# Concurrency Theory

## Lecture 3: Bisimilarity and All That

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# Towards Bisimilarity

$p \equiv q$ if, for all $a \in \mathsf{Act}$,

1. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \equiv q'$;
2. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \equiv q'$.

**Problematic:** no deadlock 😈 no *base case*

**Definition 16 (Bisimulation, Bisimilarity)** A process relation $\mathcal{R} \subseteq \mathsf{Pr} \times \mathsf{Pr}$ is called a *(strong) bisimulation* if, for all $p, q \in \mathsf{Pr}$, $p \mathrel{\mathcal{R}} q$ implies

1. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \mathrel{\mathcal{R}} q'$, and
2. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \mathrel{\mathcal{R}} q'$

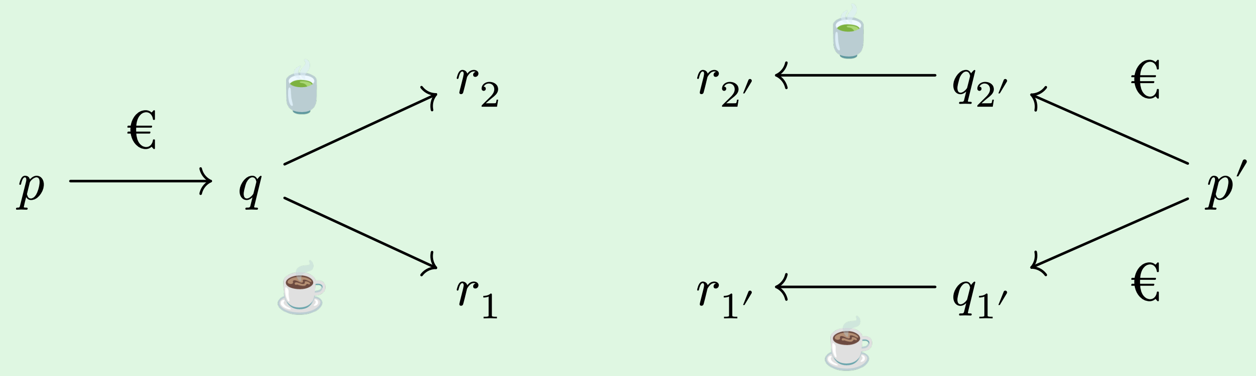
for all $a \in \mathsf{Act}$. We call $p$ and $q$ *bisimilar*, denoted $p \simeq q$, if there is a bisimulation $\mathcal{R}$ such that $p \mathrel{\mathcal{R}} q$. $\simeq$ is called *bisimilarity*.

# An Inductive Approach to Process Equivalence in Reverse

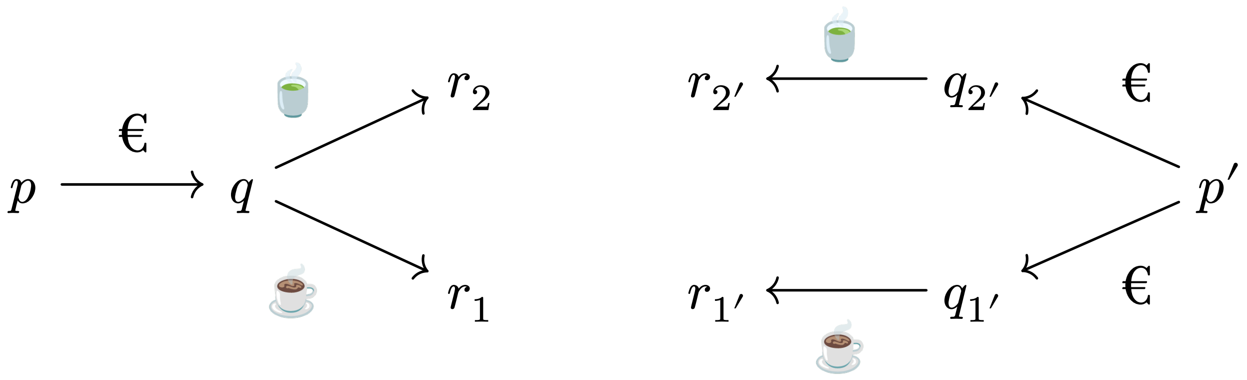
Compute $\simeq_0, \simeq_1, \ldots$ and define $\simeq_\omega := \bigcap_{i \geq 0} \simeq_i$ **for you, tj24**

1. set $\simeq_0 = \mathcal{U}$
2. $p \simeq_{n+1} q$ for $n \geq 0$ if for all $a \in \mathsf{Act}$:
   a. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \simeq_n q'$;
   b. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \simeq_n q'$.

**Example.**



# An Inductive Approach to Process Equivalence in Reverse




$$\simeq_0 = \{(p,p), \cancel{(p,q)}, \cancel{(p,r_1)}, \cancel{(p,r_2)}, (p,p'), \ldots, (q,q), \cancel{(q,q_{1'})}, \cancel{(q,q_{2'})}, \ldots\}$$

$$\simeq_1 = \{(p,p), \cancel{(p,p')}, \cancel{(p',p)}, (p',p'), (q,q), (q_{1'},q_{1'}), (q_{2'},q_{2'}), \ldots, (r_1,r_{1'}), (r_1,r_{2'}), \ldots\}$$

$$\simeq_2 = \{(p,p), (p',p'), (q,q), (q_{1'},q_{1'}), (q_{2'},q_{2'}), \ldots\} = \simeq_\omega$$

$$p \not\simeq_\omega p'$$

# Bisimilarity and Two Examples

**Definition 16 (Bisimulation, Bisimilarity)** A process relation $\mathcal{R} \subseteq \mathsf{Pr} \times \mathsf{Pr}$ is called a *(strong) bisimulation* if, for all $p, q \in \mathsf{Pr}$, $p \mathrel{\mathcal{R}} q$ implies

1. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \mathrel{\mathcal{R}} q'$, and
2. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \mathrel{\mathcal{R}} q'$

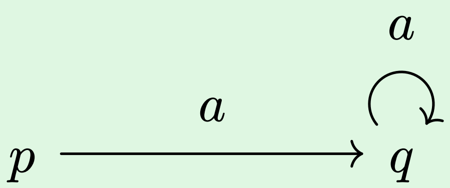
for all $a \in \mathsf{Act}$. We call $p$ and $q$ *bisimilar*, denoted $p \simeq q$, if there is a bisimulation $\mathcal{R}$ such that $p \mathrel{\mathcal{R}} q$. $\simeq$ is called *bisimilarity*.

## Consequences

1. bisimilarity $\simeq$ is the union of all bisimulations
2. showing that $p \simeq q$ holds reduces to finding a bisimulation $\mathcal{R}$ such that $p \mathrel{\mathcal{R}} q$
3. conversely, $p \not\simeq q$ can be shown by excluding the existence of any such bisimulation $\mathcal{R}$

# Bisimilarity and Two Examples

**Example.**

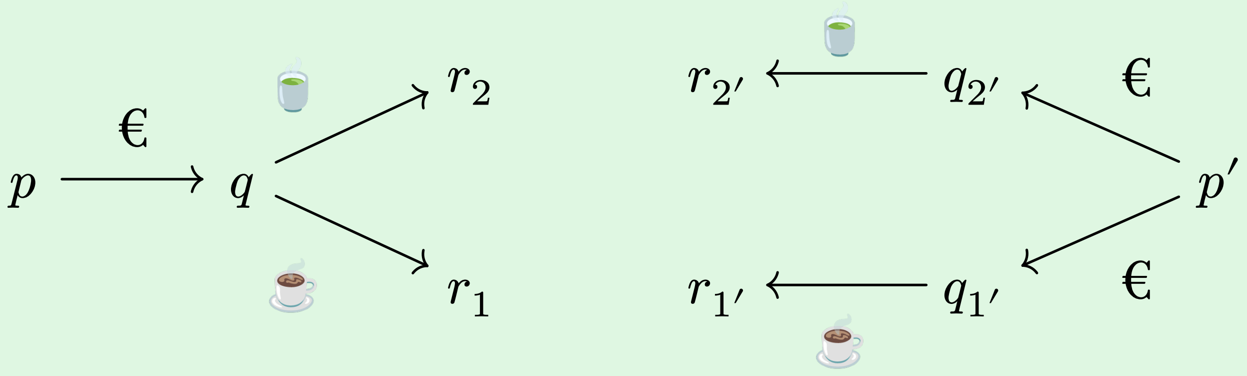



*$p \simeq q$ by $\mathcal{R} = \{(p, q), (q, q)\}$, but $\mathcal{R}' = \{(p, q), (q, p)\}$ is not a bisimulation.* $\blacksquare$

***Recall:*** $p \not\leftrightarrow q$.

# Bisimilarity and Two Examples

## Example.




*Towards a contradiction, suppose* $p \simeq p'$. *Then there is a bisimulation* $\mathcal{R}$ *with* $p \mathrel{\mathcal{R}} p'$. *As* $\mathcal{R}$ *is a bisimulation,* $q \mathrel{\mathcal{R}} q_{1'}$ *since* $p' \xrightarrow{€} q_{1'}$ *and* $p \xrightarrow{€} q$. *But* $q \mathrel{\mathcal{R}} q_{1'}$ *cannot hold since* $q \xrightarrow{🍵} r_2$ *whereas* $q_{1'} \not\xrightarrow{🍵}$. ∎

***Recall:*** $p \equiv_{\mathsf{tr}} p'$ *and* $p \equiv_{\mathsf{ctr}} p'$.

# Disecting Bisimilarity

**Definition 16 (Bisimulation, Bisimilarity)** A process relation $\mathcal{R} \subseteq \mathsf{Pr} \times \mathsf{Pr}$ is called a *(strong) bisimulation* if, for all $p, q \in \mathsf{Pr}$, $p \mathrel{\mathcal{R}} q$ implies

1. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \mathrel{\mathcal{R}} q'$, and
2. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \mathrel{\mathcal{R}} q'$

for all $a \in \mathsf{Act}$. We call $p$ and $q$ *bisimilar*, denoted $p \simeq q$, if there is a bisimulation $\mathcal{R}$ such that $p \mathrel{\mathcal{R}} q$. $\simeq$ is called *bisimilarity*.

Proofs of bisimilarity are

- *local* — checks performed on states separately
- *non-hierarchical* — no fixed temporal order
- require no **base case** — this is **not** *induction*

It is, in fact, an example of **coinduction**

(We had already seen what happens if we read Definition 16 inductively.)

# Disecting Bisimilarity

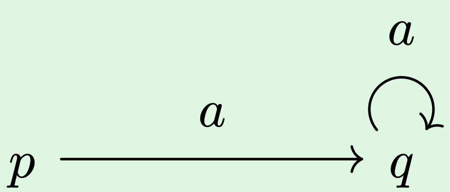
**Theorem 17** $\simeq$ is a process equivalence that is itself a bisimulation.

*Proof:* We have to show that $\simeq$ is (1) an equivalence and (2) a bisimulation.

to be continued… ∎

Not every bisimulation is an equivalence:

**Example.**




*$p \simeq q$ by $\mathcal{R} = \{(p,q),(q,q)\}$ which is **neither** reflexive **nor** symmetric.*

# Disecting Bisimilarity

**Theorem 17** $\simeq$ is a process equivalence that is itself a bisimulation.

*Proof:* We have to show that $\simeq$ is (1) an equivalence and (2) a bisimulation.

**Reflexivity** $\mathrm{id} : \mathsf{Pr} \to \mathsf{Pr}$ is, in fact, a bisimulation. For $p \;\mathrm{id}\; q$ (i.e., $\mathrm{id}(p) = q$), we get $p \xrightarrow{a} p'$ iff $q = \mathrm{id}(p) = p \xrightarrow{a} p' = \mathrm{id}(p') = q'$. The same holds for steps from $\mathrm{id}(p)$.

**Symmetry** If $\mathcal{R}$ is a bisimulation, then $\mathcal{R}^{-1} := \{(q, p) \mid p \;\mathcal{R}\; q\}$ is a bisimulation.

**Transitivity** Let $\mathcal{R}_1, \mathcal{R}_2$ be bisimulations. We subsequently show that $\mathcal{R}_1 \circ \mathcal{R}_2 := \{(x, z) \mid \exists y. x \;\mathcal{R}_1\; y \wedge y \;\mathcal{R}_2\; z\}$ is a bisimulation. For $p \;\mathcal{R}_1 \circ \mathcal{R}_2\; q$ and $p \xrightarrow{a} p'$,

1. there is an $r$ such that $x \;\mathcal{R}_1\; r$ and $r \;\mathcal{R}_2\; q$; — by definition of $\mathcal{R}_1 \circ \mathcal{R}_2$
2. there is an $r'$ such that $r \xrightarrow{a} r'$ and $p' \;\mathcal{R}_1\; r'$ — since $\mathcal{R}_1$ is a bisimulation
3. there is a $q'$ such that $q \xrightarrow{a} q'$ and $r' \;\mathcal{R}_2\; q'$ — since $\mathcal{R}_2$ is a bisimulation
4. hence, by taking that $q'$, we get $p' \;\mathcal{R}_1 \circ \mathcal{R}_2\; q'$ — by definition of $\mathcal{R}_1 \circ \mathcal{R}_2$

Since bisimulations are union-closed (by Lemma 18, cf. next slide) and $\simeq$ is the union of all bisimulations, $\simeq$ is itself a bisimulation. ∎

# Disecting Bisimilarity

**Lemma 18** Bisimulations are closed under set unions: If $\{\mathcal{R}_i\}_i$ is a (at most countable) family of bisimulations, then $\bigcup_i \mathcal{R}_i$ is a bisimulation.

Towards a special case, take two bisimulations $\mathcal{R}_1$ and $\mathcal{R}_2$ and consider $\mathcal{R}_1 \cup \mathcal{R}_2$:

Take $p \; \mathcal{R}_1 \cup \mathcal{R}_2 \; q$ and consider $p \xrightarrow{a} p'$.

1. if $p \; \mathcal{R}_1 \; q$, then there is a $q'$ such that $q \xrightarrow{a} q'$ and $p' \; \mathcal{R}_1 \; q'$ — $\mathcal{R}_1$ is a bisimulation
2. if $p \; \mathcal{R}_2 \; q$, then there is a $q'$ such that $q \xrightarrow{a} q'$ and $p' \; \mathcal{R}_2 \; q'$ — $\mathcal{R}_2$ is a bisimulation

In both cases, there is a $q'$ such that $q \xrightarrow{a} q'$ and $p \; \mathcal{R}_1 \cup \mathcal{R}_2 \; q$. Same for $q \xrightarrow{a} q'$.

*Proof:* If each $\mathcal{R}_i$ is a bisimulation, then $\mathcal{R} = \bigcup_i \mathcal{R}_i$ is a bisimulation. For each pair $p \; \mathcal{R} \; q$, there is a $\mathcal{R}_i$ such that $p \; \mathcal{R}_i \; q$.

1. if $p \xrightarrow{a} p'$, there is a $q'$ such that $q \xrightarrow{a} q'$ and $p' \; \mathcal{R}_i \; q'$ — $\mathcal{R}_i$ is a bisimulation
2. if $q \xrightarrow{a} q'$, there is a $p'$ such that $p \xrightarrow{a} p'$ and $p' \; \mathcal{R}_i \; q'$ — $\mathcal{R}_i$ is a bisimulation

In each case $p' \; \mathcal{R}_i \; q'$ and, thus, $p' \; \mathcal{R} \; q'$. ∎

# Yet Another Characterization of $\simeq$

**Theorem 19** $\simeq$ is the largest bisimulation, i.e., the largest process relation $\simeq$ such that $p \simeq q$ implies for all $a \in \mathsf{Act}$:

1. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \simeq q'$, and
2. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \simeq q'$.

*Proof:* By Theorem 17, $\simeq$ is a bisimulation. It remains to be shown that it is the **unique** largest one.

Consider two largest bisimulations $\underset{1}{\simeq}$ and $\underset{2}{\simeq}$. Since bisimulations are union-closed (by Lemma 18), $\underset{1}{\simeq} \cup \underset{2}{\simeq}$ is a bisimulation as well, implying that $\underset{1}{\simeq} = \underset{1}{\simeq} \cup \underset{2}{\simeq}$ and $\underset{2}{\simeq} = \underset{1}{\simeq} \cup \underset{2}{\simeq}$ to not contradict the assumption that $\underset{1}{\simeq}$ and $\underset{2}{\simeq}$ were chosen to be largest. Thus, $\simeq$ is the *unique* largest bisimulation. ∎

# Bisimilarity is an Example for Branching-Time

**Theorem 20**

$$\leftrightarrow \overset{(1)}{\subsetneq} \simeq \overset{(2)}{\subsetneq} \equiv_{\mathsf{ctr}} \subsetneq \equiv_{\mathsf{tr}}$$

*Proof:*

(1) Let $f : \mathsf{Pr} \to \mathsf{Pr}$ be an isomorphism. We show, $f$ is a bisimulation.
For $p\ f\ q$ (i.e., $f(p) = q$),

$$p \xrightarrow{a} p' \text{ iff } f(p) \xrightarrow{a} f(p') \quad \text{since } f \text{ is an isomorphism}$$
$$\text{iff } \exists q'.q \xrightarrow{a} q' \quad \text{by } f(p) = q \text{ take } q' = f(p')$$

We have $p'\ f\ q'$ since $f(p') = q'$. The second direction is analogous.

Towards $\leftrightarrow \neq \simeq$, $\simeq$ is insensitive to branch duplicates.

∎

# Bisimilarity is an Example for Branching-Time

**Theorem 20**

$$\leftrightarrow \overset{(1)}{\subsetneq} \simeq \overset{(2)}{\subsetneq} \equiv_{\mathsf{ctr}} \subsetneq \equiv_{\mathsf{tr}}$$

*Proof:*

(2) Let $p, q \in \mathsf{Pr}$ such that $p \simeq q$. We need to show that $p \equiv_{\mathsf{ctr}} q$ , meaning $\mathsf{ctraces}(p) = \mathsf{ctraces}(q)$. It is sufficient to show that $\mathsf{ctraces}(p) \subseteq \mathsf{ctraces}(q)$ since the other direction follows by symmetry (process equivalences are symmetric).

Let $\sigma \in \mathsf{ctraces}(p)$ with $\sigma = a_1 a_2 \ldots a_n$. Then there are states $p_1, p_2, \ldots, p_n$ such that $p \xrightarrow{a_1} p_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} p_n$ and $p_n$ is a deadlock.

Since $p \simeq q$, there are $q_1, q_2, \ldots, q_n$ such that $q \xrightarrow{a_1} q_1 \xrightarrow{a_2} \cdots \xrightarrow{a_n} q_n$ such that $p_i \simeq q_i$ $(i = 1, \ldots, n)$. In particular, $q_n$ is a deadlock. Thus, $a_1 a_2 \ldots a_n = \sigma \in \mathsf{ctraces}(q)$.

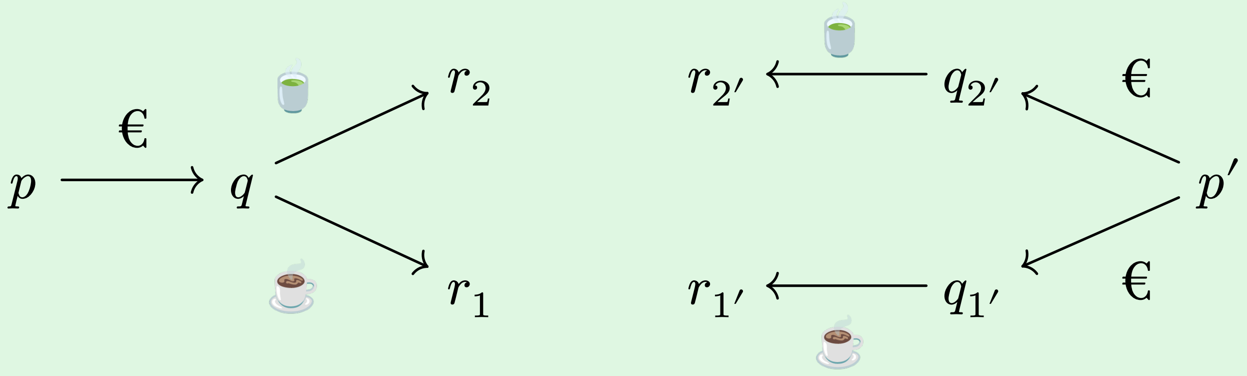
∎

# Counterexample for $\simeq = \equiv_{\mathsf{ctr}}$

## Theorem 20

$$\leftrightarrow \quad \overset{(1)}{\subsetneq} \quad \simeq \quad \overset{(2)}{\subsetneq} \quad \equiv_{\mathsf{ctr}} \quad \subsetneq \quad \equiv_{\mathsf{tr}}$$

## Example.




$$p \not\simeq p' \text{ but } p \equiv_{\mathsf{ctr}} p'$$

# What about $\simeq_\omega$?

**Definition 16 (Bisimulation, Bisimilarity)** A process relation $\mathcal{R} \subseteq \mathsf{Pr} \times \mathsf{Pr}$ is called a *(strong) bisimulation* if, for all $p, q \in \mathsf{Pr}$, $p \mathrel{\mathcal{R}} q$ implies

1. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \mathrel{\mathcal{R}} q'$, and
2. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \mathrel{\mathcal{R}} q'$

for all $a \in \mathsf{Act}$. We call $p$ and $q$ *bisimilar*, denoted $p \simeq q$, if there is a bisimulation $\mathcal{R}$ such that $p \mathrel{\mathcal{R}} q$. $\simeq$ is called *bisimilarity*.

$\simeq_\omega := \bigcap_{i \geq 0} \simeq_i$

1. set $\simeq_0 = \mathcal{U}$
2. $p \simeq_{n+1} q$ for $n \geq 0$ if for all $a \in \mathsf{Act}$:
   a. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \simeq_n q'$;
   b. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \simeq_n q'$.

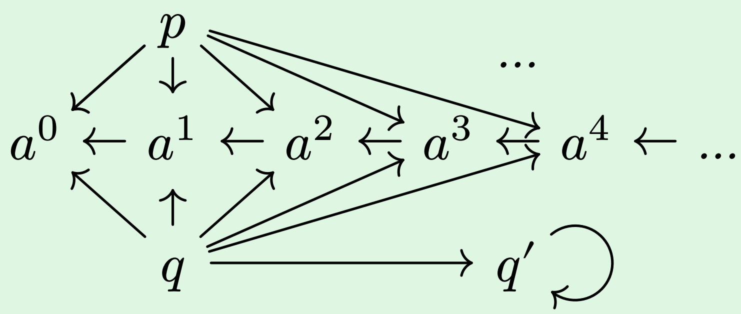
Do the two views on process equivalence, $\simeq$ and $\simeq_\omega$, coincide?

# What about $\simeq_\omega$?

$\simeq_\omega := \bigcap_{i \geq 0} \simeq_i$

1. set $\simeq_0 = \mathcal{U}$
2. $p \simeq_{n+1} q$ for $n \geq 0$ if for all $a \in \mathsf{Act}$:
   a. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \simeq_n q'$;
   b. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \simeq_n q'$.

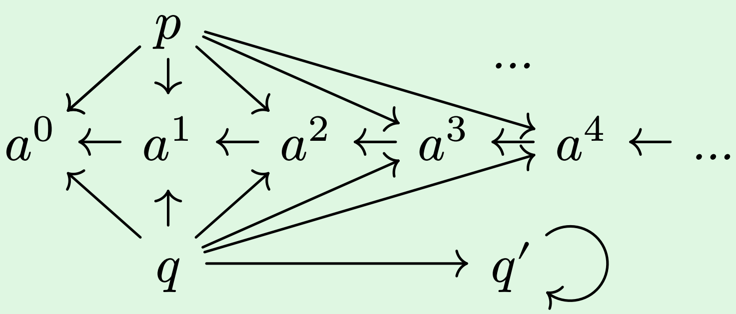
**Example.**




😔 We're going to show that $p \simeq_\omega q$ but $p \not\simeq q$ 😔

# What about $\simeq_\omega$?

**Example.**

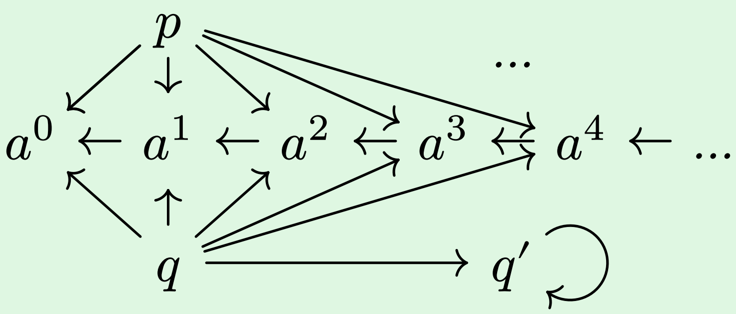


***Claim:*** *For each $n \in \mathbb{N}$, we get $p \simeq_n q$.* ***$p \simeq_\omega q$ follows (why?)***

1. *$n = 0$, $p \simeq_n q$ since $\simeq_0 = \mathsf{Pr} \times \mathsf{Pr}$ is the universal process equivalence.*
2. *$n \to n+1$,*
   - *if $q \longrightarrow q'$, $p$ answers by $p \longrightarrow a^n$;* *$a^n \simeq_n q'$ for all $n \in \mathbb{N}$.*
   - *if $q \longrightarrow a^k$, answer by $p \longrightarrow a^k$, and vice versa.* *exploit reflexivity of $\simeq_n$*

# What about $\simeq_\omega$?

**Example.**



***Claim:*** *For each* $n \in \mathbb{N}$, $a^n \simeq_n q'$

1. $n = 0$, ✓
2. $n \to n+1$, $a^{n+1}$ *still has* $n+1$ *steps to go until it deadlocks in* $a^0$.

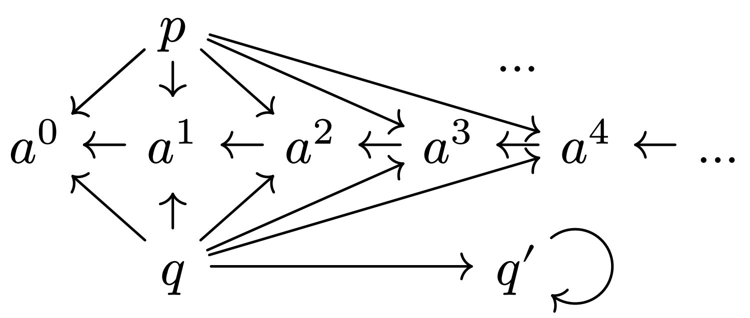
***Another Fact:*** *For each* $m, n \in \mathbb{N}$, $a^m \simeq_n q'$ *if* $m \geq n$.

# Towards $p \not\simeq q$

**Definition 16 (Bisimulation, Bisimilarity)** A process relation $\mathcal{R} \subseteq \mathsf{Pr} \times \mathsf{Pr}$ is called a *(strong) bisimulation* if, for all $p, q \in \mathsf{Pr}$, $p \mathrel{\mathcal{R}} q$ implies

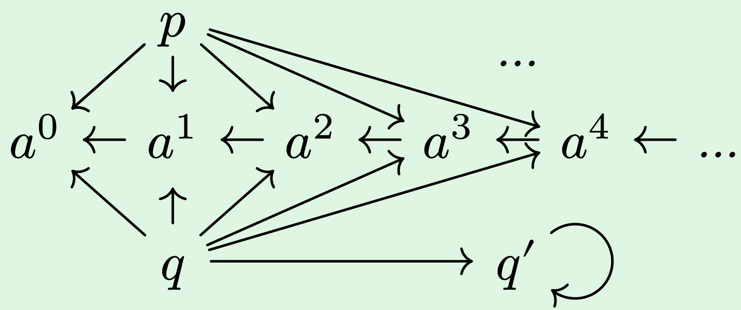
1. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \mathrel{\mathcal{R}} q'$, and
2. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \mathrel{\mathcal{R}} q'$

for all $a \in \mathsf{Act}$. We call $p$ and $q$ *bisimilar*, denoted $p \simeq q$, if there is a bisimulation $\mathcal{R}$ such that $p \mathrel{\mathcal{R}} q$. $\simeq$ is called *bisimilarity*.



# Towards $p \not\simeq q$

**Example.**



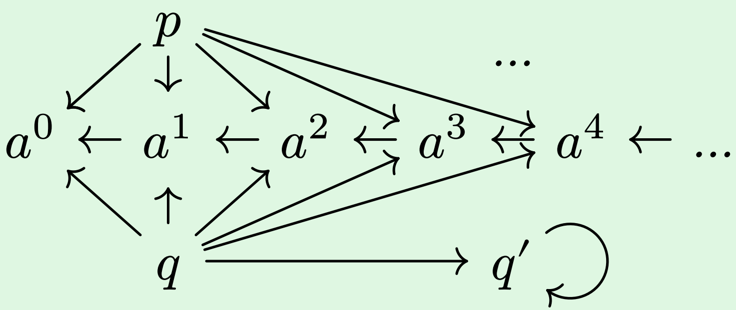

*Assume, there is a bisimulation $\mathcal{R}$ such that $p \mathrel{\mathcal{R}} q$. Then for $q \longrightarrow q'$, there is some $m \in \mathbb{N}$, so that $p \longrightarrow a^m$ and $a^m \mathrel{\mathcal{R}} q'$.*

***Claim:*** *For all $n \in \mathbb{N}$, $a^n \not\simeq q'$.*

1. $n = 0$, $a^n \not\longrightarrow$ *whereas* $q' \longrightarrow q'$.
2. $n \to n+1$, $a^{n+1} \longrightarrow a^n$. *Thus,* $a^{n+1} \simeq q'$ *if and only if* $a^n \simeq q'$. *By induction hypothesis,* $a^n \not\simeq q'$. *In conclusion,* $a^{n+1} \not\simeq q'$.

# What is Wrong with $\simeq_\omega$?

**Example.**




1. $p$ is
   - acyclic,
   - infinite-state,
   - infinitely branching, and
   - **not** even image-finite
2. $q$ is cyclic, …, and **not** even image-finite

# What is Wrong with $\simeq_\omega$?

**Theorem 21** $\simeq$ and $\simeq_\omega$ coincide on *image-finite* LTSs.

*Proof:* We prove both directions separately. Consider all processes and, in fact, the underlying LTS to be *image-finite*.

$\simeq \subseteq \simeq_\omega$ For each $n \in \mathbb{N}$, we show that $p \simeq q$ implies $p \simeq_n q$.

**$n = 0$** Since $\simeq_n = \simeq_0 = \mathsf{Pr} \times \mathsf{Pr}$, $p \simeq_n q$ holds trivially.

**Hypothesis** For $n \in \mathbb{N}$, $p \simeq q$ implies $p \simeq_n q$.

**$n \to n+1$** If $p \simeq q$ holds, we show that $p \simeq_{n+1} q$. For each $a \in \mathsf{Act}$

1. if $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \simeq q'$. By induction hypothesis, $p' \simeq q'$ implies $p' \simeq_n q'$.
2. if $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \simeq q'$. By induction hypothesis, $p' \simeq q'$ implies $p' \simeq_n q'$.

Thus, every step of $p$ ($q$, resp.) can be answered such that their successors are related by $\simeq_n$, proving that $p \simeq_{n+1} q$ holds.

# What is Wrong with $\simeq_\omega$?

$\simeq_\omega \subseteq \simeq$ We show that $\mathcal{R} = \{(p, q) \mid p \simeq_\omega q\}$ is a bisimulation. Consider a pair $(p, q) \in \mathcal{R}$.

- Suppose, $p \xrightarrow{a} p'$.
- For all $n \in \mathbb{N}$,
  as $p \simeq_{n+1} q$, there is some $q_n$ such that $q \xrightarrow{a} q_n$ and $p' \simeq_n q_n$;
- Since $q$ is image-finite, the set $Q = \left\{ q' \;\middle|\; q \xrightarrow{a} q' \right\}$ is finite;
  thus, there must be one $q' \in Q$ such that $p' \simeq_n q'$ for each $n \in \mathbb{N} \Rightarrow p' \simeq_\omega q'$

∎

# What is Right with $\simeq_\omega$?

$\simeq_\omega := \bigcap_{i \geq 0} \simeq_i$

1. set $\simeq_0 = \mathcal{U}$
2. $p \simeq_{n+1} q$ for $n \geq 0$ if for all $a \in \mathsf{Act}$:
   a. for all $p'$ with $p \xrightarrow{a} p'$, there is a $q'$ with $q \xrightarrow{a} q'$ and $p' \simeq_n q'$;
   b. for all $q'$ with $q \xrightarrow{a} q'$, there is a $p'$ with $p \xrightarrow{a} p'$ and $p' \simeq_n q'$.

**Theorem 21** $\simeq$ and $\simeq_\omega$ coincide on *image-finite* LTSs.

1. Finite LTSs are image-finite. **recall**
2. How hard is it to compute $\simeq$ on finite LTSs $(\mathsf{Pr}, \mathsf{Act}, \longrightarrow)$? i.e., $\simeq_\omega$
   - compute $\simeq_0 = \mathcal{U}$ $\mathcal{O}(|\mathsf{Pr}|^2)$
   - iteratively remove all pairs from $\simeq_i$ contradicting bisimulations $\rightsquigarrow \simeq_{i+1}$ $\mathcal{O}(|\mathsf{Pr}|^3)$
   - stop when nothing changes after at most $|\mathsf{Pr}|^2$ removals

Compare with $\equiv_{\mathsf{tr}}$ (PSPACE-complete)