Conditionals under Weak Completion Semantics

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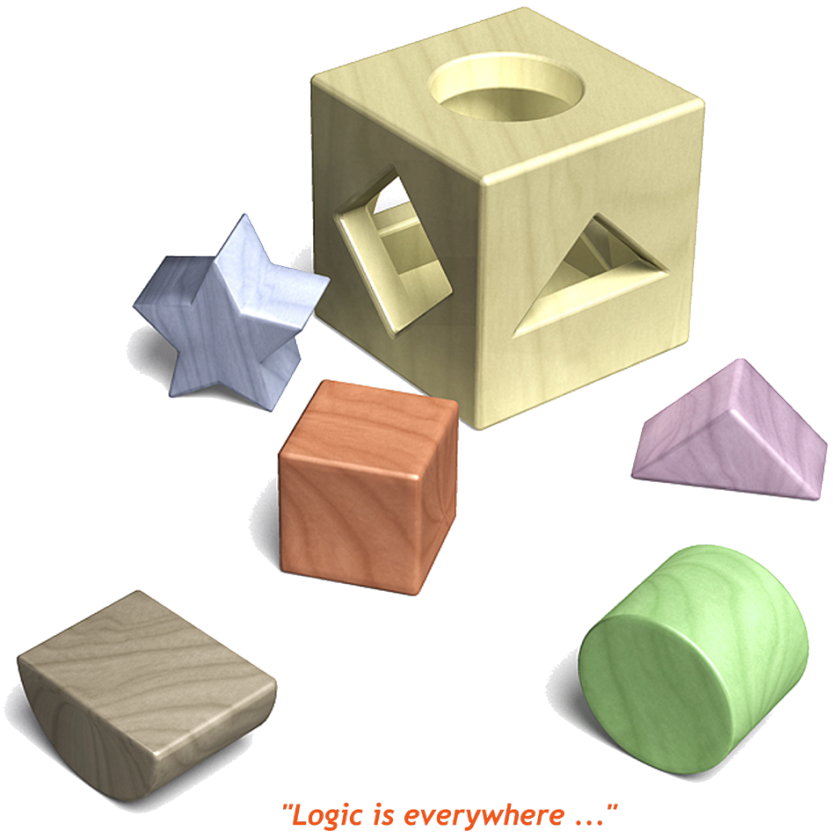

"Logic is everywhere ..."

Conditionals

- Statements of the form *if condition then consequence*
- Indicative conditionals
  - Condition and consequence can be true, false or unknown
  - If the condition is true, the consequence is asserted to be true
- Subjunctive conditionals (counterfactuals)
  - Condition is false
  - Consequence can be true, false or unknown
  - In the counterfactual circumstance of the condition being true, the consequence is asserted to be true

Evaluating conditionals

- **MRFA: Minimal Revision Followed by Abduction**
  - **Model background knowledge as a logic program**
  - **Reason wrt the least model of the weak completion of the program**
  - **Explain the conditions by revision and abduction**
  - **Evaluate the conclusion, once the conditions are explained**
  - **Minimize revision and maximize abduction**

MRFA: Minimal Revision Followed by Abduction

- Given $\mathcal{P}$ and $cond(\mathcal{C}, \mathcal{D})$
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \top$ then $cond(\mathcal{C}, \mathcal{D})$ is assigned to $\mathcal{M}_{\mathcal{P}}(\mathcal{D})$.
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \bot$, then evaluate $cond(\mathcal{C}, \mathcal{D})$ with respect to $\mathcal{M}_{rev(\mathcal{P},\mathcal{S})}$, where $\mathcal{S} = \{L \in \mathcal{C} \mid \mathcal{M}_{\mathcal{P}}(L) = \bot\}$.
  - If $\mathcal{M}_{\mathcal{P}}(\mathcal{C}) = \mathsf{U}$, then evaluate $cond(\mathcal{C}, \mathcal{D})$ with respect to $\mathcal{M}_{\mathcal{P}'}$, where
    - $\mathcal{P}' = rev(\mathcal{P}, \mathcal{S}) \cup \mathcal{E}$,
    - $\mathcal{S}$ is a smallest subset of $\mathcal{C}$ and $\mathcal{E} \subseteq \mathcal{A}_{rev(\mathcal{P},\mathcal{S})}$ is a minimal explanation for $\mathcal{C} \setminus \mathcal{S}$ such that $\mathcal{M}_{\mathcal{P}'}(\mathcal{C}) = \top$.

Examples

- **Background knowledge**
  - **If it rains, then the streets are wet and I take my umbrella**
- $\mathcal{P} = \{wet\_streets \leftarrow rain \wedge \neg ab_1,$
  $umbrella \leftarrow rain \wedge \neg ab_2,$
  $ab_1 \leftarrow \bot,$
  $ab_2 \leftarrow \bot\}$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab_1, ab_2\}\rangle$
- $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \bot\}$

Examples

- **If the streets are not wet, then it did not rain**
  - $cond(\{\neg wet\_streets\}, \{\neg rain\})$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab_1, ab_2\}\rangle$
- $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \bot\}$
- $\mathcal{P} = \{wet\_streets \leftarrow rain \wedge \neg ab_1,$
  $umbrella \leftarrow rain \wedge \neg ab_2,$
  $ab_1 \leftarrow \bot,$
  $ab_2 \leftarrow \bot\}$
- $\mathcal{E} = \{rain \leftarrow \bot\}$
- $M_{\mathcal{P}\cup\mathcal{E}} = \langle \emptyset, \{rain, wet\_streets, umbrella, ab_1, ab_2\}\rangle$
- **The conditional is true**

Examples

- **If the streets are wet, then it rained**
  - $cond(\{wet\_streets\}, \{rain\})$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab_1, ab_2\}\rangle$
- $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \top\}$
- $\mathcal{P} = \{wet\_streets \leftarrow rain \wedge \neg ab_1,$
  $umbrella \leftarrow rain \wedge \neg ab_2,$
  $ab_1 \leftarrow \bot,$
  $ab_2 \leftarrow \bot\}$
- $\mathcal{E} = \{rain \leftarrow \top\}$
- $M_{\mathcal{P}\cup\mathcal{E}} = \langle\{rain, wet\_streets, umbrella\}, \{ab_1, ab_2\}\rangle$
- **The conditional is true**

Examples

- **Background knowledge**
  - **If it rains, then the streets are wet and I take my umbrella**
- **Denying the consequent (Modus tollens)**
  - **If the streets are not wet, then it did not rain**
  - **If I did not take my umbrella, then it did not rain**
- **Affirming the consequent**
  - **If the streets are wet, then it rained**
  - **If I took my umbrella, then it rained**

Obligation vs factual conditional

If *condition*, then *consequence*

- **Background knowledge**
  - **If it rains, then the streets are wet and I take my umbrella**
- **Obligation conditional: *consequence* is obligatory**
  - **Permitted possibility: *condition* and *consequence***
  - **Forbidden possibility: *condition* and not *consequence***
  - **Example: If the streets are not wet, then it did not rain *(true)***
- **Factual conditional**
  - **Permitted possibility: *condition* and *consequence***
  - **Example: If I did not take my umbrella, then it did not rain *(not true)***

Necessity vs sufficiency

If *condition*, then *consequence*

- **Background knowledge**
  - **If it rains, then the streets are wet and I take my umbrella**
- **Necessity**
  - **The *consequence* cannot be true unless the *condition* is true**
  - **Example: If the streets are wet, then it rained *(true)***
- **Sufficiency**
  - **The *condition* to be true is adequate grounds to conclude the *consequence***
  - **Example: If I took my umbrella, then it rained *(not true)***

Handling *obligation vs factual conditionals* and *necessity vs sufficiency*

- **Modify the set of abducibles to model the difference between**
  - **obligation and facts**
  - **necessary and sufficient conditions**
- $\mathcal{A}_{\mathcal{P}} = \{A \leftarrow \top \mid \text{A is undefined in } \mathcal{P}\}$
  $\cup \{A \leftarrow \bot \mid \text{A is undefined in } \mathcal{P}\}$
  $\cup \{A \leftarrow \top \mid \text{A is the head of a sufficient conditional in } \mathcal{P}\}$
  $\cup \{ab_i \leftarrow \top \mid ab_i \text{ is in the body of a factual conditional in } \mathcal{P}\}$
- **Reason skeptically wrt the consequences**
- $\mathcal{F}$ **follows skeptically from** $\mathcal{P}$ **and** $\mathcal{O}$
  iff for all explanations $\mathcal{E}$ **for** $\mathcal{O}$ **we find** $\mathcal{P} \cup \mathcal{E} \models_{WCS} \mathcal{F}$

Examples: revisited

- **Background knowledge**
  - **If it rains, then the streets are wet *(Obligation and necessity)***
  - **If it rains, then I take my umbrella *(Factual and sufficiency)***
- $\mathcal{P} = \{wet\_streets \leftarrow rain \wedge \neg ab_1,$
  $umbrella \leftarrow rain \wedge \neg ab_2,$
  $ab_1 \leftarrow \bot,$
  $ab_2 \leftarrow \bot\}$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab_1, ab_2\}\rangle$
- $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \bot, umbrella \leftarrow \top, ab_2 \leftarrow \top\}$

Examples: revisited (Obligation and necessity)

- **If the streets are not wet, then it did not rain**
  - $cond(\{\neg wet\_streets\}, \{\neg rain\})$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab_1, ab_2\}\rangle$
- $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \bot, umbrella \leftarrow \top, ab_2 \leftarrow \top\}$
- $\mathcal{P} = \{wet\_streets \leftarrow rain \wedge \neg ab_1,$
  $umbrella \leftarrow rain \wedge \neg ab_2,$
  $ab_1 \leftarrow \bot,$
  $ab_2 \leftarrow \bot\}$
- $\mathcal{E} = \{rain \leftarrow \bot\}$
- $M_{\mathcal{P}\cup\mathcal{E}} = \langle \emptyset, \{rain, wet\_streets, umbrella, ab_1, ab_2\}\rangle$
- **The conditional is true**

Examples: revisited (Factual and sufficiency)

- **If I did not take my umbrella, then it did not rain**
  - $cond(\{\neg umbrella\}, \{\neg rain\})$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab_1, ab_2\}\rangle$
- $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \bot, umbrella \leftarrow \top, ab_2 \leftarrow \top\}$
- $\mathcal{P} = \{wet\_streets \leftarrow rain \wedge \neg ab_1,$
  $umbrella \leftarrow rain \wedge \neg ab_2,$
  $ab_1 \leftarrow \bot,$
  $ab_2 \leftarrow \bot\}$

$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}\cup\mathcal{E}}$	
$\{rain \leftarrow \bot\}$	$\langle \emptyset, \{rain, wet\_streets, umbrella, ab_1, ab_2\}\rangle$	$\mathcal{P} \cup \mathcal{E} \models_{WCS} \neg rain$
$\{ab_2 \leftarrow \top\}$	$\langle \{ab_2\}, \{umbrella, ab_1\}\rangle$	$\mathcal{P} \cup \mathcal{E} \not\models_{WCS} \neg rain$

- **The conditional is unknown**

Examples: revisited (Obligation and necessity)

- **If the streets are wet, then it rained**
  - $cond(\{wet\_streets\}, \{rain\})$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab_1, ab_2\}\rangle$
- $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \bot, umbrella \leftarrow \top, ab_2 \leftarrow \top\}$
- $\mathcal{P} = \{wet\_streets \leftarrow rain \wedge \neg ab_1,$
  $umbrella \leftarrow rain \wedge \neg ab_2,$
  $ab_1 \leftarrow \bot,$
  $ab_2 \leftarrow \bot\}$
- $\mathcal{E} = \{rain \leftarrow \top\}$
- $M_{\mathcal{P}\cup\mathcal{E}} = \langle\{rain, wet\_streets, umbrella\}, \{ab_1, ab_2\}\rangle$
- **The conditional is true**

Examples: revisited (Factual and sufficiency)

- **If I took my umbrella, then it rained**
  - $cond(\{umbrella\}, \{rain\})$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab_1, ab_2\}\rangle$
- $\mathcal{A}_{\mathcal{P}} = \{rain \leftarrow \top, rain \leftarrow \bot, umbrella \leftarrow \top, ab_2 \leftarrow \top\}$
- $\mathcal{P} = \{wet\_streets \leftarrow rain \wedge \neg ab_1,$
  $umbrella \leftarrow rain \wedge \neg ab_2,$
  $ab_1 \leftarrow \bot,$
  $ab_2 \leftarrow \bot\}$

$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}\cup\mathcal{E}}$	
$\{rain \leftarrow \top\}$	$\langle\{rain, wet\_streets, umbrella\}, \{ab_1, ab_2\}\rangle$	$\mathcal{P} \cup \mathcal{E} \models_{WCS} rain$
$\{umbrella \leftarrow \top\}$	$\langle\{umbrella\}, \{ab_1, ab_2\}\rangle$	$\mathcal{P} \cup \mathcal{E} \not\models_{WCS} rain$

- **The conditional is unknown**

Examples: revisited

- **Background knowledge**
  - **If it rains, then the streets are wet and I take my umbrella**
- **Obligation conditionals with necessary conditions: Evaluated as true**
  - **If the streets are not wet, then it did not rain**
  - **If the streets are wet, then it rained**
- **Factual conditionals with sufficient conditions: Evaluated as unknown**
  - **If I did not take my umbrella, then it did not rain**
  - **If I took my umbrella, then it rained**

The Wason Selection Task: Abstract Case (Wason 1968)

- **Consider cards with a letter on one side and a number on the other side**
- **Given the conditional**

  If there is a D on one side of the card, then there is a 3 on the other side

- **Which cards must be turned to show that the conditional holds?**

D	F	3	7
89%	16%	62%	25%

The Wason Selection Task: Social Case (Griggs and Cox 1982)

- **Consider cards with a person's age on one side and what the person is drinking on the other side**
- **Given the conditional**

  If a person is drinking beer, then the person must be over 19 years of age

- **Which cards must be turned to show that the conditional holds?**

beer	coke	22yrs	16yrs
95%	0.025%	0.025%	80%

Modelling the Wason Selection Task

- Consider cards with X on one side and Y on the other side
- Conditional: If X, then Y
- $\mathcal{P} = \{Y \leftarrow X \wedge \neg ab,$
  $ab \leftarrow \bot\}$
- Set $\mathcal{A}_{\mathcal{P}}$ of abducibles will depend on the type of the conditional
- Assume $\mathcal{O} = \{X\}$ st X can be negative or positive
- Turn the card, when
  - X and Y follow skeptically from $\mathcal{P}$ and $\mathcal{O}$, or
  - Y is the head of a rule in each $\mathcal{E} \in \{\mathcal{E} \mid \mathcal{P} \cup \mathcal{E} \models X\}$

The Wason Selection Task: Abstract Case (Wason 1968)

- **If there is a D on one side of the card, then there is a 3 on the other side**
  - **Defined as a belief by Kowalski**
  - **Factual conditional and necessity**
- $\mathcal{P} = \{3 \leftarrow D \wedge \neg ab,$
  $ab \leftarrow \bot\}$
- $M_{\mathcal{P}} = \langle \emptyset, \{ab\} \rangle$
- $\mathcal{A}_{\mathcal{P}} = \{D \leftarrow \top, D \leftarrow \bot, ab \leftarrow \top\}$

$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P} \cup \mathcal{E}}$	Experimental results
D	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, \{ab\} \rangle \rightsquigarrow$ turn	89%
F ($\neg D$)	$\{D \leftarrow \bot\}$	$\langle \emptyset, \{D, 3, ab\} \rangle$	16%
3	$\{D \leftarrow \top\}$	$\langle \{D, 3\}, \{ab\} \rangle \rightsquigarrow$ turn	62%
7 ($\neg 3$)	$\{D \leftarrow \bot\}$	$\langle \emptyset, \{D, 3, ab\} \rangle$	25%
	$\{ab \leftarrow \top\}$	$\langle \{ab\}, \{3\} \rangle$	

The Wason Selection Task: Social Case (Griggs and Cox 1982)

- **If a person is drinking beer, then the person must be over 19 years of age**
  - **Defined as a goal by Kowalski**
  - **Obligation conditional and sufficiency**
- $\mathcal{P} = \{\mathit{over\_19} \leftarrow \mathit{beer} \wedge \neg \mathit{ab},$
  $\mathit{ab} \leftarrow \bot\}$
- $M_{\mathcal{P}} = \langle \emptyset, \{\mathit{ab}\} \rangle$
- $\mathcal{A}_{\mathcal{P}} = \{\mathit{beer} \leftarrow \top, \mathit{beer} \leftarrow \bot, \mathit{over\_19} \leftarrow \top\}$

$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P} \cup \mathcal{E}}$	**Results**
beer	$\{\mathit{beer} \leftarrow \top\}$	$\langle \{\mathit{beer}, \mathit{over\_19}\}, \{\mathit{ab}\} \rangle \rightsquigarrow$ turn	95%
coke ($\neg\mathit{beer}$)	$\{\mathit{beer} \leftarrow \bot\}$	$\langle \emptyset, \{\mathit{beer}, \mathit{over\_19}, \mathit{ab}\} \rangle$	0.025%
22yrs ($\mathit{over\_19}$)	$\{\mathit{beer} \leftarrow \top\}$	$\langle \{\mathit{beer}, \mathit{over\_19}\}, \{\mathit{ab}\} \rangle$	0.025%
	$\{\mathit{over\_19} \leftarrow \top\}$	$\langle \{\mathit{over\_19}\}, \{\mathit{ab}\} \rangle$	
16yrs ($\neg\mathit{over\_19}$)	$\{\mathit{beer} \leftarrow \bot\}$	$\langle \emptyset, \{\mathit{beer}, \mathit{over\_19}, \mathit{ab}\} \rangle \rightsquigarrow$ turn	80%

Summary

- MRFA is not sufficient to evaluate all types of conditionals
- Proposed modifications solve the problem
  - Change the set of abducibles
  - Reason skeptically
- Solve the Wason Selection Task
- Future work
  - More categories of conditionals
  - Analyze more examples
  - Experimental data