The WCS Approach in the Syllogism Challenge: Reasoning Principles and their Cognitive Adequacy

Emmanuelle Dietz Saldanha
International Center for Computational Logic
TU Dresden
Germany

TU Dresden

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Syllogism Challenge

Syllogistic Reasoning Task

All *artists* are *bakers*   Some *chemists* are not *bakers*

What follows about the relation between *artists* and *chemists*?

- All *artists* are *chemists*
- No *artists* are *chemists*
- Some *artists* are *chemists*
- Some *artists* are not *chemists*
- All *chemists* are *artists*
- No *chemists* are *artists*
- Some *chemists* are *artists*
- Some *chemists* are not *artists*
- No valid conclusion

- Majority answers Some *chemists* are not *artists* and No valid conclusion (NVC)
- Who answers Some *chemists* are not *artists* doesn't answer NVC, and vice versa

Hypothesis

- Different groups of human reasoners apply different principles and heuristics

Cognitive Principles	presuppositions that humans make while reasoning
Heuristic Strategies	reasoning based on structural patterns or biases

$\downarrow$ Description

Reasoning Clusters	groups of reasoners

$\downarrow$ Translation

Program $\mathcal{P}$	cognitive principles represented by logic program clauses

$\downarrow$ Computation

Model $\mathcal{M}$	least model of the weak completion of $\mathcal{P}$

$\downarrow$ Entailment

Conclusions	All artists are chemists, Some artists are chemists, ...

$\downarrow$ Specification

Distribution	based on number of clusters that entail the conclusion

How plausible are the assumptions made for the principles and clusters?

Principles on Universally Quantified Statements

Natural language	First-order logic
All artists are bakers	$\forall X(a(X) \rightarrow b(X))$

1. Statement as Conditional (CONDITIONALS) (Stenning and van Lambalgen, 2008)
   - $\forall X(a(X) \ \rightarrow \ b(X))$
2. Licences for Inferences (LICENSES) (Stenning and van Lambalgen, 2008)
   - All a that are not abnormal, are b. By default, nothing is abnormal
3. Existential Import (IMPORT) (Grice, 1975)
   - There exists (at least) one object o such that $a(o)$ is true
4. Contraposition (CONTRAPOSITION)
   - For all X if $a(X)$ then $b(X)$ implies for all X if $\neg b(X)$ then $\neg a(X)$

$b(X)$	$\leftarrow$	$a(X) \wedge \neg ab_{ab}(X)$	(CONDITIONALS&LICENSES)
$ab_{ab}(X)$	$\leftarrow$	$\bot$	(LICENSES)
$a(o)$	$\leftarrow$	$\top$	(IMPORT)
$a'(X)$	$\leftarrow$	$\neg b(X) \wedge \neg ab_{nbna}(X)$	(TRANSFORM&CONTRAPOSITION&LICENSES)
$ab_{nbna}(X)$	$\leftarrow$	$\bot$	(LICENSES)
$a(X)$	$\leftarrow$	$\neg a'(X) \wedge \neg ab_{naa}(X)$	(TRANSFORM&LICENSES)
$ab_{naa}(X)$	$\leftarrow$	$\bot$	(LICENSES)

Under the Weak Completion Semantics, $a(o)$, $b(o)$ are true, $a'(o)$ is false.

Principles on Existentially Quantified Statements

Principles on Existentially Quantified Statements

Natural language	First-order logic
Some artists are bakers	$\exists X(a(X) \land b(X))$

By Grice (1975) theory on conversational implicatures

- *Some a are b* implies *Not all a are b*
- *not all a are b* implies *Some a are not b*

5. Unknown Generalization (UNKNOWNGEN)
   - There exists one object o_1 such that $a(o_1)$ and $b(o_1)$ is true
   - There exists one object o_2 such that $a(o_2)$ is true and $b(o_2)$ is unknown
6. Converse Implication (CONVERSE) (Parry and Hacker, 1991)
7. Search for Alternative Models (SEARCHALT) (Johnson-Laird, 1983)

Clusters

All *artists* are *bakers* Some *chemists* are not *bakers*

Five Clusters

1. Three clusters on reasoning principles
2. Two clusters on heuristic strategies

Principles	Cluster 1	Cluster 2	...
CONDITIONALS	✓	✓	
LICENSES	✓	✓	
IMPORT	✓	✓	
UNKNOWNGEN	✓	✓	
CONTRAPOSITION		✓	
...			

Cluster 1

All *artists* are *bakers* Some *chemists* are not *bakers*

The program representing this pair of syllogistic premises consists of

$$
\begin{array}{lll}
b(X) \leftarrow a(X) \wedge \neg ab_{ab}(X). & b'(X) \leftarrow c(X) \wedge \neg ab_{cnb}(X). & ab_{ab}(X) \leftarrow \bot. \\
a(o_1) \leftarrow \top. & c(o_2) \leftarrow \top. & ab_{nbb}(o_2) \leftarrow \bot. \\
 & b(X) \leftarrow \neg b'(X) \wedge \neg ab_{nbb}(X). & ab_{cnb}(o_2) \leftarrow \bot. \\
 & c(o_3) \leftarrow \top. & ab_{nbb}(o_3) \leftarrow \bot.
\end{array}
$$

Weak Completion Semantics

True	False
$a(o_1), c(o_2), c(o_3)$ $b(o_1), b'(o_2)$	$ab_{ab}(o_1), ab_{ab}(o_2), ab_{ab}(o_3), \ldots$

No valid conclusion about the relation between *a* and *c* follows from this model.

Clusters

All *artists* are *bakers*   Some *chemists* are not *bakers*

Five Clusters

1. Three clusters on reasoning principles
2. Two clusters on heuristic strategies

Principles	Cluster 1	Cluster 2	...
CONDITIONALS	✓	✓	
LICENSES	✓	✓	
IMPORT	✓	✓	
UNKNOWNGEN	✓	✓	
CONTRAPOSITION		✓	
...			
	$\Downarrow$	$\Downarrow$	
Entailment	No valid conclusion	Some *chemists* are not *artists*	

Are the principles cognitively adequate?

1. CONDITIONALS:
   Quantified Statements as conditionals (Stenning and van Lambalgen, 2008) ☑
2. LICENSES: Licenses for inferences (Stenning and van Lambalgen, 2008) ☑
3. IMPORT: Existential Import (Grice, 1975) ☑
4. CONVERSE: Converse Implication (Parry and Hacker, 1991) ☑
5. SEARCHALT: Search for alternative Models (Johnson-Laird, 1983) ☑
6. CONTRAPOSITION: Contraposition ☐
7. UNKNOWNGEN: Unknown Generalization ☐

We will consider the principles CONTRAPOSITION and UNKNOWNGEN

Contraposition

From All *artists* are *bakers* it is formally valid to derive All not *bakers* are not *artists*.

- There is little evidence that humans recognize this inference directly
- Humans derive modus tollens conclusions by simple forms of proof by contradiction (O'Brien, D. S. Braine, and Yang [1994], Rips [1994])
- Humans make inferences by means of mental models (Johnson-Laird, 1983)

Assume someone who is not *baker*

not *baker*		
not *baker*	*artist*	
not *baker*	*artist*	$\Rightarrow$ contradiction!
not *baker*	not *artist*	

Under WCS we immediately encoded contraposition as a clause in the logic program.

Unknown Generalization

- Maxim of quantity (Grice,1975): humans try to be as informative as required

Some *bakers* are *artists*

$\Downarrow$

Not all *bakers* are *artists*

$\Downarrow$

Some *bakers* are not *artists*

- For humans

Some *bakers* are *artists* is not equivalent to Some *artists* are *bakers*

Some *bakers* are *artists*	$a(X) \leftarrow b(X) \wedge \neg ab_{ba}(X)$
$\Downarrow$	
Some *baker* exists who is an *artist*	$b(o_1) \leftarrow \top$ $ab_{ba}(o_1) \leftarrow \bot$
$\Downarrow$	
Some *baker* exists, it is unknown whether it is an *artist*	$b(o_2) \leftarrow \top$

Under the Weak Completion Semantics, $b(o_1)$, $a(o_1)$, $b(o_2)$ are true, $ab_{ba}(o_1)$ is false, and $a(o_2)$, $ab_{ba}(o_2)$ are unknown.

Open Questions

- How plausible are the discussed cognitive principles?
- Is the unknown generalization prinicple a good indicator for the cognitive adequacy of three-valued logics?
- How can both assumptions be tested in an psychological experiment?
- How many different human reasoning clusters seem to cognitively adequate ?

Thank you!

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