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Representative Answer Sets: Collecting Something of Everything

Kraków, Poland, October 4th 2023

Motivation

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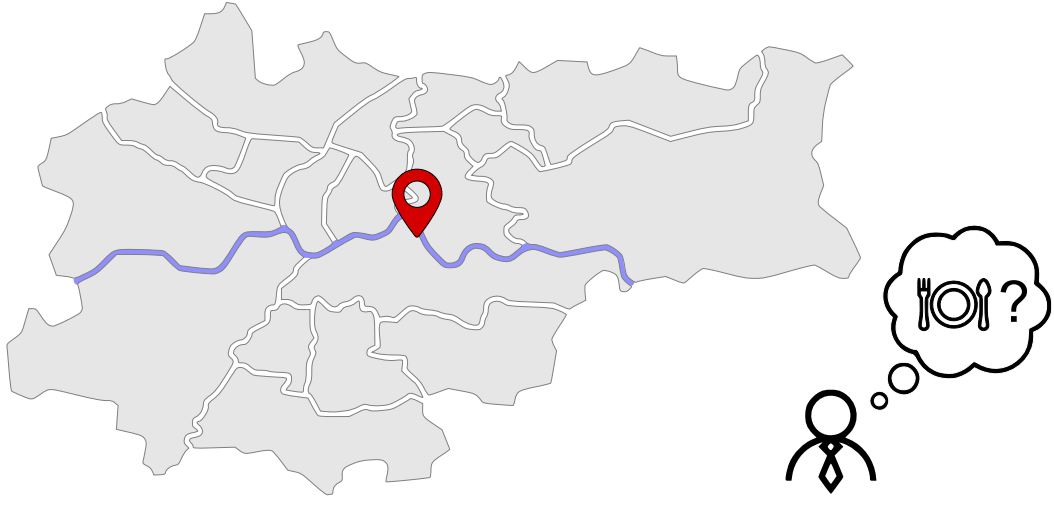

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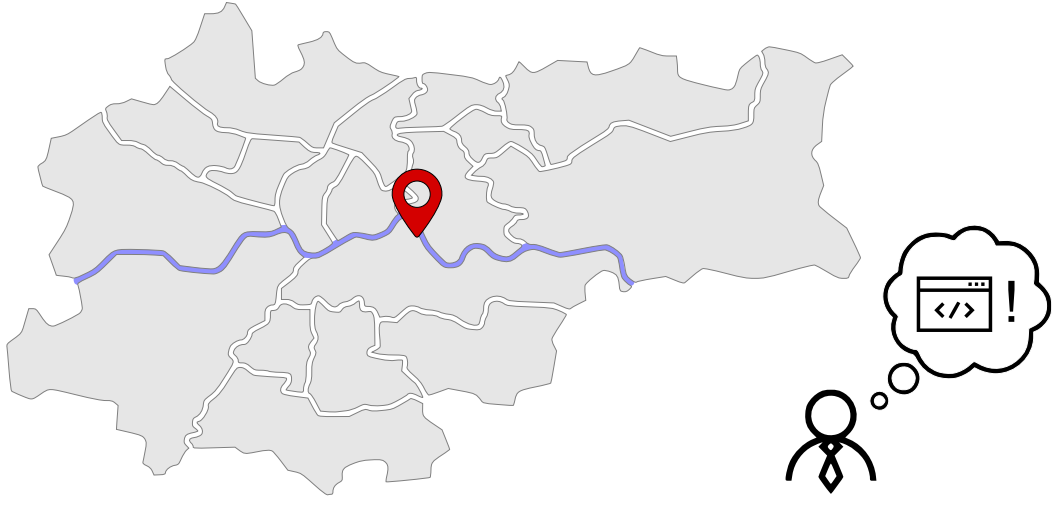
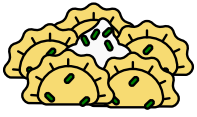
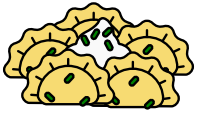


Motivation - Enumeration

1.)     

2.)     

3.)     

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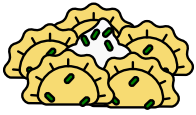
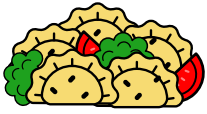
Motivation - Sampling

1.)


2.)

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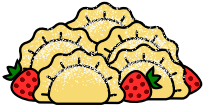

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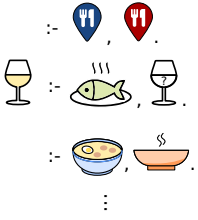
Problem description

- Given: ASP logic program.
- Search: Representative set of solutions.
- Question: What is ASP?
- Problem: How to measure representativeness?
- Problem: How to obtain highly representative collections?

Answer Set Programming

- declarative programming paradigm
- suited for configuration problems (among others)
- program Π : set of rules over atoms
- answer set: $\subseteq$-minimal model satisfying all rules, set of atoms



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Representative Collections

[1] T. Leinster, 'Entropy and diversity: the axiomatic approach', Cambridge university press, 2021

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$$H[T|_S] \coloneqq \sum_{a \in T} p_S(a) \log_2 \frac{1}{p_S(a)}$$

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- normalisation into representativeness:

$$R(T|_S) \coloneqq \frac{D(T|_S)}{|T|} \in [0, 1]$$

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Representative Collections

Target atoms a in S

Representative Collections

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Representative Collections

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Representativeness	$R(T\|_S)$	$= \frac{D(T\|_S)}{\|T\|}$	≈ 0.292

Representative Collections

Target atoms a in S

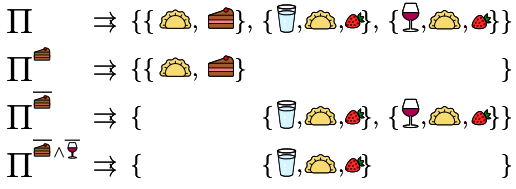
$p_S(a)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	
$\log_2 \frac{1}{p_S(a)}$	$\log_2 6$	$\log_2 6$	$\log_2 6$	$\log_2 6$	$\log_2 6$	≈ 2.585	
Entropy	$H[T\|_S]$	$= \sum_{a \in T} p_S(a) \log_2 \frac{1}{p_S(a)}$			$= 6\frac{1}{6} \log_2 6 = \log_2 6$		
Diversity	$D(T\|_S)$	$= 2^{H[T\|_S]}$			$= 2^{\log_2 6} = 6$		
Representativeness	$R(T\|_S)$	$= \frac{D(T\|_S)}{\|T\|}$			$= \frac{6}{6} = 1$		

Obtaining representative collections

- Approach: Answer Set Navigation

Answer Set Navigation

- facet[2]: atom which appears in at least one but not all answer sets of a program
- activating a facet: propagation of a truth value
- navigation through solution space
- counting facets enables to measure uncertainty
- route: finite sequence of facets, concatenation via $\wedge$

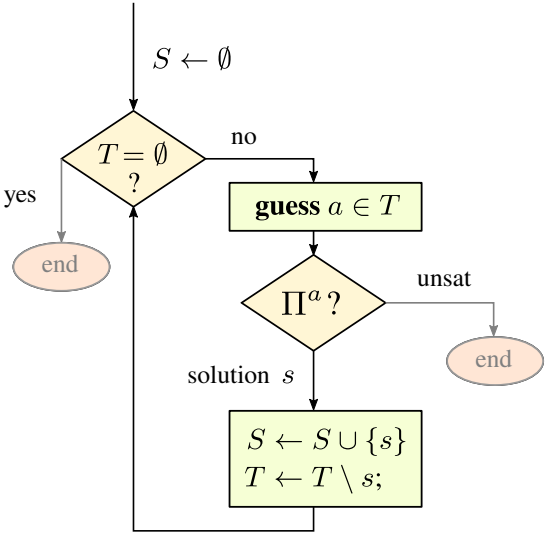


[2] J. Fichte, S. Gaggl, D. Rusovac, 'Rushing and strolling among answer sets–navigation made easy', AAAI 2022

Obtaining representative collections

- Approach: Answer Set Navigation
- Algorithms: Greedy for soundness (S) and diversity (D)

Algorithm: S-Greedy



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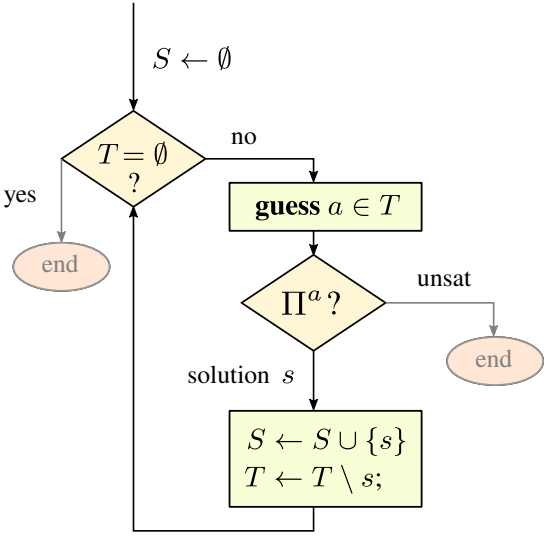
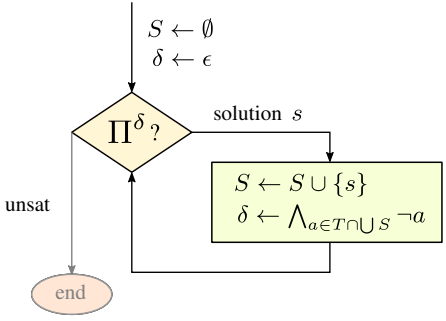
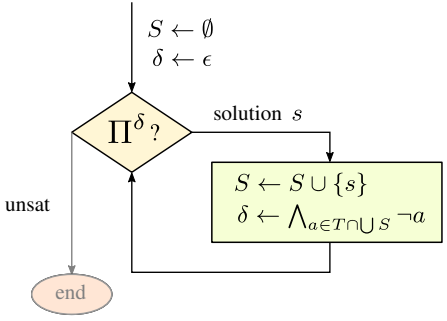


- Σ_2^{P}
- S-Greedy
- S-Greedy-Sieve

Algorithm: D-Greedy



Algorithm: D-Greedy

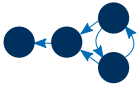


- Δ_3^{P}
- D-Greedy-max
- D-Greedy-max
- D-Greedy-all-max$_+$
- D-Greedy-all-max$_+$

Experiments

- smoke testing (S1)
- 8 base instances, 2000 program/target atom pairings
- real world application, open source Rust projects

- claim centric argumentation frameworks (S2)[3]
- 195 base instances, 1950 program/target atom pairings

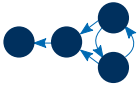


- generated for this contribution
- 300s timelimit, PC specs: single core AMD EPYC 7513, 2.6 GHz, 16 GB RAM

[3] W. Dvorák and S. Woltran; 'Complexity of abstract argumentation under a claim-centric view', Artif. Intell., 2020

Evaluation

	heuristic	mean R	mean time [s]
S1	S-Greedy	0.713	< 0.01
	S-Greedy-sieve	**0.826**	$<$ **0.01**
	D-Greedy-max	0.772	0.21
	D-Greedy-max$_+$	0.804	0.21
	D-Greedy-all-max	0.555	0.55
	D-Greedy-all-max$_+$	0.760	0.61
S2	S-Greedy	0.967	1.88
	S-Greedy-sieve	0.958	**1.58**
	D-Greedy-max	0.972	11.00
	D-Greedy-max$_+$	0.969	11.07
	D-Greedy-all-max	0.976	46.31
	D-Greedy-all-max$_+$	**0.979**	54.23



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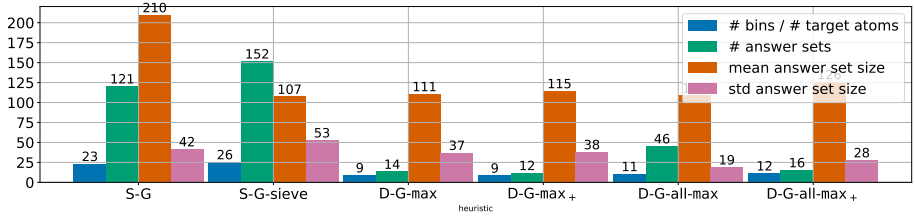
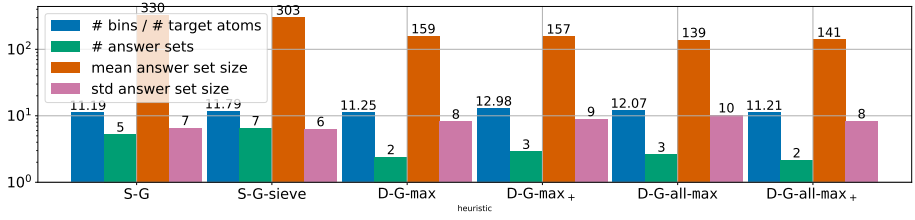


S2



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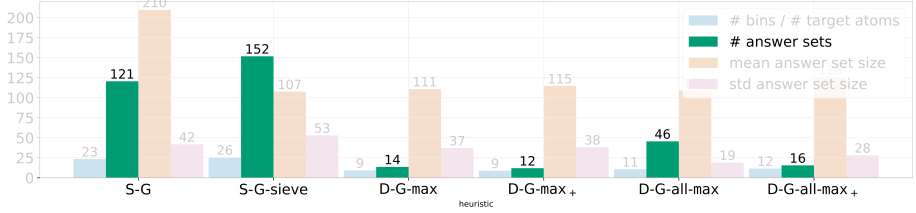
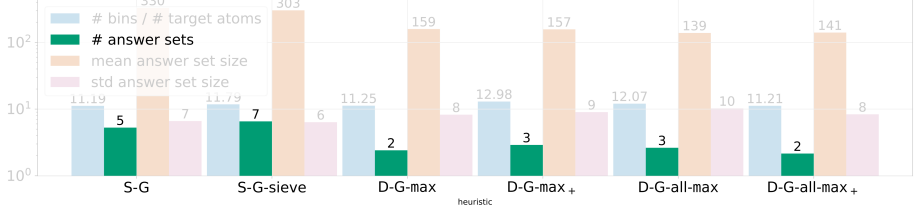


S2



Conclusion and Future Work

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- six algorithms with varying complexity, evaluated on two newly introduced benchmarks

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- further elaboration of methods to improve diversity outcomes
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