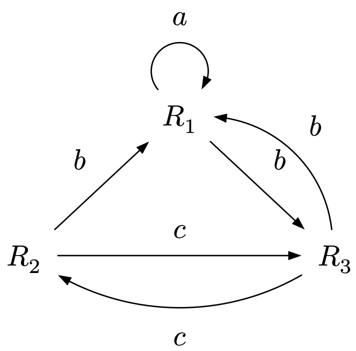
TU Dresden, Knowledge-Based Systems Group
Dr. Stephan Mennicke Summer 2024

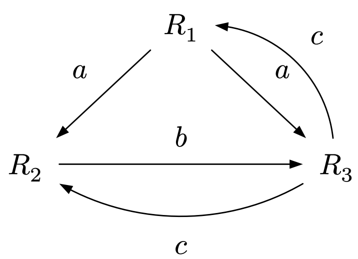
Concurrency Theory
Exercise Sheet 1: Bisimulation and Bisimilarity
17th of April 2024

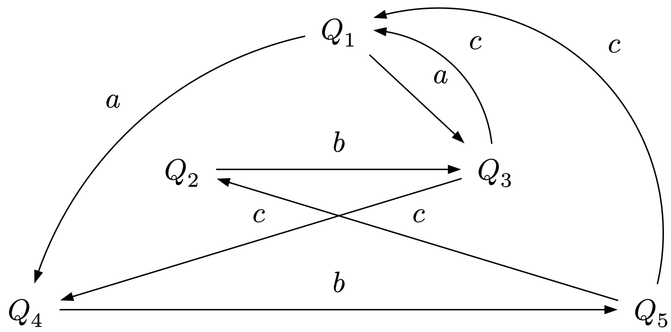
Before we get into this exercise session, we want to introduce a notation that will also be present in the respective lecture slides. As a convention, and we sometimes already did, we write binary relations like $\xrightarrow{a} \subseteq \mathrm{Pr} \times \mathrm{Pr}$ ($a \in \mathrm{Act}$) in infix notation: $P \xrightarrow{a} Q$ instead of $(P, a, Q) \in \rightarrow$. We use the same convention for bisimulations: write $P \mathcal{R} Q$ instead of $(P, Q) \in \mathcal{R}$.

Exercise 1.1. Find an LTS with only two states that is bisimilar to the following LTS:



Exercise 1.2. Show that R_1 and Q_1 are bisimilar.





Exercise 1.3. Consider the following change of clauses 1 and 2 in the definition of bisimulation, and consequently bisimilarity:

- for all P' with $P \xrightarrow{\mu} P'$, and for all Q' such that $Q \xrightarrow{\mu} Q'$, we have $P' \mathcal{R} Q'$;
- for all Q' with $Q \xrightarrow{\mu} Q'$, and for all P' such that $P \xrightarrow{\mu} P'$, we have $P' \mathcal{R} Q'$.

What would be the effect on *bisimilarity*?

Exercise 1.4. Let T be an LTS. Prove or disprove that bisimulations (on T) are closed under

(a) union;
(b) intersection;
(c) relation concatenation.

Complete the proof of Theorem 7 (2nd lecture, slide 24).

Exercise 1.5. A process relation $\mathcal{R}$ is a *simulation* if, whenever $P \mathcal{R} Q$, for all P' and μ with $P \xrightarrow{\mu} P'$, there is a Q' such that $Q \xrightarrow{\mu} Q'$ and $P' \mathcal{R} Q'$. Similarity, written $\lesssim$, is the union of all simulations; Q simulates P if $P \lesssim Q$. P and Q are *simulation equivalent*, denoted by $P \approx Q$, if $P \lesssim Q$ and $Q \lesssim P$. Show that

(a) $\mathcal{R}$ is a bisimulation if, and only if, $\mathcal{R}$ and $\mathcal{R}^{-1}$ are simulations.
(b) If P is a process without a transition, then $P \lesssim Q$ for all processes Q.
(c) $\lesssim$ is reflexive and transitive.

(d) $\Leftrightarrow$ is strictly included in $\approx$.
(e) $\approx$ is strictly included in trace equivalence.

Exercise 1.6. A process relation $\mathcal{R}$ is a *bisimulation up-to* $\Leftrightarrow$ if, whenever $P \Leftrightarrow Q$, for all μ we have

(a) for all P' with $P \xrightarrow{\mu} P'$, there is a Q' such that $Q \xrightarrow{\mu} Q'$ and $P' \Leftrightarrow \mathcal{R} \Leftrightarrow Q'$;[1]
(b) the converse on transitions from Q.

Show that if $\mathcal{R}$ is a bisimulation up-to $\Leftrightarrow$, then $\mathcal{R} \subseteq \Leftrightarrow$.

[1]Here, $\Leftrightarrow \mathcal{R} \Leftrightarrow$ is the two-fold relational composition of $\Leftrightarrow$ with $\mathcal{R}$ and, then again, with $\Leftrightarrow$.