Exercise 2 — Linear-Time Branching-Time Spectrum

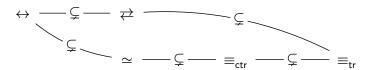


Figure 1: Our Linear-Time Branching-Time Spectrum

Task 1. — Failure Equivalence

We call a pair $\langle \sigma, X \rangle$ a failure pair of process $p \in Pr$ if there is a process $q \in Pr$ such that $p \stackrel{\circ}{\longrightarrow} q$ and $q \xrightarrow{a}$ for all $a \in X$. Denote the set of all failure pairs of p by F(p).

Processes p and q are failures equivalent, denoted by $p \equiv_{\mathsf{F}} q$, if F(p) = F(q).

- a) Discuss why \equiv_{F} is an equivalence relation.
- b) Find the position of \equiv_{F} in the spectrum (cf. Figure 1).

Task 2. — Simulations and Similarity

We call $S \subseteq \Pr{\times} \Pr{a}$ simulation if, for all $p \ S \ q$ and all $a \in \mathsf{Act}$,
• $p \overset{a}{\longrightarrow} p'$ implies that there is a $q' \in \Pr{with} \ q \overset{a}{\longrightarrow} q'$ and $p' \ S \ q'$.

 $p \in \mathsf{Pr}$ simulates $q \in \mathsf{Pr}$, denoted by $p \rightharpoonup q$, if there is a simulation S such that $p \mathrel{S} q$.

p and q are similar, denoted by $p \rightleftharpoons q$, if $p \rightleftharpoons q$ and $q \rightleftharpoons p$.

- a) Argue that \rightleftharpoons is a process equivalence?
- b) Find the position of \rightleftharpoons in the spectrum (cf. Figure 1).

Task 3. — Branching-Time in Deterministic Systems

Is it possible to find two deterministic processes p and q such that $p \equiv_{\mathsf{tr}} q$ but $p \simeq q$?

- a) If yes, write them down. What makes them non-bisimilar?
- b) If no, try to prove that $\simeq \equiv_{tr}$ in the special case of deterministic processes.
- c) What do your findings tell us about the difference between \rightleftharpoons and \equiv_F ?

Task 4. — Branching-Time in Fully Nondeterministic Processes

For this task, we consider processes that use a single action, meaning |Act| = 1 (e.g., $Act = \{a\}$). Play around with some examples and make two conjectures. Try proving these conjectures or find counterexamples for them?