TECHNISCHE UNIVERSITÄT DRESDEN

Computational Logic ∴ Group

Sebastian Rudolph (based on slides by Stefan Woltran and Sarah Gaggl)
Faculty of Computer Science, Institute of Artificial Intelligence, Computational Logic Group

Abstract Argumentation

Lecture 10, 5th Jan 2026 // Foundations of Knowledge Representation, WS 2025/26

Overview

Overall Process

Argumentation Frameworks

AFs – Semantics

AFs – Outlook

Overall Process

Introduction

Argumentation:

The study of processes "concerned with how assertions are proposed, discussed, and resolved in the context of issues upon which several diverging opinions may be held".

[Bench-Capon and Dunne, Argumentation in AI, AIJ 171:619-641, 2007]

Formal Models of Argumentation are concerned with

- representation of an argument (i.e. an expression of opinion)
- representation of the relationship between arguments
- resolving conflicts between the arguments ("acceptability")

Overall Process

The overall process of using argumentation frameworks consists of the steps listed below.

Starting point: Knowledge base

1. Form arguments
2. Identify conflicts
3. Abstract from internal structure
4. Resolve conflicts
5. Draw conclusions

Overall Process – Form Arguments

Consider the following knowledge base:

Knowledge Base

$$\Delta = \{s, r, w, s \to \neg r, r \to \neg w, w \to \neg s\}$$

From this, form arguments:

$\langle\{w, w \to \neg s\}, \neg s\rangle$

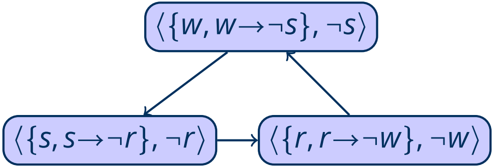
$\langle\{s, s \to \neg r\}, \neg r\rangle$

$\langle\{r, r \to \neg w\}, \neg w\rangle$

Overall Process – Identify Conflicts

Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

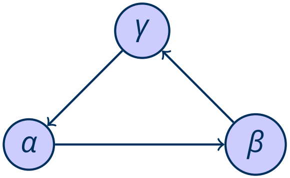


Overall Process – Abstract from Internal Structure

Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

F_Δ:



Overall Process – Resolve Conflicts

Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$

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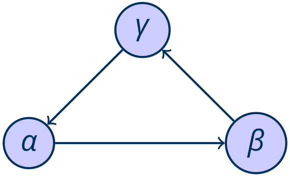
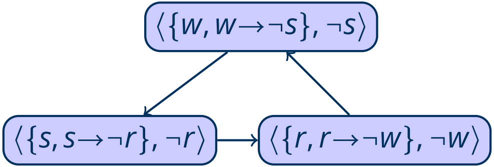


$pref(F_\Delta) = \{\emptyset\}$
$stage(F_\Delta) = \{\{\alpha\}, \{\beta\}, \{\gamma\}\}$

Overall Process – Draw Conclusions

Knowledge Base

$$\Delta = \{s, r, w, s \rightarrow \neg r, r \rightarrow \neg w, w \rightarrow \neg s\}$$



$$Cn_{pref}(F_\Delta) = Cn(\top)$$
$$Cn_{stage}(F_\Delta) = Cn(\neg r \vee \neg w \vee \neg s)$$

The Overall Process (ctd.)

Some Remarks

- Main idea dates back to the seminal work of Phan Minh Dung [1995]; has then been refined by several authors (Prakken, Gordon, Caminada, . . .)
- Separation between logical (forming arguments) and non-monotonic reasoning ("abstract argumentation frameworks")
- Abstraction allows to compare several KR formalisms on a conceptual level ("calculus of conflict")

Main Challenge

- All steps in the argumentation process are, in general, intractable.
- This calls for:
  - careful complexity analysis (identification of tractable fragments)
  - re-use of established tools for implementations (reduction method)

Approaches to Form Arguments

Classical Arguments [Besnard & Hunter, 2001]

- Given is a KB (a set of propositions) Δ
- An **argument** is a pair (Φ, α), such that $\Phi \subseteq \Delta$ is consistent, $\Phi \models \alpha$ and for no $\Psi \subsetneq \Phi$, $\Psi \models \alpha$
- An argument (Φ, α) **attacks** argument (Φ', α') iff $\Phi' \cup \{\alpha\}$ is inconsistent

Example

$$\langle\{s, s \rightarrow \neg r\}, \neg r\rangle \longrightarrow \langle\{r, r \rightarrow \neg w\}, \neg w\rangle$$

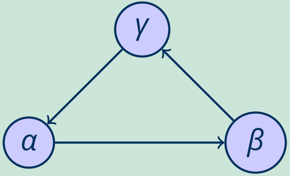
Other Approaches:

- arguments are trees (or directed acyclic graphs) of statements
- claims are obtained via strict and defeasible rules
- different notions of conflict: rebuttal, undercut, etc.

Argumentation Frameworks

Dung's Abstract Argumentation Frameworks

Example



Main Properties

- Abstract from the concrete content of arguments;
  only consider the relation between them
- Semantics select subsets of arguments respecting certain criteria
- Simple, yet powerful formalism
- Most active research area in the field of argumentation

Dung's Abstract Argumentation Frameworks

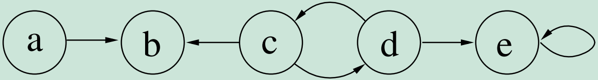
Definition

An **argumentation framework** (AF) is a pair $F = (A, R)$ where

- A is a set of arguments,
- $R \subseteq A \times A$ is a relation representing the conflicts ("attacks").

Example

$$F = (\{a, b, c, d, e\}, \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\})$$



Basic Properties (1)

Definition (Conflict-Free Sets)

Let $F = (A, R)$ be an AF.

A set $S \subseteq A$ is **conflict-free** in F iff for each $a, b \in S$ we have $(a, b) \notin R$.

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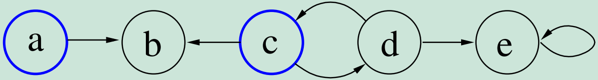

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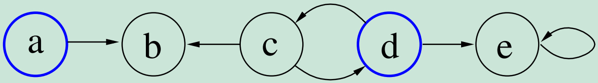

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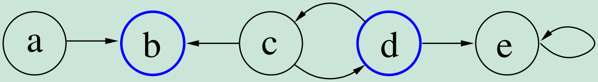

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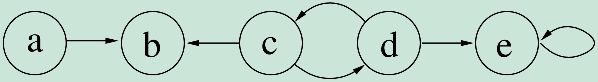

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Basic Properties (2)

Definition (Admissible Sets [Dung, 1995])

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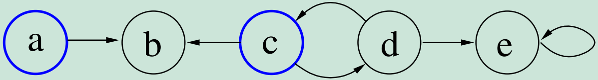

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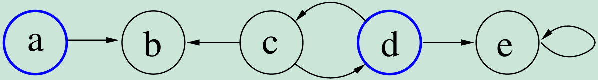

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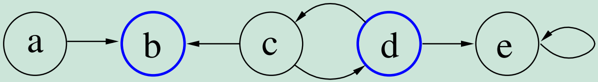

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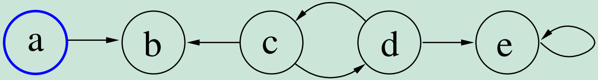

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Basic Properties (3)

Dung's Fundamental Lemma

Let S be admissible in an AF F and a, a' arguments in F defended by S in F. Then,

1. $S' = S \cup \{a\}$ is admissible in F.
2. a' is defended by S' in F.

AFs – Semantics

Naive

Definition (Naive Sets/Extensions)

Let $F = (A, R)$ be an AF.

A set $S \subseteq A$ is **naive** in F iff

- S is conflict-free in F,
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Naive sets are subset-maximally conflict-free sets.

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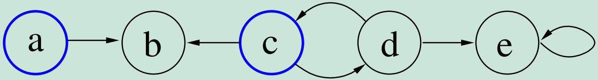

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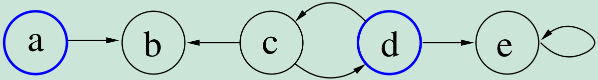

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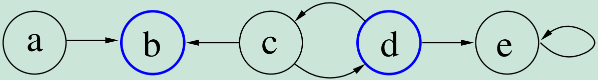

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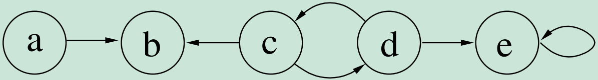

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Grounded

Definition (Grounded Extension [Dung, 1995])

Let $F = (A, R)$ be an AF. The unique **grounded extension** of F is defined as the outcome S (initially empty) of the following "algorithm":

1. put each argument $a \in A$ that is not attacked in F into S; if no such arguments exist, return S;
2. remove from F all (new) arguments in S and all arguments attacked by them (together with all adjacent attacks); and continue with Step 1.

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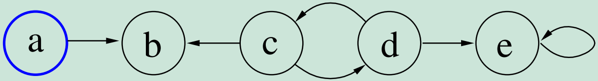

$ground(F) = \{\{a\}\}$

Complete

Definition (Complete Extension [Dung, 1995])

Let (A, R) be an AF.

A set $S \subseteq A$ is **complete** in F iff

- S is admissible in F,
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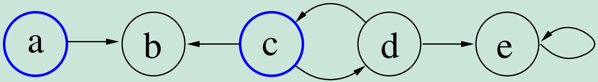

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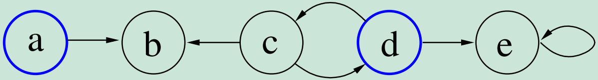

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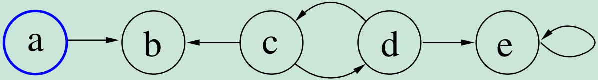

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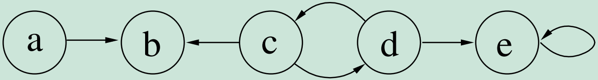

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Grounded vs. Complete

Properties of the Grounded Extension

For any AF F, the grounded extension of F is the subset-least complete extension of F.

Grounded vs. Complete

Properties of the Grounded Extension

For any AF F, the grounded extension of F is the subset-least complete extension of F.

Remark

Since there exists exactly one grounded extension for each AF F, we often write $\mathit{ground}(F) = S$ instead of $\mathit{ground}(F) = \{S\}$.

Preferred

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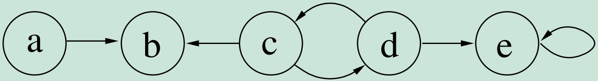

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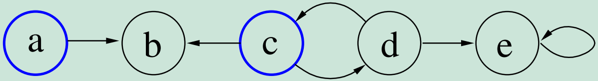

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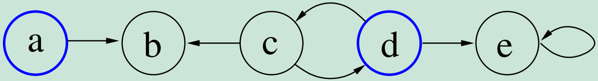

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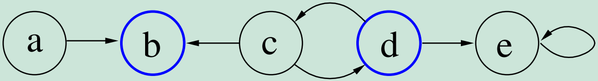

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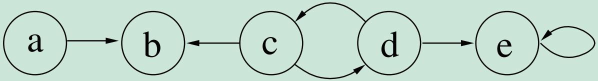

$stable(F) = \{$~~$\{a, c\}$~~, $\{a, d\}$, ~~$\{b, d\}$~~, ~~$\{a\}$~~, ~~$\{b\}$~~, ~~$\{c\}$~~, ~~$\{d\}$~~, ~~$\emptyset$~~, $\}$

Relationships Between Semantics

Proposition

For any AF F the following relations hold:

1. Each stable extension of F is also a preferred one;
2. Each preferred extension of F is also a complete one;
3. Each complete extension of F is admissible in F.

Semi-Stable

Definition (Semi-Stable Extensions [Caminada, 2006])

Let $F = (A, R)$ be an AF.

A set $S \subseteq A$ is a **semi-stable extension** of F iff

- S is admissible in F,
- there is no admissible $T \subseteq A$ in F such that $S^+ \subsetneq T^+$, where
  - for $S \subseteq A$, define $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$.

Defined as admissible stages by Verheij [1996].

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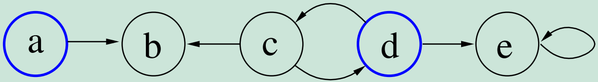


$semi(F) = \{$ ~~$\{a, c\}$~~, $\{a, d\}$, ~~$\{a\}$, $\{c\}$, $\{d\}$, $\emptyset$~~ $\}$

Stage

Definition (Stage Extensions [Verheij, 1996])

Let $F = (A, R)$ be an AF.

A set $S \subseteq A$ is a **stage extension** of F iff

- S is conflict-free in F,
- there is no conflict-free $T \subseteq A$ in F such that $S^+ \subsetneq T^+$.
  - Recall: $S^+ = S \cup \{a \mid \exists b \in S \text{ with } (b, a) \in R\}$.

S^+ is also called the range of S. Thus:

- Semi-stable extensions are range-maximally admissible sets.
- Stage extensions are range-maximally conflict-free sets.

Ideal

Definition (Ideal Extension [Dung, Mancarella & Toni 2007])

Let $F = (A, R)$ be an AF.

A set $S \subseteq A$ is an **ideal extension** of F iff

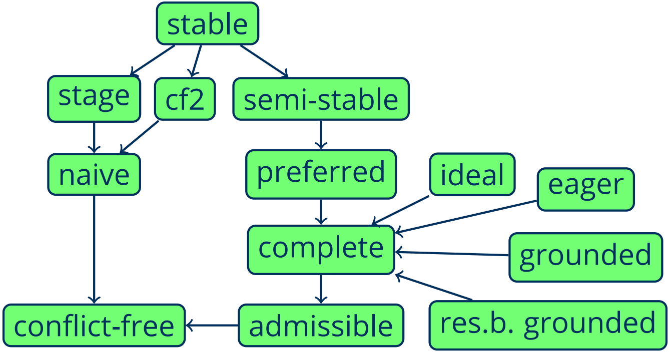
- S is admissible in F and contained in each preferred extension of F,
- there is no $T \supsetneq S$ admissible in F and contained in each of $pref(F)$.

Properties of Ideal Extensions

For any AF F the following observations hold:

1. There exists exactly one ideal extension of F.
2. The ideal extension of F is also a complete one.

Relations Between Semantics



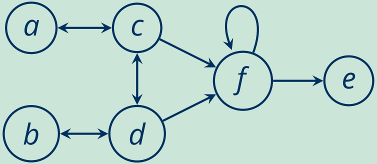
An arrow from semantics σ to semantics τ means that each σ-extension is also a τ-extension.

AFs – Outlook

Characteristics of Argumentation Semantics

Example

$pref(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b\}\}$
$naive(F) = \{\{a, d, e\}, \{b, c, e\}, \{a, b, e\}\}$



Natural Questions $\rightsquigarrow$ Realizability

- How to change the AF if we want $\{a, b, e\}$ instead of $\{a, b\}$ in $pref(F)$?
- How to change the AF if we want $\{a, b, d\}$ instead of $\{a, b\}$ in $pref(F)$?
- Can we have equivalent AFs without argument f?

Some Properties ...

Theorem

For any AFs F and G, we have

- $adm(F) = adm(G)$ implies $\sigma(F) = \sigma(G)$, for $\sigma \in \{pref, ideal\}$;
- $comp(F) = comp(G)$ implies $\theta(F) = \theta(G)$, for $\theta \in \{pref, ideal, ground\}$;
- no other such relation between the different semantics (*adm*, *pref*, *ideal*, *semi*, *ground*, *comp*, *stable*) in terms of standard equivalence holds.

Decision Problems on AFs

Credulous Acceptance

Cred_σ: Given AF $F = (A, R)$ and $a \in A$;
is a contained in at least one σ-extension of F?

Skeptical Acceptance

Skept_σ: Given AF $F = (A, R)$ and $a \in A$;
is a contained in every σ-extension of F?

If no extension exists then all arguments are skeptically accepted and no argument is credulously accepted.[1]

[1] This is only relevant for stable semantics.

Decision Problems on AFs

Hence we are also interested in the following problem:

Skeptically and Credulously Accepted

Skept'_σ: Given AF $F = (A, R)$ and $a \in A$;
is a contained in every and at least one σ-extension of F?

Further Decision Problems

Verifying an extension

Ver_σ: Given AF $F = (A, R)$ and $S \subseteq A$;
is S a σ-extension of F?

Does there exist an extension?

Exists_σ: Given AF $F = (A, R)$;
Does there exist a σ-extension for F?

Does there exist a nonempty extensions?

$\text{Exists}_\sigma^{\neg\emptyset}$: Given AF $F = (A, R)$;
Does there exist a non-empty σ-extension for F?

Conclusion

- Abstract argumentation frameworks are constructed from KBs.
- Edges (attacks) between nodes (arguments) express (directed) conflicts.
- A variety of semantics for AFs try to make sense of acceptability:
  - Complete
  - Grounded
  - Preferred
  - Stable
  - ...
- Various inclusion relationships between the semantics hold, as they build on similar notions.