

# Restricted Chase Termination

You Want More than Fairness

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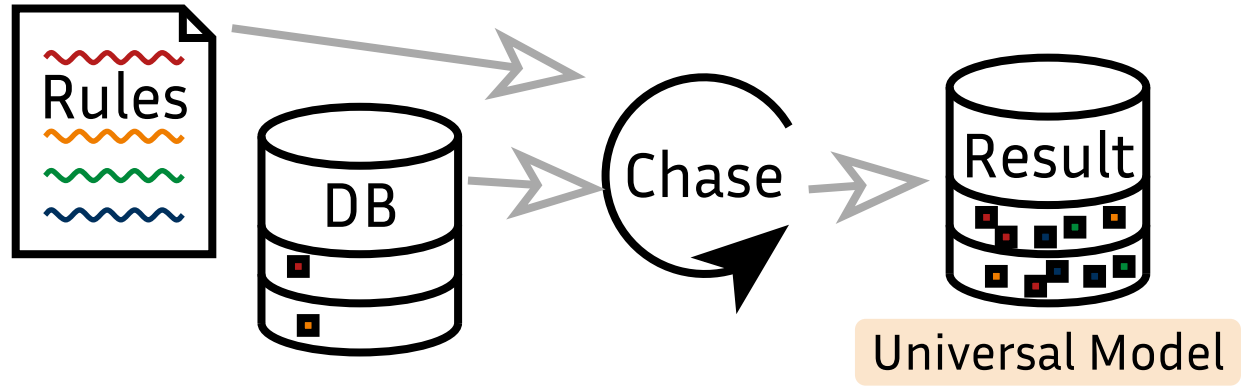


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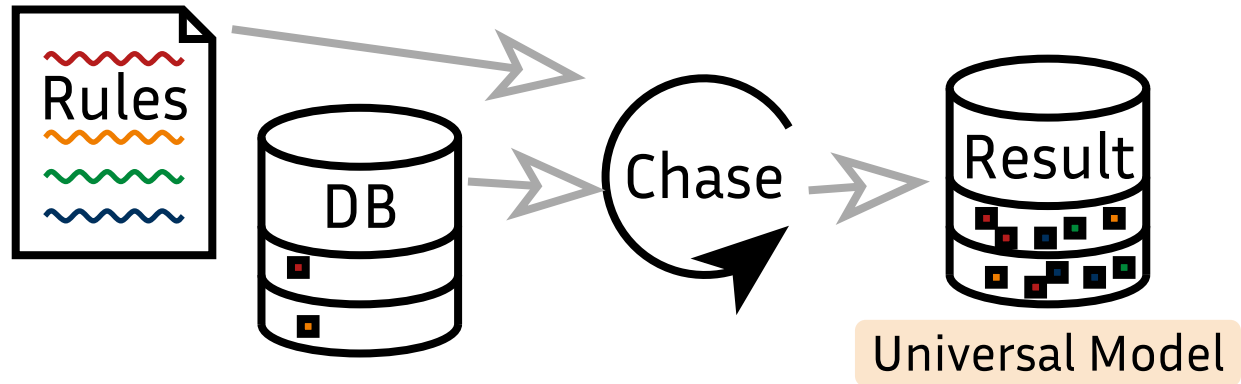


# Chase Crash Course

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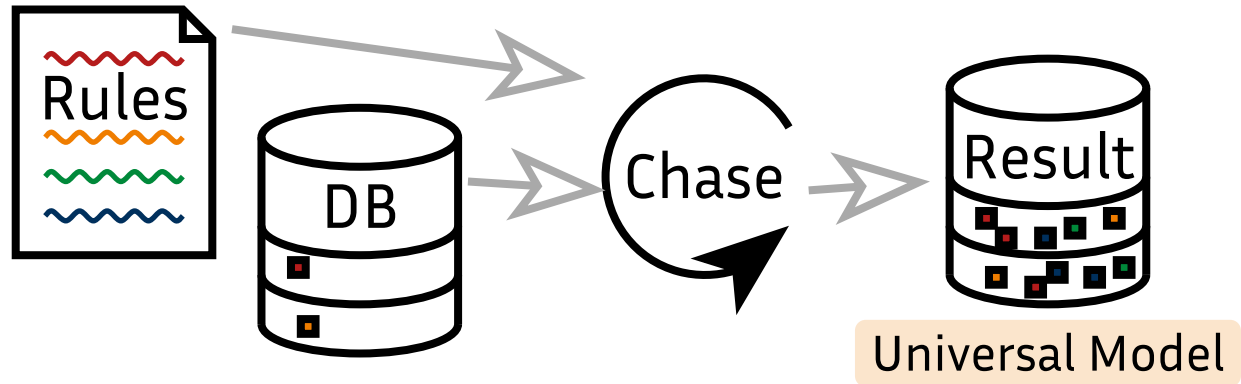


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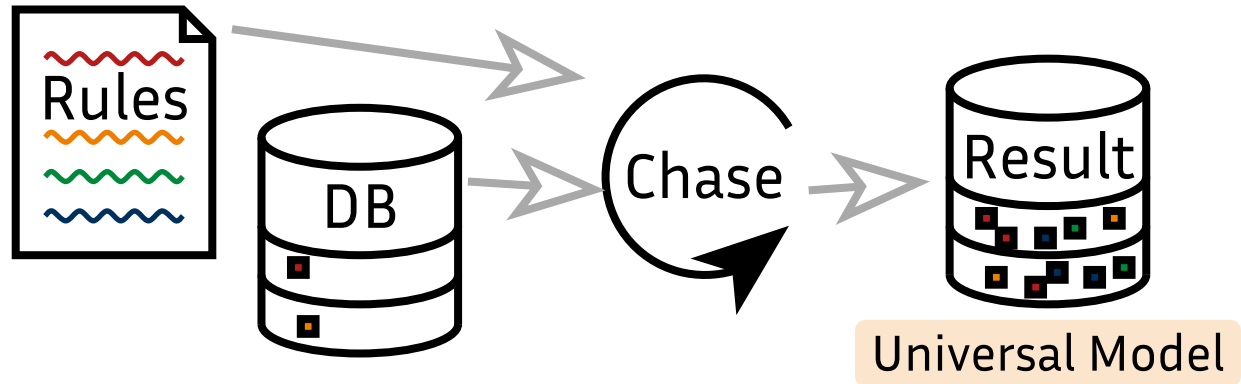
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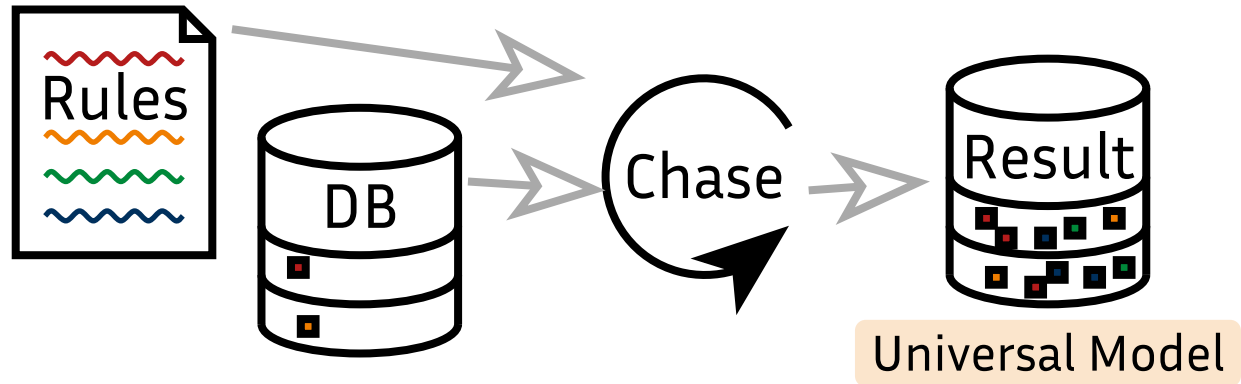
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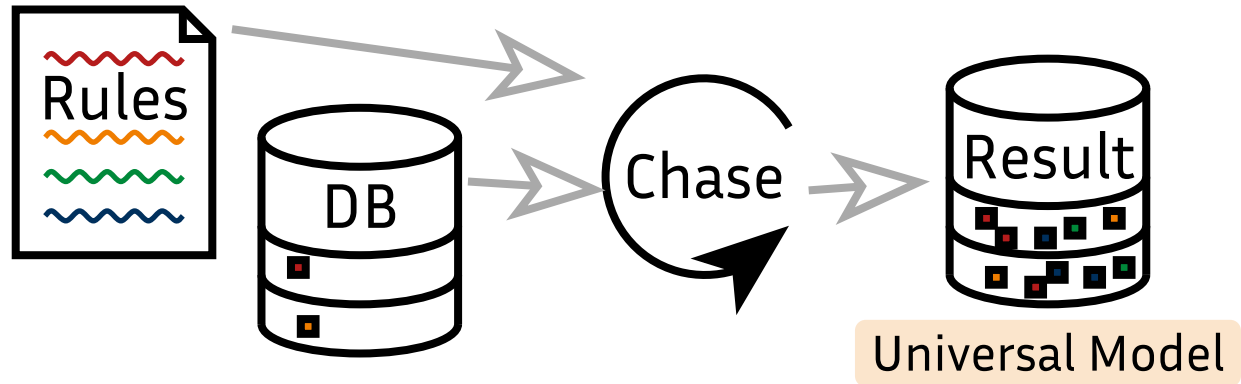
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*Universal Models are “most general” and can answer conjunctive queries.*

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$\text{CTK}_{\forall}^r$  is the set of all knowledge bases, i.e. pairs of rule sets, and databases, on which every ( $\forall$ ) restricted ( $r$ ) chase sequence terminates.

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$\text{CTR}_{\forall}^r$  is analogous for rule sets by  $\forall$ -quantifying over all databases.

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$\Sigma_1^0$  - Semi-Decidable Languages  
(e.g. Halting Problem)

$\Pi_2^0$  - Co-Semi-Decidable with  
Semi-Decision Oracle  
(e.g. Universal Halting Problem)



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*Membership:* Run all Chase Sequences in Parallel

*Hardness:* TM can be simulated with Existential Rules

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*Membership:* Use  $\text{CTK}_{\exists}^r$  oracle.

*Hardness:* (more involved)  
*see (Grahne and Onet 2018)*

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This does not work because of *fairness*!

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*The upper bound was still unknown! (Until slide 6)*

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Possible Fix: We could demand something stronger, e.g. “breadth-first”.

# Fairness for NTMs - based on (Harel 1986)

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Decide if a given non-deterministic Turing machine admits a run on  $w$  that visits a designated state  $q_r$  recurringly after finitely many steps.

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*This problem is  $\Sigma_1^1$ -complete - first analytical hierarchy level - beyond infinitely many Turing jumps. (Harel 1986)*

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Run visiting the design. state infinitely often **iff** infinite fair chase sequence.  
Hence: Complement of  $\text{CTK}_{\forall}^r$  is in  $\Sigma_1^1$ ; therefore  $\text{CTK}_{\forall}^r$  is in  $\Pi_1^1$ .

# A Better Lower Bound for $\text{CTK}_{\forall}^r$ ?

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We know that we can simulate (N)TMs with existential rules.

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*Idea:* With emergency brakes, we can force the chase to terminate after finitely many steps. If the designated state is visited, we create a new brake.



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# Summing up...

Fairness makes Chase Termination highly Undecidable.

Stricter, finitely verifiable conditions could solve this (e.g. breadth-first).

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 *Talk to me about Lean and Typst* 



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