Exercise 4: Calculus of Communicating Systems

Concurrency Theory

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The SOS of CCS in One Slide

$$[\text{Pre}]\frac{}{a.P \xrightarrow{a} P} \qquad [\text{Res}]\frac{P \xrightarrow{\mu} P'}{(\nu a)P \xrightarrow{\mu} (\nu a)P'}\mu \notin \{a, \overline{a}\}$$

$$[\text{SumL}]\frac{P \xrightarrow{\mu} P'}{P + Q \xrightarrow{\mu} P'} \qquad [\text{SumR}]\frac{Q \xrightarrow{\mu} Q'}{P + Q \xrightarrow{\mu} Q'}$$

$$[\text{ParL}]\frac{P \xrightarrow{\mu} P'}{P \mid Q \xrightarrow{\mu} P' \mid Q} \qquad [\text{ParR}]\frac{Q \xrightarrow{\mu} Q'}{P \mid Q \xrightarrow{\mu} P \mid Q'}$$

$$[\text{Com}]\frac{P \xrightarrow{\mu} P' \quad Q \xrightarrow{\overline{\mu}} Q'}{P \mid Q \xrightarrow{\tau} P' \mid Q'}\mu \neq \tau$$

Exercise 4.1

With respect to bisimilarity, prove or disprove that parallel composition is

1. commutative
2. associative
3. neutral w.r.t. $\mathbf{0}$
4. idempotent.

Same task for summation.

Exercise 4.2

Does $(\nu a)(P \mid Q) \Leftrightarrow (\nu a)P \mid (\nu a)Q$ hold? What else does hold for the restriction operator?

Exercise 4.3

Definition 1: Let I be a finite set of indexes. A process of the form $\sum_{i\in I}\mu_i.P_i$ is in *head standard form*.

If $P = \sum_i \mu_i.P_i$ and $P' = \sum_j \mu'_j.P'_j$ are in head standard form, then

$$P \mid P' \Leftrightarrow \sum_i \mu_i.(P_i \mid P') + \sum_j \mu'_j.(P \mid P'_j) + \sum_{\overline{\mu_i}=\mu'_j} \tau.(P_i \mid P'_j).$$

Exercise 4.4

Which of the following equivalences hold? Why (not)?

1. $\mu.(P + Q) \Leftrightarrow \mu.P + \mu.Q$
2. $(P + Q) \mid R \Leftrightarrow (P \mid R) + (Q \mid R)$

Exercise 4.5

Let us call a function $f : \mathsf{Act} \to \mathsf{Act}$ a *relabeling function* if $f\,\overline{\alpha} = \overline{f\,\alpha}$ and $f\,\tau = \tau$. A relabeling of process P via f is written $P[f]$.

$$\text{[Rel]}\,\frac{P \xrightarrow{\mu} P'}{P[f] \xrightarrow{f\,\mu} P'[f]}$$

1. Show that $\Leftrightarrow$ is preserved by relabeling.
2. Show that $(P + Q)[f] \Leftrightarrow P[f] + Q[f]$
3. Is it true that $(P \mid Q)[f] \Leftrightarrow P[f] \mid Q[f]$? If not, how do we have to restrict f such that the equation holds?
4. What about $((\nu a)P)[f] \Leftrightarrow (\nu a)(P[f])$?

Exercise 4.6

Let $A \subseteq \mathcal{N}$ be a finite set of names (i.e., $A = \{a_1, \ldots, a_n\}$). Consider the operator $P \parallel_A Q$ defined as

$$P \parallel_A Q := (\nu a_1 \ldots a_n)(P \mid Q)$$

Show that $\parallel_A$ is commutative but not associative.