TECHNISCHE UNIVERSITÄT DRESDEN

Artificial Intelligence, Computational Logic

PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

Lecture 5 Tabu Search

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DRESDEN concept
Exzellenz aus Wissenschaft und Kultur

Agenda

1. Introduction
2. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
3. Local Search, Stochastic Hill Climbing, Simulated Annealing
4. Tabu Search
5. Answer-set Programming (ASP)
6. Constraint Satisfaction (CSP)
7. Structural Decomposition Techniques (Tree/Hypertree Decompositions)
8. Evolutionary Algorithms/ Genetic Algorithms

Tabu Search

Main Idea

- A memory forces the search to explore new areas of the search space
- Memorize solutions that have been examined recently. They become tabu points in next steps
- Tabu search is deterministic

Tabu Search and SAT

- SAT problem with $n = 8$ variables
- Initial (random) assignment $\mathbf{x} = (0, 1, 1, 1, 0, 0, 0, 1)$
- Evaluation function: weighted sum of number of satisfied clauses. Weights depend on the number of variables in the clause
- Maximize evaluation function (i.e. we're trying to satisfy all clauses)
- Random assignment provides $eval(\mathbf{x}) = 27$
- Neighborhood of $\mathbf{x}$ consists of 8 solutions. Evaluate them and select best
- **At this stage, it is the same as hill-climbing**
- Suppose flipping 3rd variable generates best evaluation ($eval(\mathbf{x}') = 31$)
- Memory keeps track of actions

Recency-based Memory

- Index of flipped variable + time when it was flipped
- **Differentiate between older and more recent flips**
- SAT: time stamp for each position of solution vector M (initialized to 0)
- Value of time stamp provides information on recency of flip at position

Memory Vector

$M(i) = j$ (when $j \neq 0$)
j is most recent iteration when i-th bit was flipped

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Assume information is stored for at most 5 iterations.

Alternative Interpretation

$M(i) = j$ (when $j \neq 0$)
i-th bit was flipped $5 - j$ iterations ago

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Example

0	0	5	0	0	0	0	0

Memory after one iteration. 3rd bit is **tabu** for next 5 iterations.

Different Interpretations

1st Variant

- Stores iteration number of most recent flip
- Requires a current iteration counter t which is compared with memory values
- If $t - M(i) > 5$ forget
- Only requires updating a single entry, and increase the counter
- **Used in most implementations**

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2nd Variant

- Values are interpreted as number of iterations for which a position is not available
- **All** nonzero entries are decreased by one at every iteration

Example ctd.

- Initial assignment $\mathbf{x} = (0, 1, 1, 1, 0, 0, 0, 1)$
- After 4 additional iterations M :

3	0	1	5	0	4	2	0

- Most recent flip $M(4) = 5$
- Current solution: $\mathbf{x} = (1, 1, 0, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$

Example ctd.

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Neighborhood of $\mathbf{x}$

$$\mathbf{x}_1 = (0, 1, 0, 0, 0, 1, 1, 1) \qquad \mathbf{x}_5 = (1, 1, 0, 0, 1, 1, 1, 1)$$
$$\mathbf{x}_2 = (1, 0, 0, 0, 0, 1, 1, 1) \qquad \mathbf{x}_6 = (1, 1, 0, 0, 0, 0, 1, 1)$$
$$\mathbf{x}_3 = (1, 1, 1, 0, 0, 1, 1, 1) \qquad \mathbf{x}_7 = (1, 1, 0, 0, 0, 1, 0, 1)$$
$$\mathbf{x}_4 = (1, 1, 0, 1, 0, 1, 1, 1) \qquad \mathbf{x}_8 = (1, 1, 0, 0, 0, 1, 1, 0)$$

Example ctd.

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TABU, best evaluation $eval(\mathbf{x}_5) = 32$, **decrease!**

Example ctd.

- Current solution: $\mathbf{x} = (1, 1, 0, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$
- New solution: $\mathbf{x}_5 = (1, 1, 0, 0, 1, 1, 1, 1)$ with $eval(\mathbf{x}_5) = 32$

3	0	1	5	0	4	2	0

changes to:

2	0	0	4	5	3	1	0

Example ctd.

- Current solution: $\mathbf{x} = (1, 1, 0, 0, 0, 1, 1, 1)$ with $eval(\mathbf{x}) = 33$
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Policy might be too restrictive

- What if tabu neighbor $\mathbf{x}_6$ provides excellent evaluation score?
- Make search more flexible: override tabu classification if solution is outstanding

$\Longrightarrow$ **aspiration criterion**

Long-term Memory

Question

1. What is stored in long-term memory (think of SAT as an example)?
2. How can we escape local optima with help of a long-term memory?

Frequency-based Memory

- Operates over a longer horizon
- SAT: vector H serves as long-term memory.
  - Initialized to 0, at any stage of the search

$$H(i) = j$$

    interpreted as: during last h (horizon) iterations, the i-th bit was flipped j times
  - Usually horizon is large
  - After 100 iterations with $h = 50$, long-term memory H might have the following values

5	7	11	3	9	8	1	6

  - Shows distribution of moves throughout the last 50 iterations

Diversity of Search

Frequency-based memory provides information about which flips have been under-represented or not represented.
$\Longrightarrow$ we can diversify the search by exploring these possibilities

Use of Long-term Memory

Special Circumstances

- Situations where all non-tabu moves lead to worse solution
- To make a meaningful decision about which direction to explore next
- Typically: most frequent moves are less attractive
- Value of evaluation score is decreased by some penalty measure that depends on frequency, final score implies the winner

Example SAT

- Assume value of current solution is $eval(\mathbf{x}) = 35$
- Non-tabu flips $2, 3$ and 7 have values $30, 33, 31$
- None of tabu moves provides value greater than 37 (highest value so far)
  $\Longrightarrow$ we can't apply aspiration criterion

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- Non-tabu flips $2, 3$ and 7 have values $30, 33, 31$
- None of tabu moves provides value greater than 37 (highest value so far) $\implies$ we can't apply aspiration criterion
- Frequency based-memory and evaluation function for new solution $\mathbf{x}'$ is

$$eval(\mathbf{x}') - penalty(\mathbf{x}')$$

- $penalty(\mathbf{x}') = 0.7 \times H(i)$, where 0.7 coefficient, $H(i)$ value from long-term memory H :

7	for solution created by flipping 2nd bit
11	for solution created by flipping 3nd bit
1	for solution created by flipping 7nd bit

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- New scores are:

$30 - 0.7 \times 7 = 25.1$	2nd bit
$33 - 0.7 \times 11 = 25.3$	3nd bit
$31 - 0.7 \times 1 = 30.3$	7th bit

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Diversify Search

Including frequency values in a penalty measure for evaluating solutions.

Further Options to Diversify Search

We migth add additional rules:

- Aspiration by default: select the oldest of all considered
- Aspiration by search direction: memorize whether or not the performed moves generated any improvement
- Aspiration by influence: measures the degree of change of the new solution
  a) in terms of the distance between old and new solution
  b) change in solution's feasibility, if we deal with a constraint problem
  – Intuition: particular move has a larger influence if a larger step was made from old to new solution

Tabu Search and the TSP

- Move: swap two cities in a particular solution
- Current solution: $(2, 4, 7, 5, 1, 8, 3, 6)$
- 28 neighbors $\binom{8}{2} = \frac{7 \cdot 8}{2} = 28$
- Recency-based memory: swap of cities i and j in i-th row and j-th column (for $i < j$)
- Maintain number of remaining iterations for which swap stays on tabu list
- Frequency-based memory: same structure; indicate totals of all swaps within horizon $h = 50$

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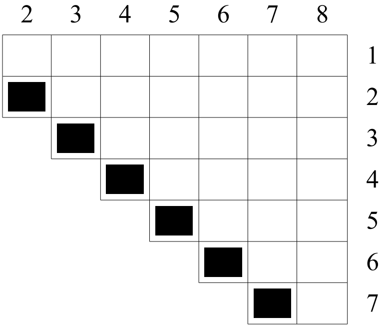

Tabu Search and the TSP ctd.

- Assume both memories initialized to zero and 500 iterations have been completed
- Current solution: $(7, 3, 5, 6, 1, 2, 4, 8)$ with length: 173, best solution so far 171

2	3	4	5	6	7	8	
0	0	1	0	0	0	0	1
	0	0	0	5	0	0	2
		0	0	0	4	0	3
			3	0	0	0	4
				0	0	2	5
					0	0	6
						0	7

2	3	4	5	6	7	8	
0	2	3	3	0	1	1	1
	2	1	3	1	1	0	2
		2	3	3	4	0	3
			1	1	2	1	4
				4	2	1	5
					3	1	6
						6	7

left: recency-based memory; right: frequency-based memory

Particular Implementation of Tabu Search for TSP

Algorithm Tabu Search for TSP [Knox, J. (1994)]

```
tries ← 0
repeat
    generate a tour
    count ← 0
    repeat
        identify a set T of 2-interchange moves
        select the best admissible move from T
        make appropriate 2-interchange
        update taub list and other variables
        if the new tour is the best-so-far for a given 'tries' then
            update local best tour information
        else
            count ← count + 1
        end if
    until count = ITER
    tries ← tries + 1
    if the current tour is the best-so-far (for all 'tries') then
        update global best tour information
    end if
until tries = MAX-TRIES
```

Particular Implementation of Tabu Search for TSP ctd.

- A tour is tabu if both added edges of interchange were on tabu list
- Tabu list update: placing added edges on list (deleted edges are ignored)
- Tabu list is of fixed size
- Whenever it is full, the oldest element in list is replaced by new deleted edge
- Initially, list is empty and all elements of aspiration list are set to large values
- Note: Algorithm examines all neighbors, i.e. all 2-*interchange* tours

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- Whenever it is full, the oldest element in list is replaced by new deleted edge
- Initially, list is empty and all elements of aspiration list are set to large values
- Note: Algorithm examines all neighbors, i.e. all 2-*interchange* tours
- Best results were achieved when
  - Length of tabu list was $3n$ (n number of cities of problem)
  - Candidate tour could override tabu status if both edges passed aspiration test. Compared length of tour with aspiration values for both added edges. If length of tour was better than both aspiration values, the test was passed.
  - Values present on aspiration list were tour costs prior to the interchange
  - MAX-TRIES and ITER (# of interchanges) depend on size of the problem. For 100 cities or less, MAX-TRIES was 4, and ITER was set to $0.0003 \cdot n^4$.

Summary

- Simulated annealing and tabu search are both design to escape local optima
- Tabu search makes uphill moves only when it is stuck in local optima
- Simulated annealing can make uphill moves at any time
- Simulated annealing is stochastic, tabu search is deterministic
- Compared to classic algorithms, both work on complete solutions. One can halt them at any iteration and obtain a possible solution
- Both have many parameters to worry about

References

Zbigniew Michalewicz and David B. Fogel.
How to Solve It: Modern Heuristics, volume 2. Springer, 2004.

Knox, J.
Tabu Search Performance on the Symmetric Traveling Salesman Problem, Computer Operations Research, Vol.21, No.8, pp.867–876, 1994.