

# Deduction Systems

## Tutorial 2

Sebastian Rudolph

SS 2015

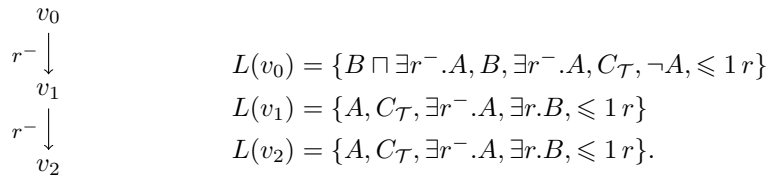
**Exercise 2.1.** Transform the following concepts into negation normal form:

- (a)  $\neg(A \sqcap \forall r.B)$
- (b)  $\neg\forall r.\exists s.(\neg B \sqcup \exists r.A)$
- (c)  $\neg((\neg A \sqcap \exists r.\top) \sqcup \geq 3 s.(A \sqcup \neg B))$

**Exercise 2.2.** Apply the tableau algorithm in order to check if the axiom  $A \sqsubseteq B$  is a logical consequence of the TBox  $\{\neg C \sqsubseteq B, A \sqcap C \sqsubseteq \perp\}$ .

**Exercise 2.3.** Apply the tableau algorithm in order to check satisfiability of the concept  $A \sqcap \forall r.B$  w.r.t. the TBox  $\{A \sqsubseteq \exists r.A, B \sqsubseteq \exists r^-.C, C \sqsubseteq \forall r.\forall r.B\}$ .

**Exercise 2.4.** Markus wants to apply the tableau algorithm for checking the satisfiability of the concept  $B \sqcap \exists r^-.A$  w.r.t. the TBox  $\{A \sqsubseteq \exists r^-.A \sqcap \exists r.B, \top \sqsubseteq \leq 1 r\}$ . He arrives at the situation depicted below and concludes that no further rules are applicable, since  $v_2$  is blocked by  $v_1$ . What is Markus' error? Continue the algorithm until its termination. (You don't have to illustrate all intermediate steps, just provide the final state.)



**Exercise 2.5.** Extend the  $\leq 1$  rule in a way that also qualified functionality axioms of the form  $\top \sqsubseteq \leq 1 r.A$  can be treated correctly, where  $A$  is an atomic concept. Can you also treat arbitrary axioms of the form  $C \sqsubseteq \leq 1 r.D$ ?