

# Science of Computational Logic

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## Problem 6.1

Show that first-order logic is monotonic.

## Problem 6.2

Show that reasoning with CWA is non-monotonic.

## Problem 6.3

Consider the language  $\mathcal{L}(\mathcal{R}, \mathcal{F}, \mathcal{V})$ , with  $\mathcal{R} = \{p/0, q/0\}$ .

Given the set of formulas  $\mathcal{S} = \{p \leftarrow \neg q, q \leftarrow \neg p\}$

Compute  $C_{CWA}(\mathcal{S})$ .

## Problem 6.4

Prove that the closed world assumption eliminated non-least Herbrand models:

If  $F$  is a formula and  $I$  is a non-least Herbrand model  $I$  of  $F$ , then  $I \not\models C_{CWA}(F)$ .

## Problem 6.5

Proof the following proposition:

Let  $\mathcal{F}$  be a satisfiable set of Skolem formulas. Then it holds:

$C_{CWA}(\mathcal{F})$  is satisfiable  $\Leftrightarrow \mathcal{F}$  admits a least Herbrand model.

## Problem 6.6

Reconsider the theorem from the lectures proved in the preceding problem.

1. Show that the condition that  $\mathcal{F}$  a set of formulas in Skolem normal form is necessary for  $\Rightarrow$ -direction.
2. Show for the  $\Leftarrow$ -direction that without the condition that  $\mathcal{F}$  a set of formulas in Skolem normal form the existence of a least Herbrand model of  $\mathcal{F}$  does not entail the existence of a Herbrand model of  $C_{CWA}(\mathcal{F})$ .