Exercise Sheet 1: Getting to Know Graphs

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Exercise 1.1. Show that the number of vertices of odd degree is even in every simple graph.

Exercise 1.2. Write a program that reads a directed graph from a file in the format of Exercise 0.2 and prints out the graph in METIS graph format:

$$\begin{array}{cccccc} n & m & & & \\ v_1 & n_1^1 & n_1^2 & \cdots & n_1^{d_1} \\ v_2 & n_2^1 & n_2^2 & \cdots & n_2^{d_1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ v_n & n_n^1 & n_n^2 & \cdots & n_n^{d_1} \end{array}$$

The first line consists of two integers n and m, separated by a space, where n is the number of vertices, and m is the total number of edges. Each of the following lines specifies the neighbours $n_i^1, n_i^2, \ldots, n_i^{d_i}$ of vertex v_i.

As an example, the directed star S_3 would be encoded as:

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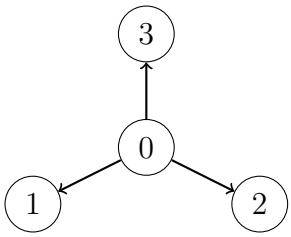
4 3
0 1 2 3
```



Data files are available at https://github.com/knowsys/Course-Knowledge-Graphs/tree/master/exercises/01-graph-basics/data/12-worldcup-2018.

Exercise 1.3. A triangle in a directed graph is a simple directed path $v_1 \xrightarrow{e_1} v_2 \xrightarrow{e_2} v_3 \xrightarrow{e_3} v_1$.

Write a program that reads a directed graph G from a file in the format of Exercise 0.2 and prints out the number of triangles in G. How does the runtime of your program scale with the size of the input graph?

Data files are available at https://github.com/knowsys/Course-Knowledge-Graphs/tree/master/exercises/01-graph-basics/data/13-spacecraft.

∗ **Exercise 1.4.** Let $G = \langle V, E \rangle$ be an undirected graph. Show that if G is triangle-free (i.e., there are no triangles in G), then

$$|E| \leq \left\lfloor \frac{|V|^2}{4} \right\rfloor .$$