Problem 1.1

In the lectures the following example from Description Logics was presented:

\[ \mathcal{K}_T : \]
- woman \sqsubseteq \text{person},
- man \sqsubseteq \text{person},
- mother = woman \cap \exists \text{child} : \text{person},
- father = man \cap \exists \text{child} : \text{person},
- parent = mother \sqcup \text{father},
- grandparent = parent \cap \exists \text{child} : \text{parent},
- \text{father_without_son} = \text{father} \cap \forall \text{child} : \neg \text{man} \]

\[ \mathcal{K}_A : \]
- parent(\text{carl}), parent(\text{conny}),
- child(\text{conny}, \text{joe}), child(\text{conny}, \text{carl}),
- \text{man(\text{joe})}, \text{man(\text{carl})}, \text{woman(\text{conny})}. \]

Are the following consequences valid? **Justify** your answers.

1. \( \mathcal{K}_T \cup \mathcal{K}_A \models \text{grandparent} (\text{conny}) \)
2. \( \mathcal{K}_T \cup \mathcal{K}_A \models \text{father} (\text{carl}) \)
3. \( \mathcal{K}_T \cup \mathcal{K}_A \models \text{father_without_son} (\text{carl}) \)

Problem 1.2

Prove that \( F \sqsubseteq G \equiv F \cap \neg G = \bot \)

Problem 1.3

Show that \( \text{grandparent} \sqsubseteq_{\mathcal{K}_T} \text{parent} \) by reducing subsumption into concept satisfiability, where \( \mathcal{K}_T \) is the T-Box from Problem 1.1.

Problem 1.4

Is the concept \( (\text{father} \cap \text{mother}) \) satisfiable w.r.t. \( \mathcal{K}_T \) from Problem 1.1?

Problem 1.5

1. Which generalized concept axioms must be added to prevent that a person is female and male?
2. Is there a single generalized concept axiom that prevents that a person is female and male?

**Problem 1.6**

Give an equivalent concept of \( \text{woman} \sqcap \exists \text{child}. \text{person} \) without using the constructors \( \sqcap \) and \( \exists r.C \).

**Problem 1.7**

Prove the following:

If \( (\forall r.C)(a) \in \mathcal{A} \), and \( r(a, b) \in \mathcal{A} \), then \( \mathcal{A} \models C(b) \).