

$$\neg A \cap \exists R^- . A \cap (\leq 1 R) \cap \forall (R^-)^+ . (\exists R^- . A \cap (\leq 1 R)) \quad (1)$$

$$\begin{aligned} & \backslash \text{begin}\{\text{equation}\} \\ & \quad \backslash \text{neg } A \backslash \text{sqcap } \backslash \text{exists } R^{\{-\}} . A \backslash \text{sqcap } (\backslash \text{leq } 1 \backslash R) \backslash \text{sqcap} \\ & \quad \backslash \text{forall } (R^{\{-\}})^{\{+\}} . (\backslash \text{exists } R^{\{-\}} . A \backslash \text{sqcap } (\backslash \text{leq } 1 \backslash R)) \\ & \backslash \text{end}\{\text{equation}\} \end{aligned}$$

$$\pi_x(\leq n R) = \exists^{\leq n} y . R(x, y) = \exists y_1, \dots, y_n . \bigwedge_{i \neq j} y_i \neq y_j \supset \bigvee_i \neg R(x, y_i) \quad (2)$$

$$\begin{aligned} & \backslash \text{begin}\{\text{equation}\} \\ & \quad \backslash \text{pi}_{\{x\}}(\backslash \text{leq } n \backslash R) = \backslash \text{exists}^{\{\backslash \text{leq } n\}} y . R(x, y) = \backslash \text{exists } y_{\{1\}}, \\ & \quad \backslash \text{dotsc}, y_{\{n\}} . \backslash \text{bigwedge}_{\{i \backslash \text{neq } j\}} y_{\{i\}} \backslash \text{neq } y_{\{j\}} \backslash \text{supset} \\ & \quad \backslash \text{bigvee}_{\{i\}} \backslash \text{neg } R(x, y_{\{i\}}) \\ & \backslash \text{end}\{\text{equation}\} \end{aligned}$$

$$\text{Tree} \equiv \mu X . (\text{EmptyTree} \sqcup (\text{Node} \cap \leq 1 \text{child}^- \cap \exists \text{child} . \top \cap \forall \text{child} . X)) \quad (3)$$

$$\begin{aligned} & \backslash \text{begin}\{\text{equation}\} \\ & \quad \backslash \text{mathsf}\{\text{Tree}\} \backslash \text{equiv } \backslash \mu X . (\backslash \text{mathsf}\{\text{EmptyTree}\} \backslash \text{sqcup} (\backslash \text{mathsf}\{\text{Node}\} \\ & \quad \backslash \backslash \text{sqcap } \backslash \text{leq } 1 \backslash \backslash \text{mathsf}\{\text{child}\}^{\{-\}} \backslash \text{sqcap } \backslash \text{exists} \\ & \quad \backslash \backslash \text{mathsf}\{\text{child}\} . \backslash \text{top } \backslash \text{sqcap } \backslash \text{forall } \backslash \backslash \text{mathsf}\{\text{child}\} . X)) \\ & \backslash \text{end}\{\text{equation}\} \end{aligned}$$

$$(\mu X . C)_\rho^{\mathcal{I}} = \bigcap \{ \mathcal{E} \subseteq \Delta^{\mathcal{I}} \mid C_{\rho[X/\mathcal{E}]}^{\mathcal{I}} \subseteq \mathcal{E} \} \quad (4)$$

$$\begin{aligned} & \backslash \text{begin}\{\text{equation}\} \\ & \quad (\backslash \mu X . C)_{\rho}^{\{\backslash \text{rho}\}^{\{\backslash \text{mathcal}\{I\}\}}} = \backslash \text{bigcap } \{ \backslash \backslash \text{mathcal}\{E\} \backslash \text{subteq} \\ & \quad \backslash \backslash \text{Delta}^{\{\backslash \text{mathcal}\{I\}\}} \backslash \text{mid } C_{\rho[X/\backslash \text{mathcal}\{E\}]}^{\{\backslash \text{mathcal}\{I\}\}} \\ & \quad \backslash \backslash \text{subteq } \backslash \backslash \text{mathcal}\{E\} \backslash \} \\ & \backslash \text{end}\{\text{equation}\} \end{aligned}$$

$$s \rightarrow_E t \text{ iff } \exists (l, r) \in E, p \in \text{Pos}(s), \sigma \in \text{Sub}. s|_p = \sigma(l) \text{ and } t = s[\sigma(r)]_p \quad (5)$$

$$\begin{aligned} & \backslash \text{begin}\{\text{equation}\} \\ & \quad s \backslash \text{to}_{\{E\}} t \backslash \text{iff} \backslash \backslash \text{exists } (l, r) \backslash \text{in } E, p \backslash \text{in} \\ & \quad \backslash \backslash \text{mathcal}\{P\}\text{os}(s), \backslash \text{sigma } \backslash \text{in } \backslash \backslash \text{mathcal}\{S\}\text{ub}. \backslash \text{left}. s \backslash \text{right}|_{\{p\}} = \\ & \quad \backslash \text{sigma}(l) \backslash \text{and} \backslash t = s[\backslash \text{sigma}(r)]_{\{p\}} \\ & \backslash \text{end}\{\text{equation}\} \end{aligned}$$

$$\mathbb{K}[\mathcal{E}]_r := (G \cup \mathcal{E}_{\min}, M \cup \mathcal{E}_{\max}, I_{\mathcal{E}} \cap (G \cup \mathcal{E}_{\min}) \times (M \cup \mathcal{E}_{\max})) \quad (6)$$

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\begin{equation}
\mathbb{K}[\mathfrak{C}]_r \coloneqq (G \mathbf{cup} \mathfrak{C}_{\min}, M \mathbf{cup}
\mathfrak{C}_{\max}),
I_{\mathfrak{C}} \mathbf{cap} (G \mathbf{cup} \mathfrak{C}_{\min}) \mathbf{times} (M \mathbf{cup} \mathfrak{C}_{\max}))
\end{equation}

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$$0 = \int_{\{s_n(u) > \frac{1}{k} + \mathbf{E}^{A_n} u\}} (s_n(u) - \mathbf{E}^{A_n} u) d\mu \geq \frac{1}{k} \mu \left( \left\{ s_n(u) > \frac{1}{k} + \mathbf{E}^{A_n} u \right\} \right) \quad (7)$$

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\begin{equation}
0 = \int_{\left\{ s_n(u) > \frac{1}{k} + \mathbf{E}^{A_n} u \right\}} (s_n(u) - \mathbf{E}^{A_n} u) d\mu \geq \frac{1}{k} \mu \left( \left\{ s_n(u) > \frac{1}{k} + \mathbf{E}^{A_n} u \right\} \right)
\end{equation}

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