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Iterated contraction of propositions and conditionals under the principle of conditional preservation

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Motivation

Classical AGM belief change theory deals only with changes of propositional belief sets[1] K by propositional information A; major operations are

- revision $K * A$ ("integrate A in your beliefs"), and
- contraction $K - A$ ("give up belief in A").

Revision and contraction are connected (e.g., via the Levi identity), they are dual in some sense.

[1] A belief set is a deductively closed theory

Motivation (cont'd)

However, AGM theory is limited wrt iterated revision because it does not provide a full revision strategy independent of the prior K:

$$(K *_K A) *_{K*A} B$$

$\rightarrow$ iterated revision [Darwiche & Pearl, AIJ 1997]: "double-*" postulates, basic idea was a "principle of conditional preservation".

[Konieczny & Pino Perez, SUM 2017] did nearly the same for iterated contraction, but without mentioning conditional preservation.

Contributions of this talk

- We show that the same basic principle of conditional preservation can guide both iterated revision and contraction
  - in the sense of [Darwiche & Pearl, 1997] and [Konieczny & Pino Perez, 2017].
- More precisely, an axiomatized principle of conditional preservation for iterated contraction in the context of OCFs implies all postulates of [Konieczny & Pino Perez 2017], together with few other mild and obvious postulates (e.g., "success").
- Our approach can handle revisions of epistemic states (represented by Spohn's ranking functions) by sets of conditional beliefs and hence is much more general.

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AGM revision in a nutshell

The basic theory for belief revision is the AGM theory [Alchourron, Gärdenfors, and Makinson 1985].

The AGM postulates are recommendations for rational belief change within classical propositional logic:

- The beliefs of the agent should be deductively closed, i.e., the agent should apply logical reasoning whenever possible.
- The change operation should be successfull. (This does not mean that the agent should believe everything!)
- In case of consistency, belief change should be performed via expansion, i.e., by just adding beliefs.
- The result of belief change should only depend upon the semantical content of the new information.
- and more . . .

A basic theorem for AGM revision

Proposition ([Darwiche & Pearl, 1997])

A revision operator $\star$ that assigns a posterior epistemic state $\Psi \star A$ to a prior epistemic state Ψ and a proposition A is an AGM revision operator for epistemic states iff there exists a (total) preorder $\preceq_\Psi$ for an epistemic state Ψ with associated belief set $K = \mathit{Bel}(\Psi)$, such that for every proposition A it holds that:

$$K \star A = \mathit{Bel}(\Psi \star A) = \mathcal{T}(\min(\preceq_\Psi, A))$$

- $\rightarrow$ In the context of belief revision, epistemic states can be represented by a total preorder on possible worlds
- $\rightarrow$ Iterated revision is on strategies how to revise total preorders so that a new total preorder is available after each revision step

Belief revision and conditionals

An epistemic state Ψ may also contain conditional beliefs $(B|A)$ – "If A then (usually, probably, plausibly …) B":

$$\Psi \models (B|A) \quad \text{iff} \quad AB \textit{ is more plausible than } A\overline{B},$$
$$\text{iff} \quad A \mid\!\sim B,$$

where plausibility is measured with respect to a total preorder $\preceq_\Psi$.

Via the Ramsey test

$$\Psi \models (B|A) \quad \text{iff} \quad \Psi * A \models B,$$

revision can be encoded by conditionals $(B|A)$.

Ranking functions (OCF)

Ranking functions are a popular means to implement total preorders and provide semantics for conditionals conveniently:

An Ordinal Conditional Function (OCF) or ranking function κ [Spohn 1988] assigns a degree of (im)plausibility to any possible world $\omega \in \Omega$.

Definition (OCF κ)

$\kappa := \Omega \to \mathbb{N}_0^\infty$ such that:

$$\kappa^{-1}(0) \neq \emptyset$$

$$\kappa(A) = \min_{\omega \models A} \kappa(\omega)$$

$$\kappa \models (B|A) \quad \text{iff} \quad \kappa(AB) < \kappa(A\overline{B})$$

Example (ranked flyers)

$\kappa(\omega) = 4$	$p\bar{b}f$
$\kappa(\omega) = 2$	pbf $\quad p\bar{b}\,\overline{f}$
$\kappa(\omega) = 1$	$pb\overline{f}$ $\quad \overline{p}b\overline{f}$
$\kappa(\omega) = 0$	$\overline{p}bf$ $\quad \overline{p}\bar{b}f$ $\quad \overline{p}\bar{b}\,\overline{f}$

$\kappa \models (\overline{f}|p)$, *resp.* $p \mid\!\sim_\kappa \overline{f}$

Rankings can be understood as qualitative abstractions of probabilities.

Iterated revision acc. to [Darwiche & Pearl, 1997]

Proposition ([Darwiche & Pearl, 1997])

Let $\star$ be an AGM revision operator for epistemic states Ψ with corresponding faithful preorder $\preceq_\Psi$. Then $\star$ is an iterative revision operator in the sense of [Darwiche & Pearl, 1997] iff it satisfies the postulates (CR1)–(CR4).

(CR1,2) as double-$*$ postulates: If $B \models (\neg)A$, then

$$\mathit{Bel}((\Psi * A) * B) \equiv \mathit{Bel}(\Psi * B)$$

(CR1,2) for conditional preservation: If $B \models (\neg)A$, then

$$\Psi \models (C|B) \text{ iff } \Psi * A \models (C|B)$$

(CR1,2) in its semantic form: If $\omega_1, \omega_2 \models (\neg)A$, then

$$\omega_1 \leqslant_\Psi \omega_2 \text{ iff } \omega_1 \leqslant_{\Psi * A} \omega_2$$

Iterated contraction

Similarly to [Darwiche & Pearl, 1997], [Konieczny & Pino Perez, 2017] proposed that

- iterative contraction operators – for epistemic states Ψ should satisfy four postulates (IC1)–(IC4),

the semantic version of the first two postulates being as follows:

(IC1,2) if $\omega_1, \omega_2 \models (\neg)A$, then $\omega_1 \preceq_\Psi \omega_2$ iff $\omega_1 \preceq_{\Psi - A} \omega_2$.

They built upon a result by [Caridroit et al., 2015] proving that total preorders are also fundamental for AGM contraction.

However, Konieczny and Pino Perez did not talk about conditional preservation ...

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Conditional structures

Let

$\mathcal{R} = \{(B_1|A_1), \ldots, (B_n|A_n)\}$ be a finite set of conditionals
$\mathbf{a}_1^+, \mathbf{a}_1^-, \ldots, \mathbf{a}_n^+, \mathbf{a}_n^-$ be distinct algebraic symbols

$$\sigma_{\mathcal{R}}(\omega) := \prod_{1 \leqslant i \leqslant n} \sigma_i(\omega) = \prod_{\substack{1 \leqslant i \leqslant n \\ \omega \models A_i B_i}} \mathbf{a}_i^+ \prod_{\substack{1 \leqslant i \leqslant n \\ \omega \models A_i \overline{B}_i}} \mathbf{a}_i^-$$

describes the all-over effect of $\mathcal{R}$ on ω and is called the conditional structure of ω wrt $\mathcal{R}$ [GKI, 2001, 2004].

A generic principle of conditional preservation

($\mathrm{PCP}_{\circ}^{ocf}$) Let $\circ$ be a change operation on OCFs, and let $\mathcal{R} = \{(B_1|A_1), \dots, (B_n|A_n)\}$.
If two multisets of possible worlds $\Omega = \{\omega_1, \dots, \omega_m\}$ and $\Omega' = \{\omega'_1, \dots, \omega'_m\}$ fulfill

$$\prod_{j=1}^{m} \sigma_{\mathcal{R}}(\omega_j) = \prod_{j=1}^{m} \sigma_{\mathcal{R}}(\omega'_j),$$

i.e., for each conditional $(B_i|A_i)$ in $\mathcal{R}$, Ω and Ω' show the same number of verifications resp. falsifications, then prior κ and posterior $\kappa^\circ = \kappa \circ \mathcal{R}$ are balanced by:

$$\begin{aligned}(\kappa(\omega_1) + \ldots + \kappa(\omega_m)) - (\kappa(\omega'_1) + \ldots + \kappa(\omega'_m)) \\ = (\kappa^\circ(\omega_1) + \ldots + \kappa^\circ(\omega_m)) - (\kappa^\circ(\omega'_1) + \ldots + \kappa^\circ(\omega'_m))\end{aligned}$$

Example

$\prod_{j=1}^{m} \sigma_{\mathcal{R}}(\omega_j) = \prod_{j=1}^{m} \sigma_{\mathcal{R}}(\omega'_j)$ implies

$$\begin{aligned}&(\kappa(\omega_1) + \ldots + \kappa(\omega_m)) - (\kappa(\omega'_1) + \ldots + \kappa(\omega'_m)) \\ &\quad = (\kappa^\circ(\omega_1) + \ldots + \kappa^\circ(\omega_m)) - (\kappa^\circ(\omega'_1) + \ldots + \kappa^\circ(\omega'_m))\end{aligned}$$

Let $\mathcal{R} = \{r_1 = (c|a), r_2 = (c|b)\}$; consider

ω	$\sigma_{\mathcal{R}}(\omega)$	ω'	$\sigma_{\mathcal{R}}(\omega')$
$\omega_1 = a\overline{b}c$	$\mathbf{a}_1^+$	$\omega'_1 = \overline{a}\overline{b}c$	1
$\omega_2 = \overline{a}bc$	$\mathbf{a}_2^+$	$\omega'_2 = abc$	$\mathbf{a}_1^+\mathbf{a}_2^+$
$\sigma_{\mathcal{R}}(\omega_1)\sigma_{\mathcal{R}}(\omega_2) = \sigma_{\mathcal{R}}(\omega'_1)\sigma_{\mathcal{R}}(\omega'_2)$			

Hence

$$(\kappa(\omega_1) + \kappa(\omega_2)) - (\kappa(\omega'_1) + \kappa(\omega'_2)) = (\kappa^\circ(\omega_1) + \kappa^\circ(\omega_2)) - (\kappa^\circ(\omega'_1) + \kappa^\circ(\omega'_2)).$$

Characterizing $(\mathrm{PCP}_{\circ}^{ocf})$-compatible belief changes

Theorem

Let $\mathcal{R} = \{(B_1|A_1), \ldots, (B_n|A_n)\}$ be a finite set of conditionals, and let $\kappa \circ \mathcal{R} = \kappa^{\circ}$ be a belief change of κ by $\mathcal{R}$. Then this change satisfies $(PCP_{\circ}^{ocf})$ iff there are rational[a] numbers $\kappa_0, \gamma_i^+, \gamma_i^-$, $1 \leqslant i \leqslant n$ such that

$$\kappa^{\circ}(\omega) = \kappa_0 + \kappa(\omega) + \sum_{\substack{1 \leqslant i \leqslant n \\ \omega \models A_i B_i}} \gamma_i^+ + \sum_{\substack{1 \leqslant i \leqslant n \\ \omega \models A_i \overline{B_i}}} \gamma_i^- . \qquad (1)$$

Iterated belief change operators of the form (1) are called c-change operators.

[a] Note that indeed, $\kappa_0, \gamma_i^+, \gamma_i^-$ can be rational, but κ° has to satisfy the requirements for OCF, in particular, all $\kappa^{\circ}(\omega)$ must be non-negative integers.

This provides a clear and simple schema for complex iterated change operators.

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Iterated contraction by a single conditional

$\mathcal{R} = \{(B|A)\}$:

Then a c-contracted ranking function $\kappa^{\ominus} = \kappa \ominus (B|A)$ has the form

$$\kappa^{\ominus}(\omega) = \kappa_0 - \min\{\gamma^- + \kappa(A\overline{B}), \kappa(\overline{A})\} + \begin{cases} \gamma^+ & \textit{if } \omega \models AB \\ \gamma^- & \textit{if } \omega \models A\overline{B} \\ 0 & \textit{if } \omega \models \overline{A} \end{cases},$$

where

$$\gamma^- - \gamma^+ \leqslant \kappa(AB) - \kappa(A\overline{B}).$$

to ensure

$$\kappa \ominus (B|A) \not\models (B|A) \qquad \textit{(Success)}.$$

Iterated contraction by a single proposition

Via $A \equiv (A|\top)$, we obtain easily an iterative strategy for contracting by a single proposition:

$$\kappa \ominus A(\omega) = \kappa(\omega) - \kappa(\overline{A}) + \begin{cases} \gamma^{+} - \gamma^{-} & \textit{if } \omega \models A \\ 0 & \textit{if } \omega \models \overline{A} \end{cases}, \tag{2}$$

such that $\gamma^{-} - \gamma^{+} \leqslant \kappa(A) - \kappa(\overline{A})$.

$\rightarrow$ propositional c-contractions

Propositional c-contractions with AGMes

Theorem

A propositional c-contraction $\kappa^{\ominus} = \kappa \ominus A$ *fulfills the epistemic AGM postulates for contraction [Konieczny & PinoPerez, 2017] iff*

- *for* $\kappa(A) > 0$*, it has the form*

$$\kappa^{\ominus}(\omega) = \kappa(\omega) + \begin{cases} \gamma^{+} - \gamma^{-} & \text{if } \omega \models A \\ 0 & \text{if } \omega \models \overline{A} \end{cases} \tag{3}$$

  with $\gamma^{+} - \gamma^{-} > \kappa(\overline{A}) - \kappa(A)$*,*

- *while for* $\kappa(A) = 0$*, it has the form*

$$\kappa^{\ominus}(\omega) = \kappa(\omega) + \begin{cases} 0 & \text{if } \omega \models A \\ -\kappa(\overline{A}) & \text{if } \omega \models \overline{A} \end{cases} \tag{4}$$

C-contractions – example

$\mathcal{R}$: $(f|b)$ birds (usually) fly
$(b|p)$ penguins are birds

$\kappa \models \mathcal{R}^2$:

ω	$\kappa(\omega)$	ω	$\kappa(\omega)$
$p\,b\,f$	0	$\overline{p}\,b\,f$	0
$p\,b\,\overline{f}$	1	$\overline{p}\,b\,\overline{f}$	1
$p\,\overline{b}\,f$	1	$\overline{p}\,\overline{b}\,f$	0
$p\,\overline{b}\,\overline{f}$	1	$\overline{p}\,\overline{b}\,\overline{f}\,f$	0

$\kappa \models (f|p)$ – penguins fly, $\kappa \not\models \dot{p}, \dot{b}$ – indifference wrt penguins and birds.

[2] One can also use (PCP) to build up an OCF inductively from $\mathcal{R}$.

C-contractions – example (cont'd)

We want to forget that penguins can fly $\rightarrow \kappa \ominus (f|p) = \kappa^{\ominus}$;

using c-contractions of type β (see paper) with minimal $\gamma^{+} = 1$, we obtain

$$\kappa \ominus (f|p)(\omega) = \kappa(\omega) + \begin{cases} 1 \ \textit{if} \ \omega \models pf \\ 0 \ \textit{if} \ \omega \models p\overline{f} \\ 0 \ \textit{if} \ \omega \models \overline{p} \end{cases}$$

We find that

- $\kappa^{\ominus} \models (\overline{f}|p\overline{b})$, whereas $\kappa \not\models (\dot{f}|p\overline{b})$;
- still $\kappa^{\ominus} \not\models \dot{b}$, but $\kappa^{\ominus} \models \overline{p}$ – penguins are exceptional!

C-contractions – example (cont'd)

Now, we want to forget that penguins are exceptional:

$$\kappa^{\ominus,\ominus} = \kappa^{\ominus} \ominus \overline{p}$$

There is just one AGM-compatible c-contraction:

$$\kappa^{\ominus,\ominus}(\omega) = \kappa^{\ominus}(\omega) + \begin{cases} 0 & \textit{if } \omega \models \overline{p} \\ -1 & \textit{if } \omega \models p \end{cases}$$

- As expected, we find $\kappa^{\ominus,\ominus} \not\models \dot{p}, \dot{b}$, as in κ;
- however, also $(f|b), (b|p)$ have been forgotten – $\kappa^{\ominus,\ominus} \not\models (\dot{f}|b), (\dot{b}|p)$!! What about preserving conditional beliefs ?!
- $\kappa^{\ominus,\ominus}$ still remembers the original conditionals, but in a refined form:

$$\kappa^{\ominus,\ominus} \models (f|b\overline{p}) \text{ and } \kappa^{\ominus,\ominus} \models (b|pf)$$

C-contractions – example (cont'd)

ω	$\kappa(\omega)$	$\kappa^{\ominus}(\omega)$	$\kappa^{\ominus,\ominus}$
$p\,b\,f$	0	1	0
$\overline{p}\,b\,f$	0	0	0
$p\,b\,\overline{f}$	1	1	0
$\overline{p}\,b\,\overline{f}$	1	1	1
$p\,\overline{b}\,f$	1	2	1
$\overline{p}\,\overline{b}\,f$	0	0	0
$p\,\overline{b}\,\overline{f}$	1	1	0
$\overline{p}\,\overline{b}\,\overline{f}$	0	0	0

PCP also as a cornerstone for iterated contraction

Theorem

The axioms $(PCP_{\ominus}^{ocf})$, *(5), and the epistemic AGM contraction postulates imply the axioms of [Konieczny & Pino Perez, 2017] in the context of OCFs.*

$$\kappa \ominus A(\omega) \geqslant \kappa(\omega) \text{ for all } \omega \models A. \tag{5}$$

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In the context of OCF (a major framework for belief change), the principle of conditional preservation (PCP) makes iterated belief change easy:

- It follows a simple schema for setting up the changed κ,
- and can be adapted to different belief change operations by imposing supplementary, characteristic postulates (most importantly, a proper notion of success, and AGM compatibility).
- PCP belief change has a solid algebraic foundation based on conditional logic,
- and (basically) implies both [Darwiche & Pearl, 1997] and [Konieczny & Pino Perez, 2017].
- It solves much more complex iterated change tasks (changing an OCF by a set of conditional beliefs) than any other approach,
- and can also be used for inductive reasoning from knowledge bases.