
Definition (Term algebra, [23])

Let Ω be a signature and ar its type, and let X be a set (we say that the elements of X are *variables*). We define the set of *terms* over Ω with variables X , denoted by $T_\Omega(X)$, as follows:

- ▶ $X \subseteq T_{\Omega}(X)$, i.e., any variable is a term, and
- ▶ for any $n \in \mathbb{N}$, any $\omega \in \text{ar}^{-1}[n]$, and any $t_1, t_2, \dots, t_n \in T_{\Omega}(X)$, we have $\omega(t_1, t_2, \dots, t_n) \in T_{\Omega}(X)$, i.e., we obtain terms by applying operations to terms.

The *term algebra* over Ω with variables X is the algebra

$$\mathcal{T}_\Omega(X) := (T_\Omega(X), (f_\omega)_{\omega \in \Omega}),$$

where

$$f_\omega : T_\Omega(X)^{\text{ar}(\omega)} \rightarrow T_\Omega(X) : (t_1, t_2, \dots, t_{\text{ar}(\omega)}) \mapsto \omega(t_1, t_2, \dots, t_{\text{ar}(\omega)}).$$



Bands Named After Things From a Fictional Universe

	band/artist	universe	genre
Q: w02000073	Harry and the Potters	Harry Potter universe	alternative rock, indie rock, Wizard rock
Q: w020122950	Nighty	Noir mythology	black metal
Q: w020209056	Acidgung	Tolkien's legendarium	black metal
Q: w020388839	Acidgung	Tolkien's legendarium	black metal
Q: w020320690	Crith Gongor	Tolkien's legendarium	black metal
Q: w020352761	Gergaroth	Tolkien's legendarium	black metal
Q: w020318702	Ivergand	Tolkien's legendarium	black metal
Q: w020343479	Rivewind	Tolkien's legendarium	black metal, folk metal
Q: w0201197502	Pandora	The Witcher universe	contemporary folk music, pagan metal
Q: w02017386608	Medievalis	Tolkien's legendarium	death metal
Q: w02021281	Morgoth	Tolkien's legendarium	death metal
Q: w020162603	Arwen Amarth	Tolkien's legendarium	death metal, Viking metal
Q: w020066759	Tarixia	Kalevala	folk metal
Q: w020060349	Bombard	Tolkien's legendarium	folk music
Q: w020700854	Aurhey Horne	Tim Franks	hard rock
Q: w0201660329	Mountainade	Elric saga	heavy metal
Q: w020200606	Crith Ungol	Tolkien's legendarium	heavy metal
Q: w020770203	The Mulhounds	Harry Potter universe	indie rock
Q: w020209687	Gordal	Tolkien's legendarium	melodic death metal
Q: w020394707	Green Blaxy	Marvel Universe	metalcore
Q: w020209101	T'Pau	Star Trek: nextdoor	pop music
Q: w020470610	Ivergand	Tolkien's legendarium	power metal
Q: w020470610	Marillon	Tolkien's legendarium	progressive rock, art rock, neo progressive rock
Q: w020117027	Epimeli's Beest	Star Trek: nextdoor	progressive rock, progressive metal
Q: w0201152749	Deigobis	Star Wars expanded to other media, Star Wars universe	punk rock
Q: w020767769	Baba Yaga	Russian fairy tales	rock music
Q: w020080052	Sen Gula	The World universe	rock music
Q: w02021440	Shagrat	Tolkien's legendarium	rock music
Q: w0201006008	Elizabeth Sables	Tolkien's legendarium	speed metal
Q: w0201006310	Cielack Angren	Tolkien's legendarium	symphonic black metal
Q: w020340590	Dorredin	Tolkien's legendarium	traditional/heavy metal
Q: w0204007308	Scratch Bandits Crew	Crash-Bandooit universe	troubadour, breakbeat
Q: w0206769968	Chiv'Nile	Narnia universe	alt/indie
Q: w0201702877	Drusus and the Malloys	Harry Potter universe	Wizard rock
Q: w020060612	Mothers of Moon	Harry Potter universe	Wizard rock

[illegible]

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spikes to the environment. **acc** will now send out a final **spike**, bringing the total to 2^n (note that $2^0 = 1$, so this also works for $n = 0$). None of the **rules** in d_1 and d_2 are applicable (in fact, no **rule** in the **system** at all is applicable), so the computation stops. Clearly, the language generated by Π working according to the terminating protocol in the maximally-parallel mode and taking the total number of **spikes** sent to the environment is

$$C_{\text{tot}} \mathcal{N}_t^{\text{maxpar}}(\Pi) = \{2^n \mid n \in \mathbb{N}\} = P.$$

As the only source of non-determinism in the **system** is the selection of the **rule** to be fired from among the applicable **rules**, it is exactly this mechanism we need to exploit to allow the **system** to decide which number to generate, the idea being that, depending on the **rule** chosen, the computation is either stopped or continues.

We depict such a module in Figure 4.2. Initially, the k_1 **neuron** contains two **spikes** and the **kill neuron** starts with four **spikes**, whereas k_2 remains empty. If k_1 emits two **spikes**, these two are then forgotten in **kill**. Control transfers to the second **component**, k_2 emits the two **spikes**, control transfers back to the first **component**, and the module has returned to its initial configuration. If k_1 emits only a single **spike**, however, **kill** emits five **spikes**, and computation then stops as none of the **neurons** contains an applicable **rule** anymore.

4.4. Comparing the two approaches

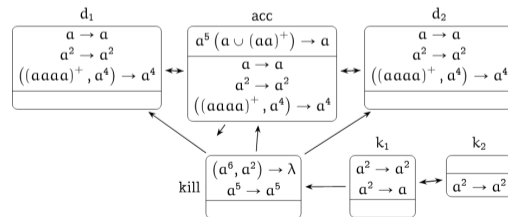


Figure 4.3.: The complete system Π

Expansion rules

Use	Expansion
$Ts \dots$	$T1, \dots, Tn$
$Ts\&\&\dots$	$T1\&\&, \dots, Tn\&\&$
$x<Ts, Y>::z \dots$	$x<T1, Y>::z, \dots, x<Tn, Y>::z$
$x<Ts\&, Us>\dots$	$x<T1\&, U1>, \dots, x<Tn\&, Un>$
$func(5, vs) \dots$	$func(5, v1), \dots, func(5, vn)$

- (Please note: ellipses on the right are in a different font)

Abstract

- It was a simple choice, really, on an IBM 370 in the 70's, between APL, Fortran, Lisp 1.5, PL/1, COBOL, and Simula'67. Nothing could come close to Simula's combination of strong typing, garbage collection, and proper string processing. Separate compilation (prefix classes) and coroutines were nice bonuses. And then there were these ... "objects" but, well, nothing is perfect. Hot topics in those days were the freshly invented denotational semantics (which Simula didn't have), formal type systems (which objects didn't have), and abstract data types (which seemed to have confusingly little to do with classes). Still, Simula was the obvious choice to get something done comfortably because, after all, it was an improved Algol. It even supported the functional programming feature of call-by-name. So, it became my first favorite language, for every reason other than it being object-oriented.
- The story I am going to tell is the very, very slow realization that Simula was the embodiment of a radically different philosophy of programming, and the gradual and difficult efforts to reconcile that philosophy with the formal methods that were being developed for procedural and functional programming. Along the way, domain theory helped rather unexpectedly, at least for a while. Type theory had to be recast for the task at hand. Landin's lambda-reductionism had to be partially abandoned. Always, there seemed to be a deep fundamental mismatch between objects and procedures, well described by Reynolds, that made any unification impossibly complicated. But in the end, both object-oriented and procedural programming have benefited from the clash of cultures. And the story is far from over yet, as witnessed by the still blooming area of program verification for both procedural and object-oriented languages.