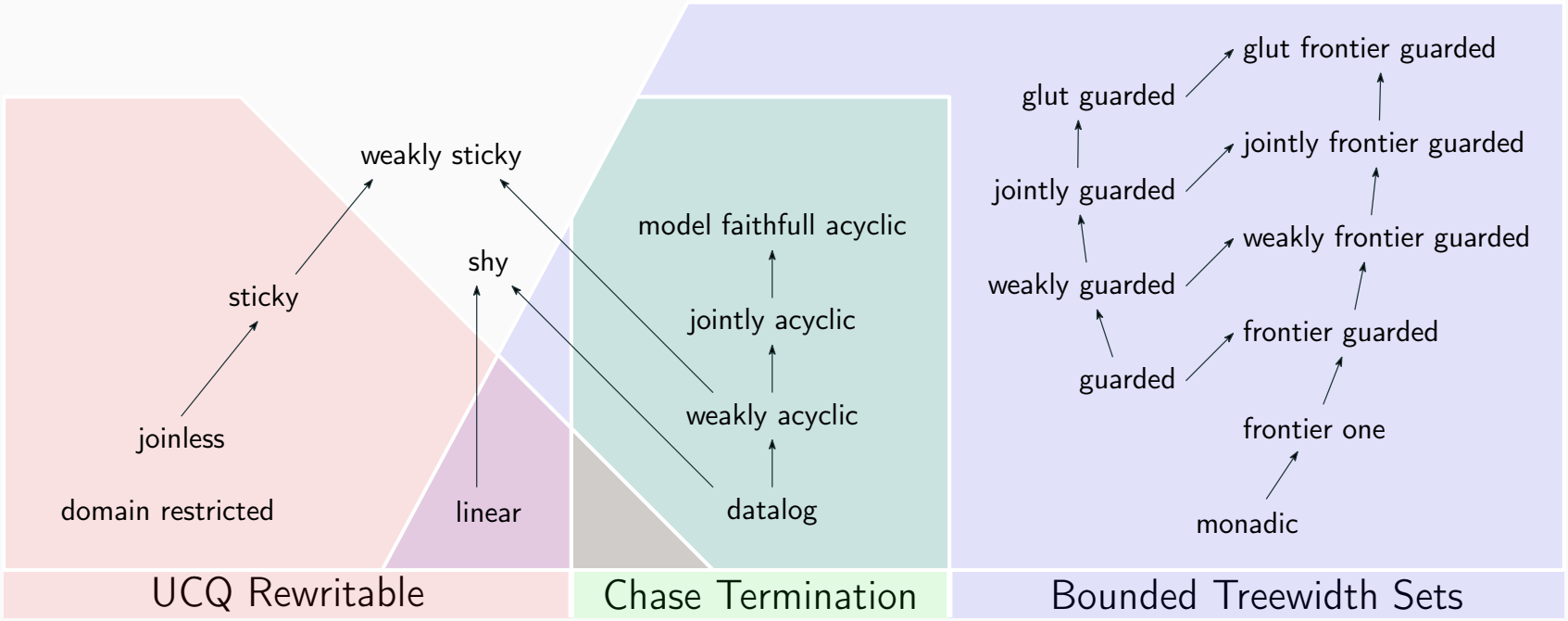


Classes of Existential Rules: CQ Landscape



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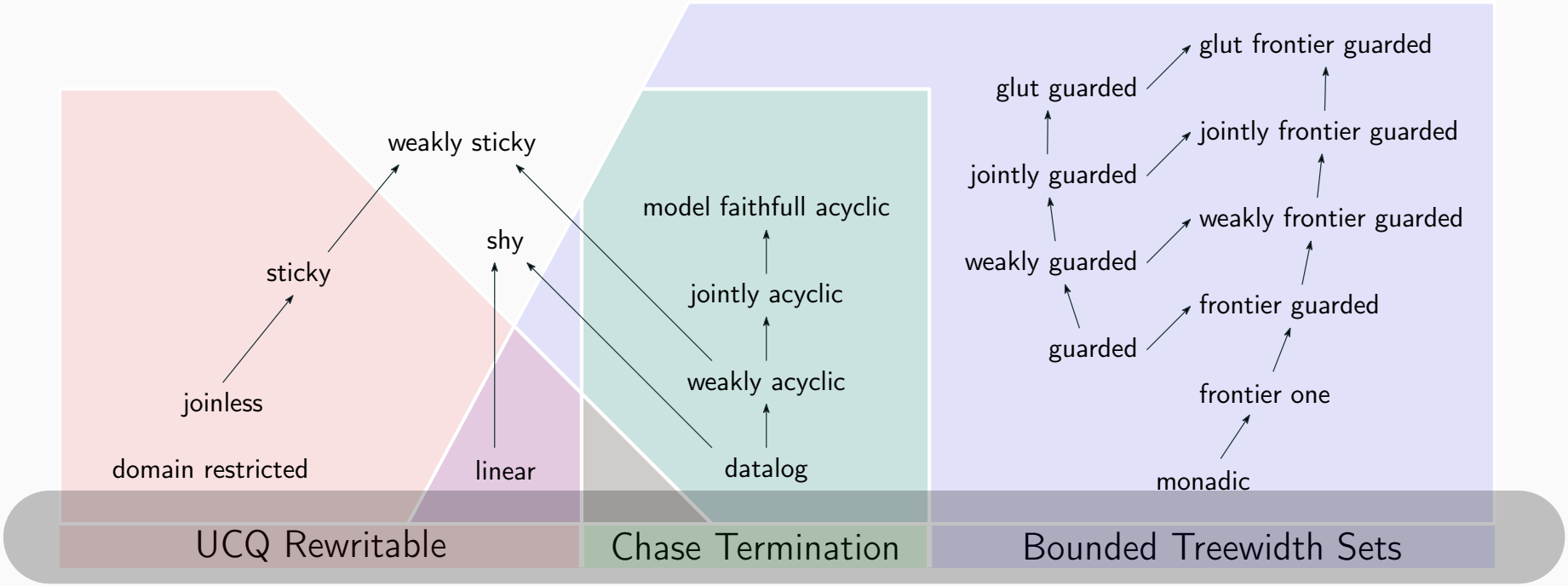

bottom row classes: decidable conjunctive query entailment

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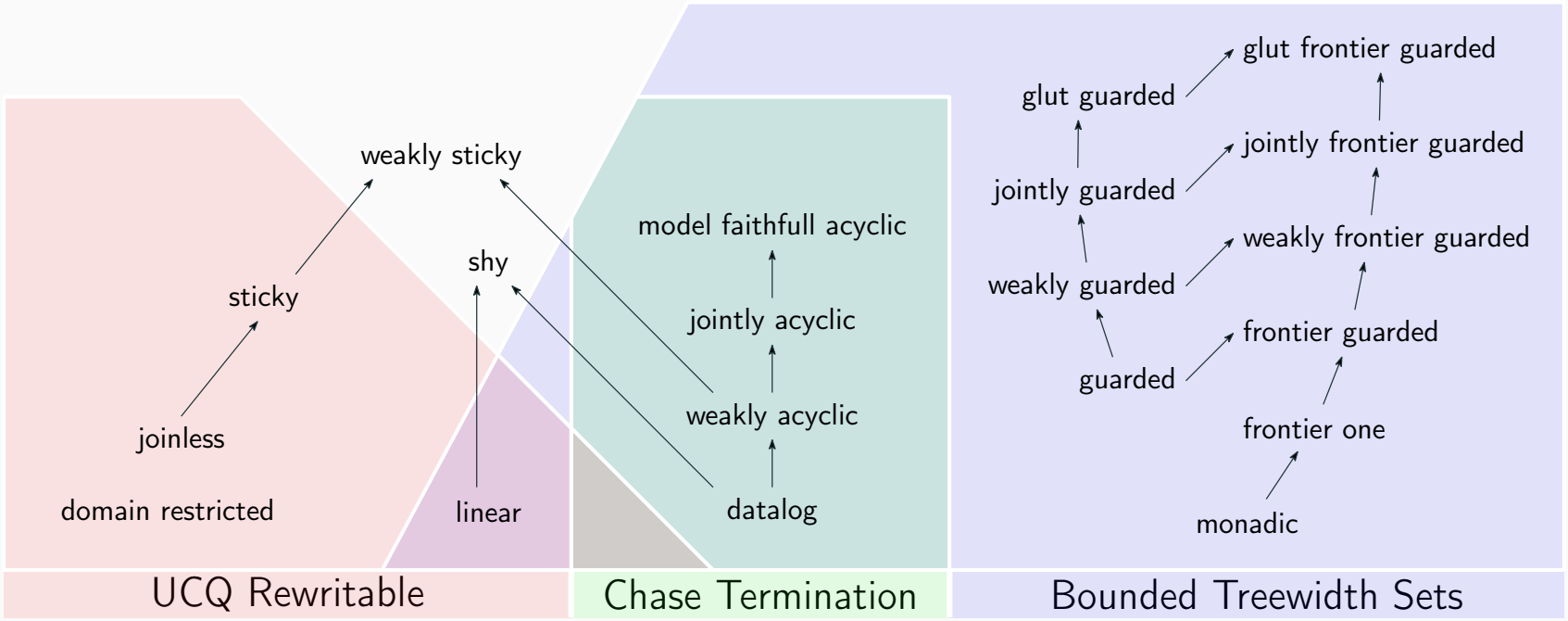

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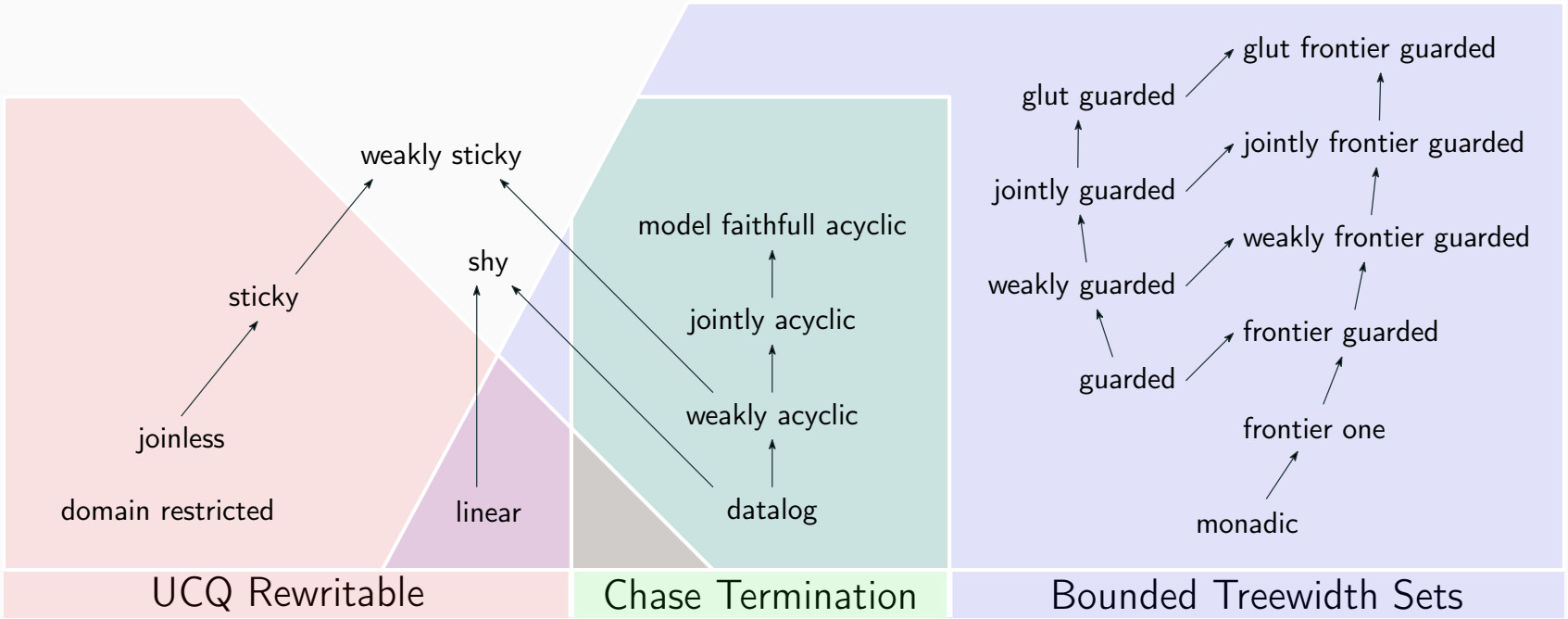

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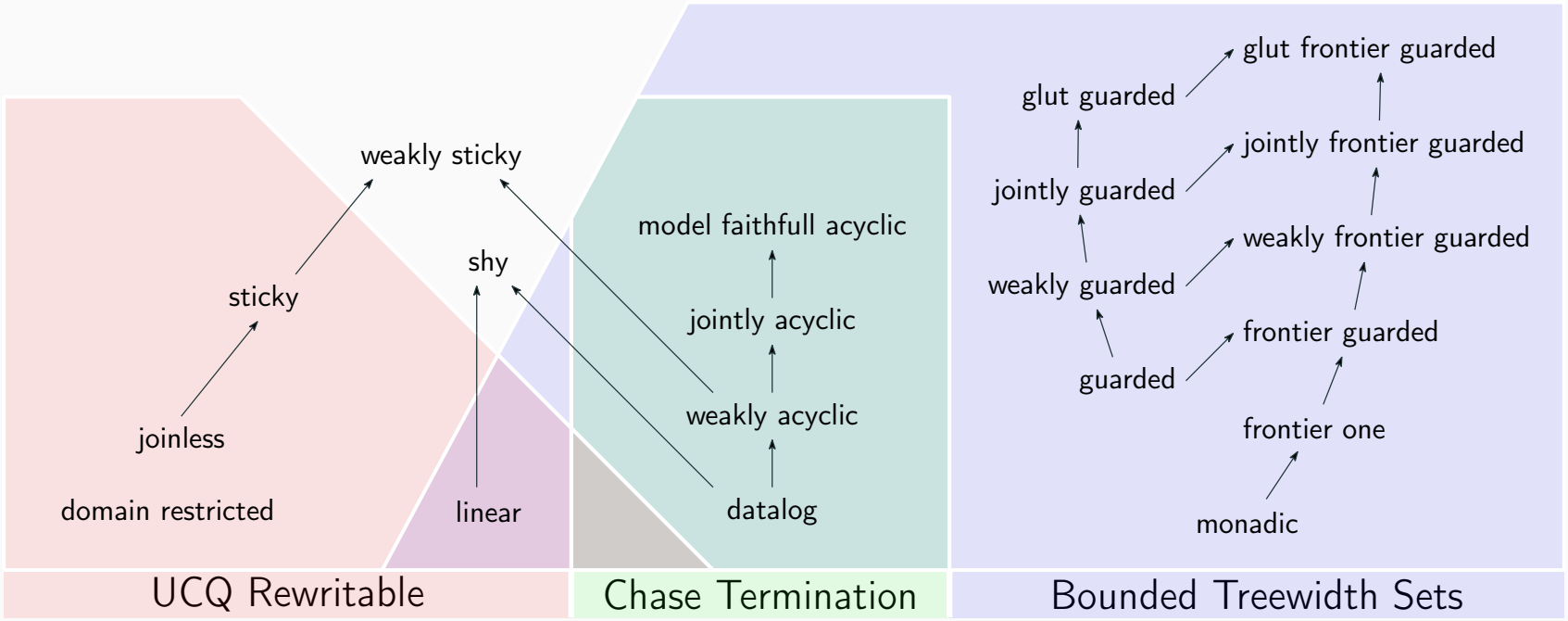

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small classes: decidable conjunctive query entailment and membership

This paper:
RPQs and Existential Rules

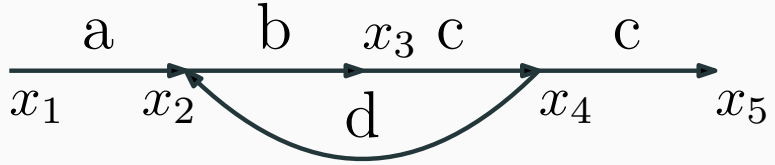
We want to focus on existential rules and Regular Path Queries with respect to the landscape.

Regular Path Queries

are defined by regular languages

$$Q = ab(c + d)^*$$

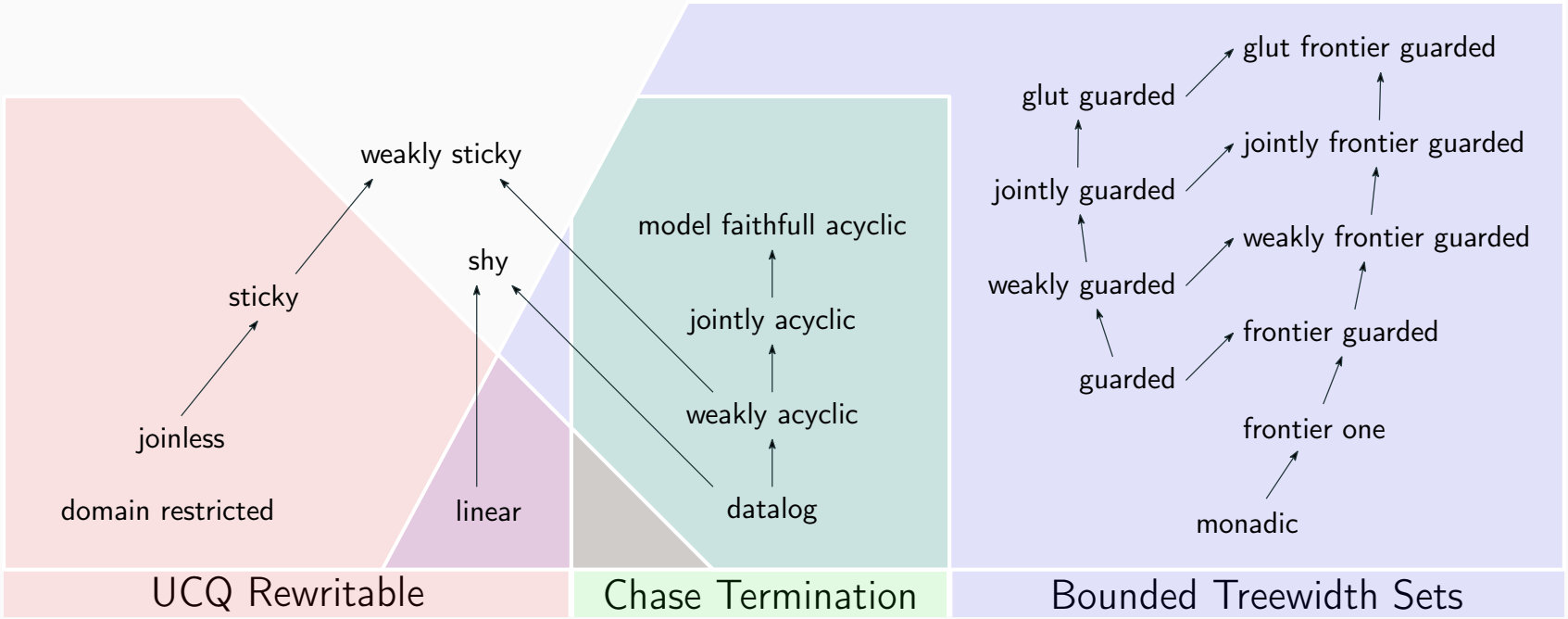
$D =$



$$Q(D) = (\langle x_1, x_3 \rangle, \langle x_1, x_4 \rangle, \langle x_1, x_5 \rangle, \langle x_1, x_2 \rangle)$$

Question?

How does the landscape translate to the RPQ case?



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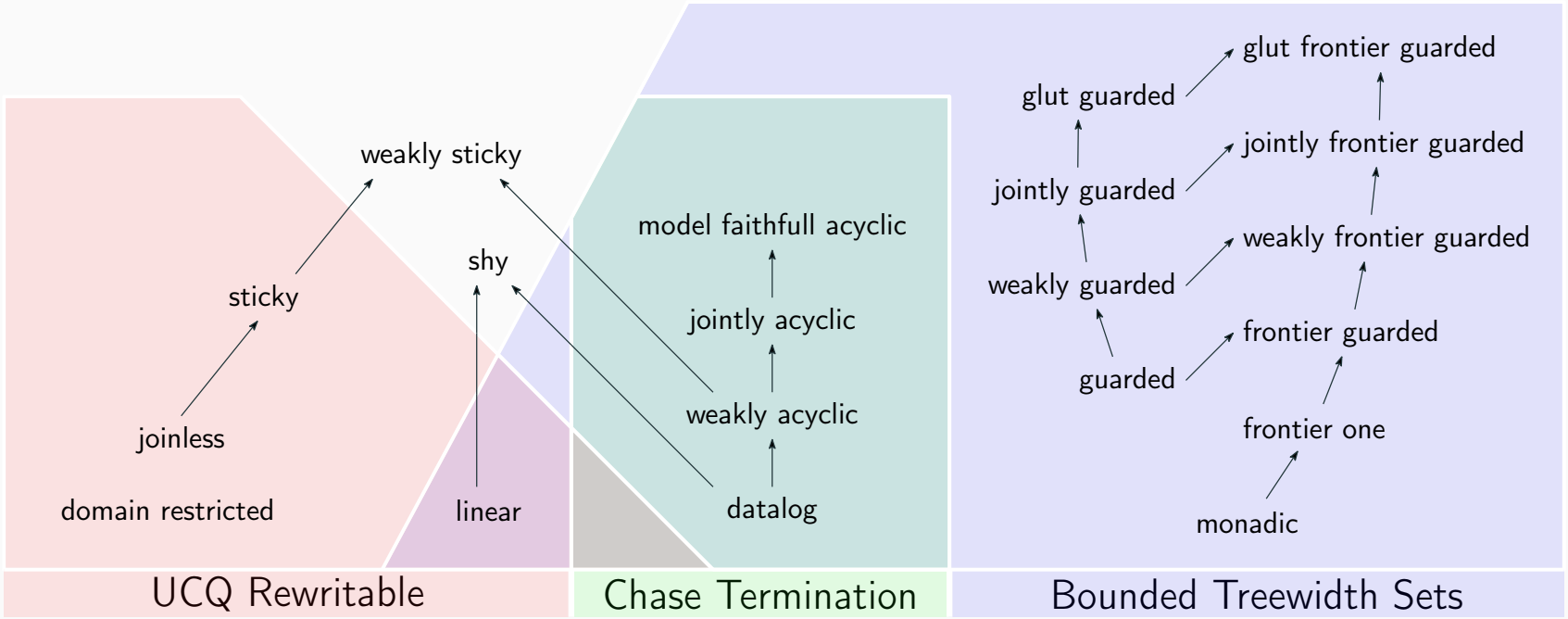

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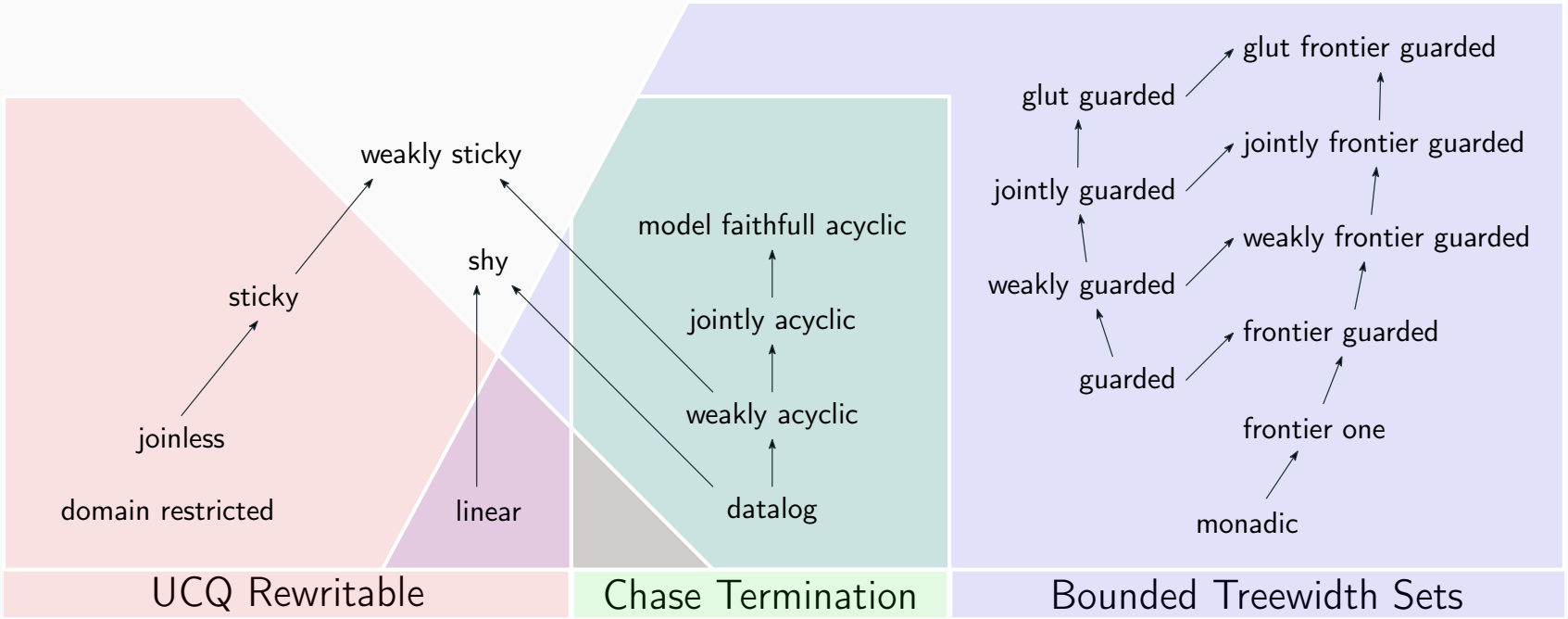

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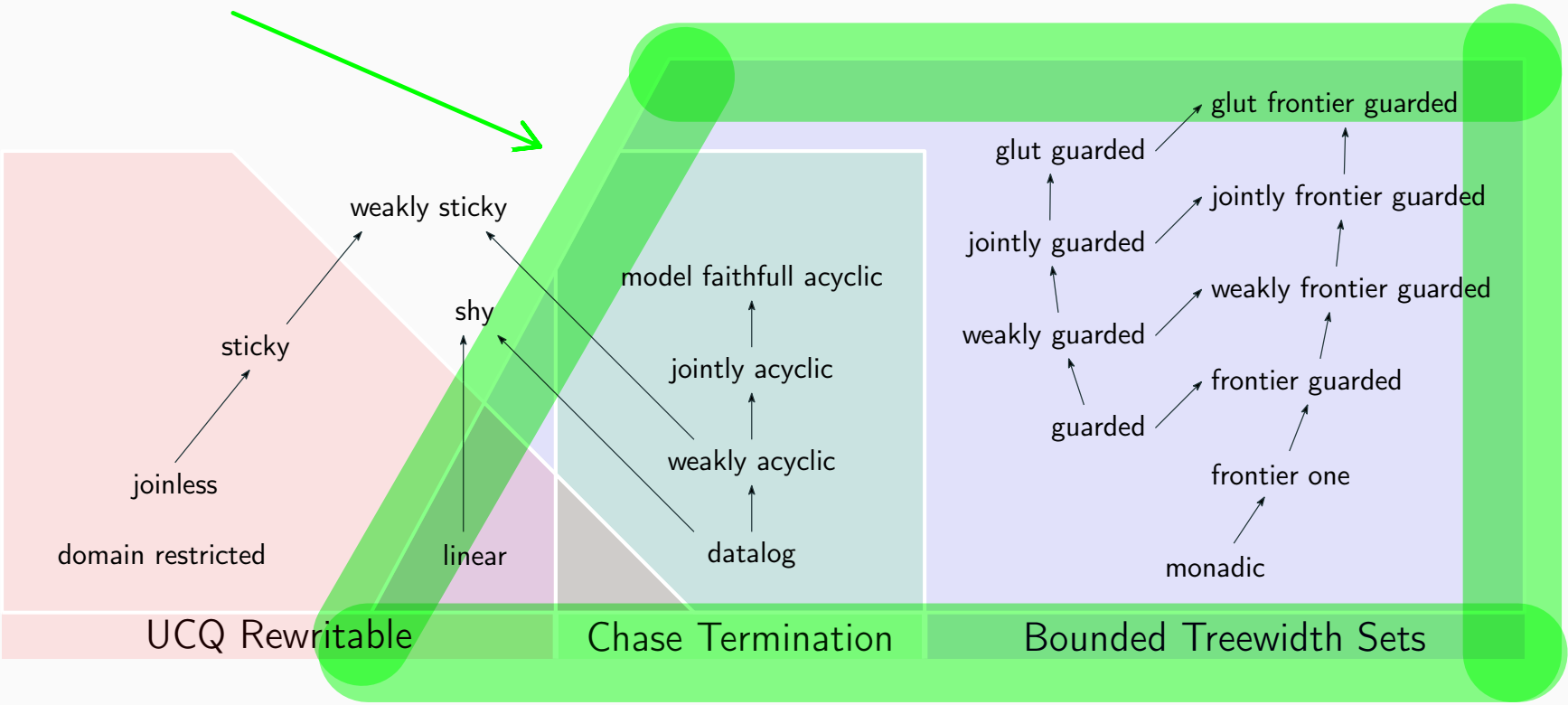

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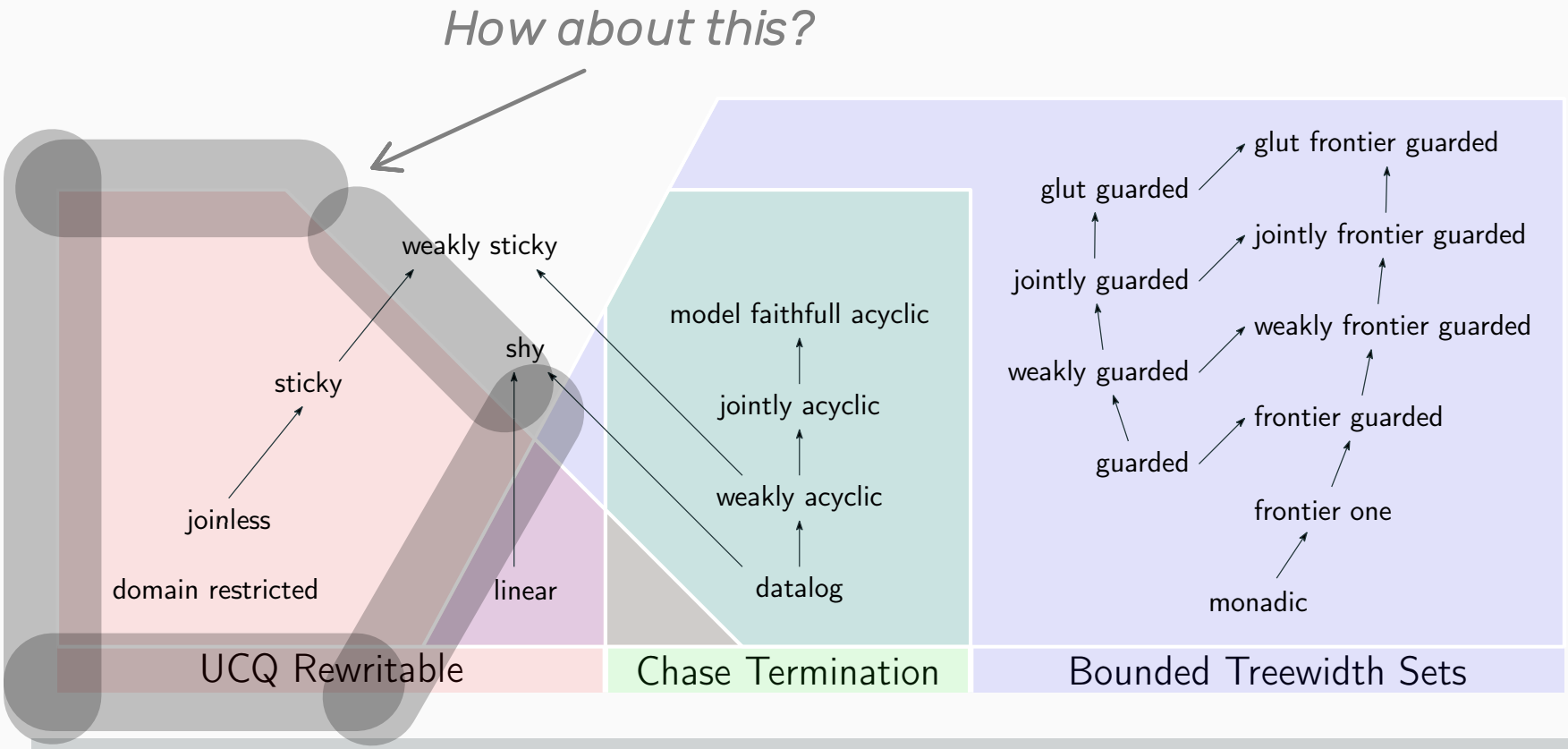

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If a ruleset is UCQ-rewritable then the entailment problem for CQs is decidable.

This paper:
Main Results

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Undecidability

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Decidability

The entailment problem for RPQs and the *sticky* subclass of UCQ-rewritable rulesets is decidable.

Decidability Gap

By tweaking the undecidability proof we can almost close the gap between decidable and undecidable!

Undecidability Proof

Source of Undecidability

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Determining whether a TCA $\mathcal{M}$ reaches the halting state q_{fin} from the starting state q_0 with both counters empty is **undecidable**.

The Reduction

Take a TCA $\mathcal{M}$ and produce $\mathcal{D}, \mathcal{R}$, and $Q_{\mathcal{M}}$

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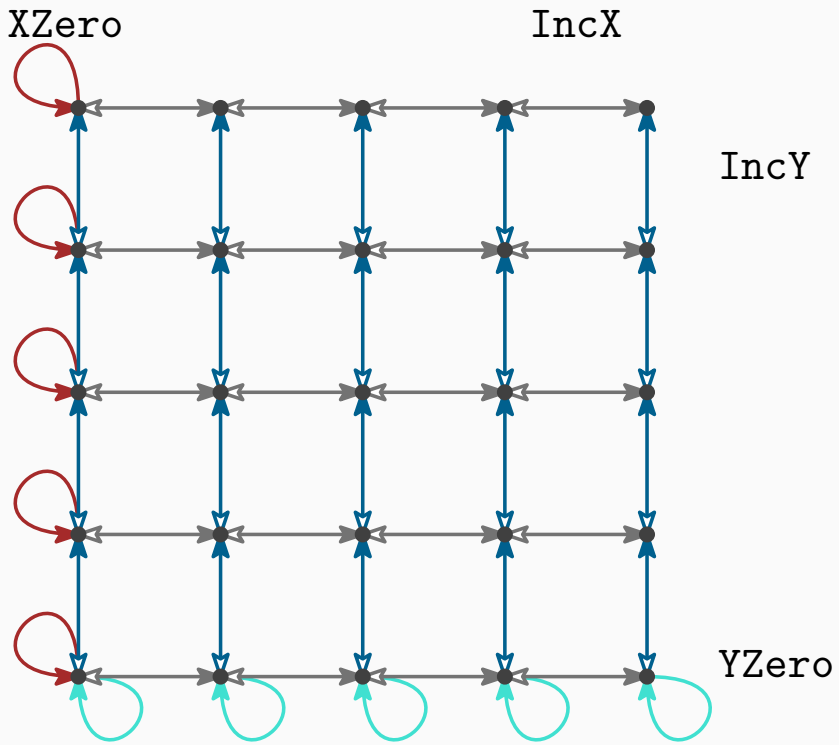

Chase of $\mathcal{D}$ and $\mathcal{R}$

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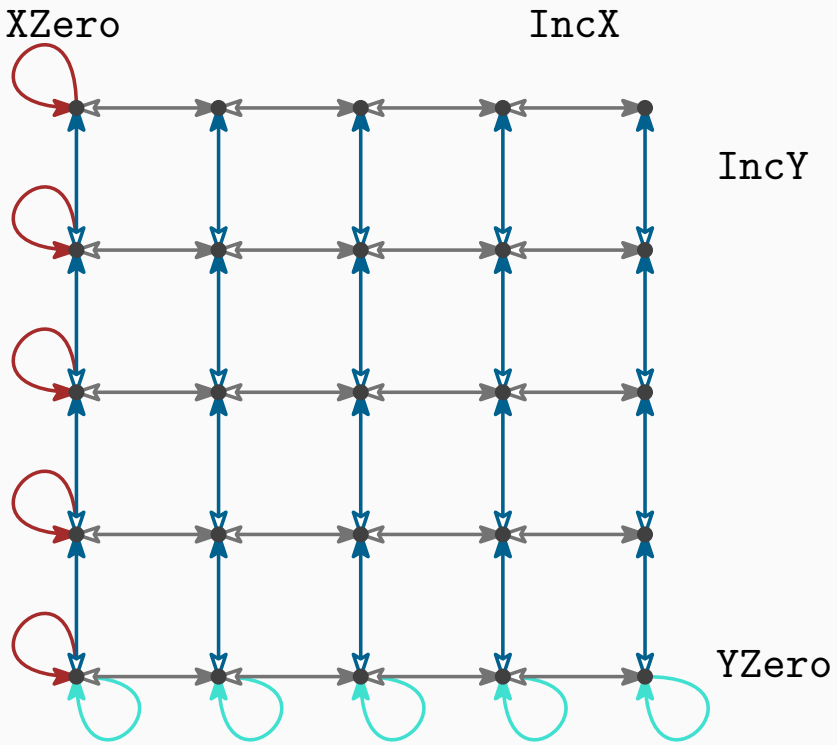
Regular Path Query $Q_{\mathcal{M}}$:

- Depends on $\mathcal{M}$!



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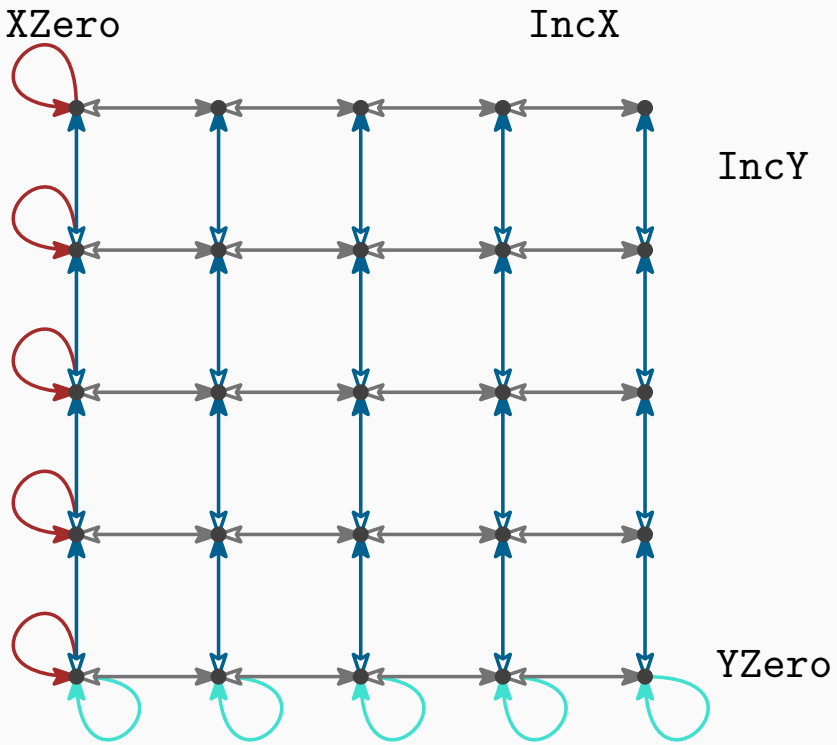

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Regular Path Query $Q_{\mathcal{M}}$:

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- Explained on the next slide

Simulating TCAs with RPQs

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Take TCA $\mathcal{M}$ and the instruction assigned to state q

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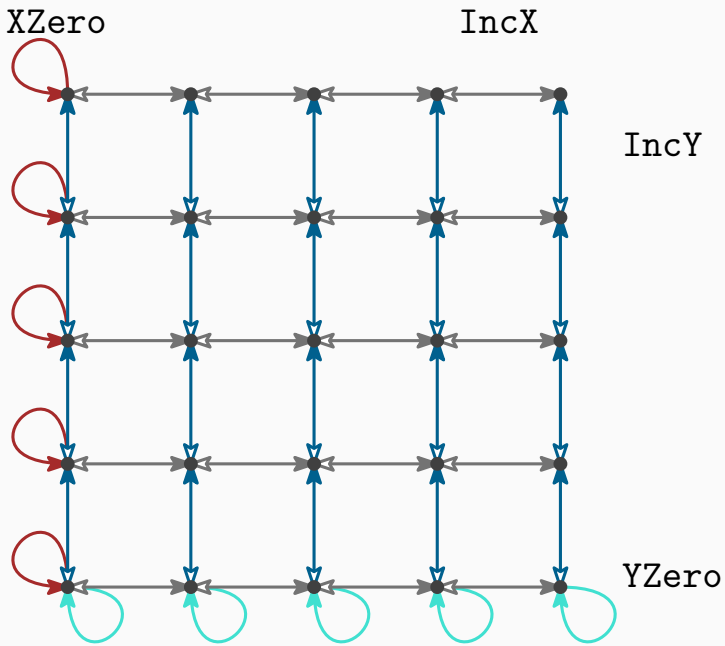
How RPQ reflects TCA

TCA in q with $\langle x, y \rangle$-counters $\cong$
RPQ in q at $\langle x, y \rangle$ in the grid



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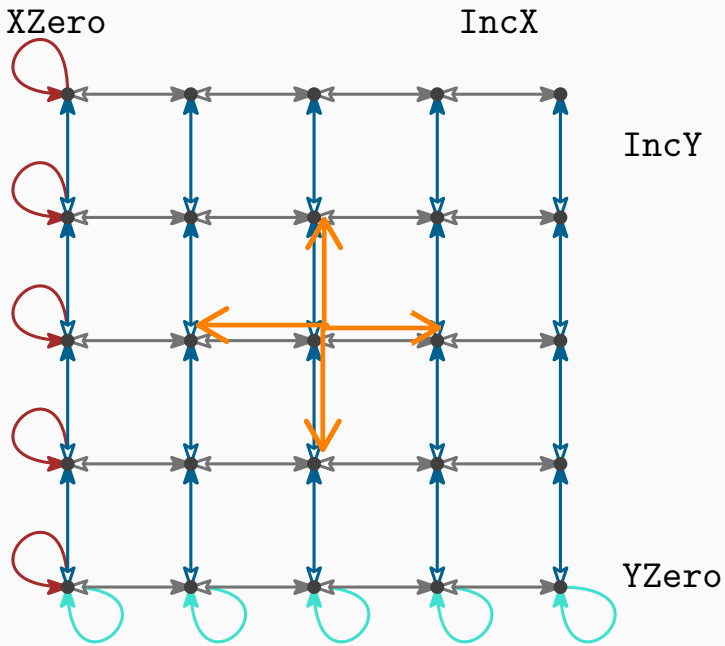

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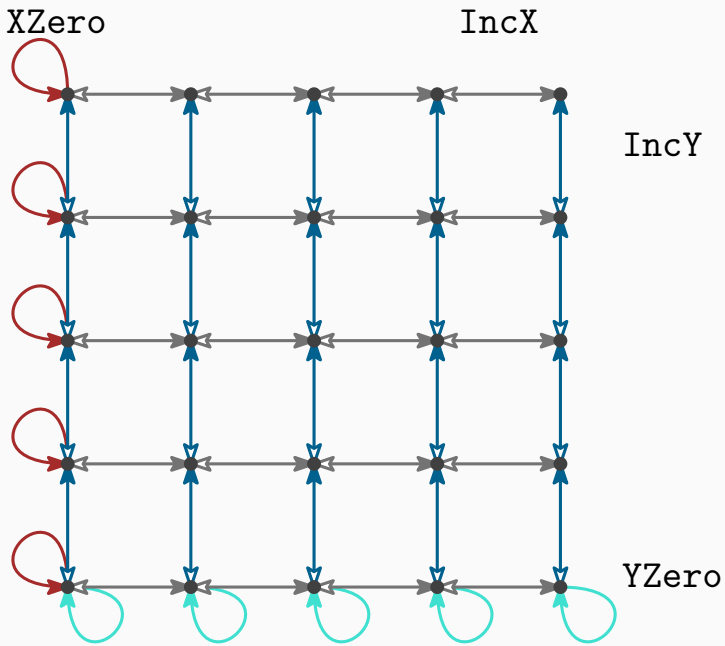

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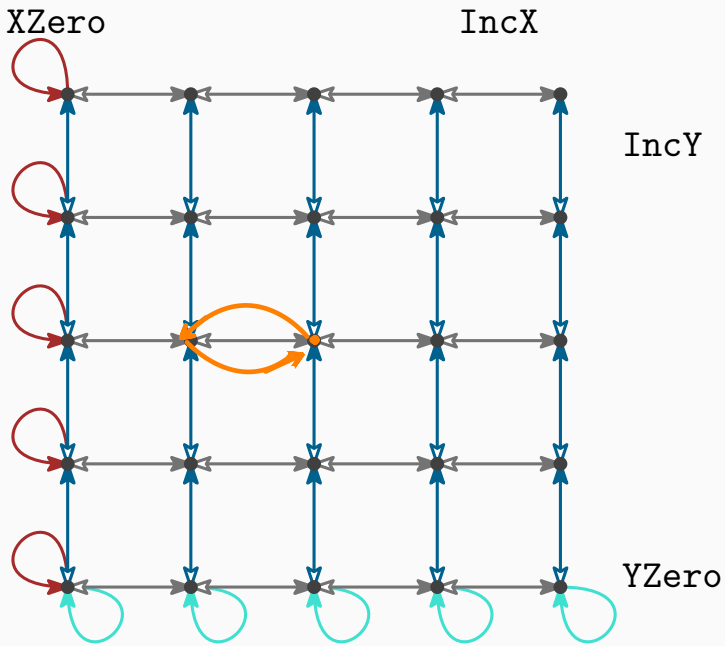

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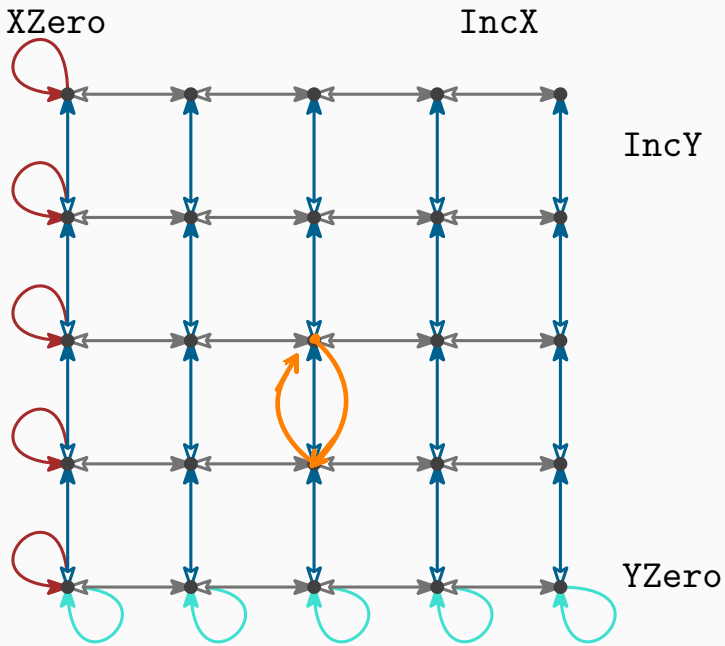

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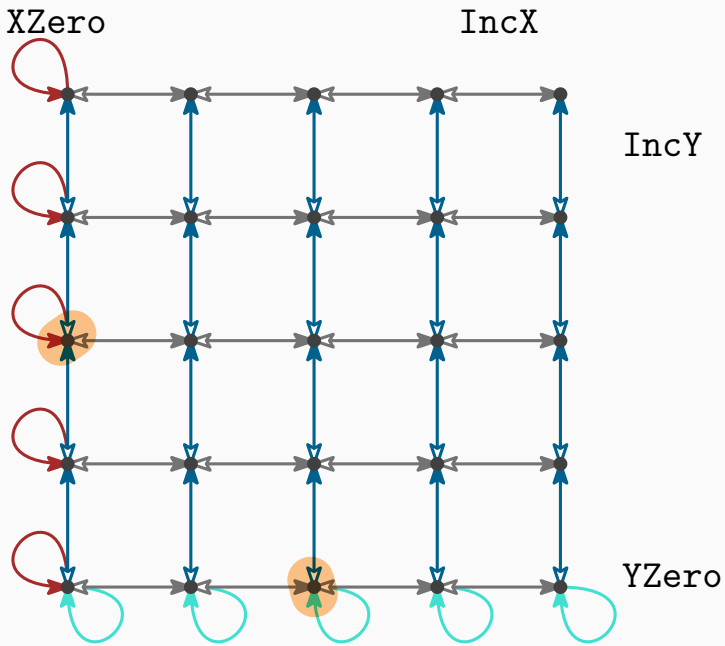

Decidability Proof

Overview of the Decidability Proof

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Given a database $\mathcal{D}$, a sticky ruleset $\mathcal{R}$ and an RPQ Q we do:

Recursive Enumerability

1. Encode Q in Datalog as Π_Q.
2. Enumerate FO consequences of $\mathcal{D} \wedge \mathcal{R} \wedge \Pi_Q$

Co-Recursive Enumerability

1. Define new *stellar* class of rulesets.
2. Stellar class is RPQ-finitely controllable.
3. Reduce sticky to stellar.
   - Extensive rewritability techniques,
   - Lots of ruleset surgeries,
   - *Heavily* relies on stickiness.

Main Results

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Decidability Gap

By tweaking the undecidability proof we can almost close the gap between decidable and undecidable!

Thank you!

Questions?