Conditionals under Weak Completion Semantics

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- Introduction
- Evaluating conditionals
- Obligation vs Factual conditionals
- Necessity vs sufficiency
- The Wason Selection Task

“Logic is everywhere …”
Conditionals

► **Statements of the form** *if condition then consequence*

► **Indicative conditionals**
  ▶ Condition and consequence can be true, false or unknown
  ▶ If the condition is true, the consequence is asserted to be true

► **Subjunctive conditionals (counterfactuals)**
  ▶ Condition is false
  ▶ Consequence can be true, false or unknown
  ▶ In the counterfactual circumstance of the condition being true, the consequence is asserted to be true
Evaluating conditionals

► MRFA: Minimal Revision Followed by Abduction
  ▶ Model background knowledge as a logic program
  ▶ Reason wrt the least model of the weak completion of the program
  ▶ Explain the conditions by revision and abduction
  ▶ Evaluate the conclusion, once the conditions are explained
  ▶ Minimize revision and maximize abduction
MRFA: Minimal Revision Followed by Abduction

- Given $\mathcal{P}$ and $\text{cond}(\mathcal{C}, \mathcal{D})$
  - If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \top$ then $\text{cond}(\mathcal{C}, \mathcal{D})$ is assigned to $\mathcal{M}_\mathcal{P}(\mathcal{D})$.
  - If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \bot$, then evaluate $\text{cond}(\mathcal{C}, \mathcal{D})$ with respect to $\mathcal{M}_{\text{rev}(\mathcal{P}, S)}$, where $S = \{L \in \mathcal{C} \mid \mathcal{M}_\mathcal{P}(L) = \bot\}$.
  - If $\mathcal{M}_\mathcal{P}(\mathcal{C}) = \cup$, then evaluate $\text{cond}(\mathcal{C}, \mathcal{D})$ with respect to $\mathcal{M}_{\mathcal{P}'}$, where
    - $\mathcal{P}' = \text{rev}(\mathcal{P}, S) \cup \mathcal{E}$,
    - $S$ is a smallest subset of $\mathcal{C}$ and $\mathcal{E} \subseteq \mathcal{A}_{\text{rev}(\mathcal{P}, S)}$ is a minimal explanation for $\mathcal{C} \setminus S$ such that $\mathcal{M}_{\mathcal{P}'}(\mathcal{C}) = \top$. 
Examples

- **Background knowledge**
  - If it rains, then the streets are wet and I take my umbrella

- \( \mathcal{P} = \{ \text{wet streets} \leftarrow \text{rain} \land \neg \text{ab}_1, \text{umbrella} \leftarrow \text{rain} \land \neg \text{ab}_2, \text{ab}_1 \leftarrow \bot, \text{ab}_2 \leftarrow \bot \} \)

- \( M_\mathcal{P} = \langle \emptyset, \{ \text{ab}_1, \text{ab}_2 \} \rangle \)

- \( \mathcal{A}_\mathcal{P} = \{ \text{rain} \leftarrow \top, \text{rain} \leftarrow \bot \} \)
Examples

- If the streets are not wet, then it did not rain
  \[ \text{cond}(\{\neg\text{wet streets}\}, \{\neg\text{rain}\}) \]
- \[ M_P = \langle \emptyset, \{ab_1, ab_2\} \rangle \]
- \[ A_P = \{\text{rain} \leftarrow \top, \text{rain} \leftarrow \bot\} \]
- \[ P = \{\text{wet streets} \leftarrow \text{rain} \land \neg ab_1, \text{umbrella} \leftarrow \text{rain} \land \neg ab_2, ab_1 \leftarrow \bot, ab_2 \leftarrow \bot\} \]
- \[ E = \{\text{rain} \leftarrow \bot\} \]
- \[ M_P \cup E = \langle \emptyset, \{\text{rain, wet streets, umbrella, } ab_1, ab_2\} \rangle \]
- The conditional is true
Examples

► If the streets are wet, then it rained
  ▶ \( \text{cond} \{ \text{wet \_ streets} \}, \{ \text{rain} \} \) 
  ▶ \( M_P = \langle \emptyset, \{ ab_1, ab_2 \} \rangle \) 
  ▶ \( A_P = \{ \text{rain} \leftarrow \top, \text{rain} \leftarrow \top \} \) 
  ▶ \( P = \{ \text{wet \_ streets} \leftarrow \text{rain} \land \neg ab_1, \text{umbrella} \leftarrow \text{rain} \land \neg ab_2, \ ab_1 \leftarrow \bot, \ ab_2 \leftarrow \bot \} \) 
  ▶ \( E = \{ \text{rain} \leftarrow \top \} \) 
  ▶ \( M_{P \cup E} = \langle \{ \text{rain, wet \_ streets, umbrella} \}, \{ ab_1, ab_2 \} \rangle \) 
  ▶ The conditional is true
Examples

- **Background knowledge**
  - If it rains, then the streets are wet and I take my umbrella

- **Denying the consequent (Modus tollens)**
  - If the streets are not wet, then it did not rain
  - If I did not take my umbrella, then it did not rain

- **Affirming the consequent**
  - If the streets are wet, then it rained
  - If I took my umbrella, then it rained
Obligation vs factual conditional

If \textit{condition}, then \textit{consequence}

- **Background knowledge**
  - If it rains, then the streets are wet and I take my umbrella

- **Obligation conditional**: \textit{consequence} is obligatory
  - Permitted possibility: \textit{condition} and \textit{consequence}
  - Forbidden possibility: \textit{condition} and not \textit{consequence}
  - Example: If the streets are not wet, then it did not rain (\textit{true})

- **Factual conditional**
  - Permitted possibility: \textit{condition} and \textit{consequence}
  - Example: If I did not take my umbrella, then it did not rain (\textit{not true})
Necessity vs sufficiency

If condition, then consequence

► Background knowledge
  ▶ If it rains, then the streets are wet and I take my umbrella
► Necessity
  ▶ The consequence cannot be true unless the condition is true
  ▶ Example: If the streets are wet, then it rained (true)
► Sufficiency
  ▶ The condition to be true is adequate grounds to conclude the consequence
  ▶ Example: If I took my umbrella, then it rained (not true)
Handling *obligation vs factual conditionals* and *necessity vs sufficiency*

- Modify the set of abducibles to model the difference between
  - obligation and facts
  - necessary and sufficient conditions

\[
\mathcal{A}_P = \{ A \leftarrow \top | A \text{ is undefined in } P \} \\
\quad \cup \{ A \leftarrow \bot | A \text{ is undefined in } P \} \\
\quad \cup \{ A \leftarrow \top | A \text{ is the head of a sufficient conditional in } P \} \\
\quad \cup \{ ab_i \leftarrow \top | ab_i \text{ is in the body of a factual conditional in } P \}
\]

- Reason skeptically wrt the consequences

- $\mathcal{F}$ follows skeptically from $P$ and $O$
  - iff for all explanations $\mathcal{E}$ for $O$ we find $P \cup \mathcal{E} \models_{WCS} \mathcal{F}$
Examples: revisited

► Background knowledge

▶ If it rains, then the streets are wet (Obligation and necessity)
▶ If it rains, then I take my umbrella (Factual and sufficiency)

► $\mathcal{P} = \{ \text{wet streets} \leftarrow \text{rain} \land \neg \text{ab}_1, \text{umbrella} \leftarrow \text{rain} \land \neg \text{ab}_2, \text{ab}_1 \leftarrow \bot, \text{ab}_2 \leftarrow \bot \}$

► $M_{\mathcal{P}} = \langle \emptyset, \{ \text{ab}_1, \text{ab}_2 \} \rangle$

► $\mathcal{A}_{\mathcal{P}} = \{ \text{rain} \leftarrow \top, \text{rain} \leftarrow \bot, \text{umbrella} \leftarrow \top, \text{ab}_2 \leftarrow \top \}$
Examples: revisited (Obligation and necessity)

- If the streets are not wet, then it did not rain
  - \( \text{cond}(\{\neg \text{wet\_streets}\}, \{\neg \text{rain}\}) \)
- \( M_P = \langle \emptyset, \{ \text{ab}_1, \text{ab}_2 \} \rangle \)
- \( A_P = \{ \text{rain} \leftarrow \top, \text{rain} \leftarrow \bot, \text{umbrella} \leftarrow \top, \text{ab}_2 \leftarrow \top \} \)
- \( P = \{ \text{wet\_streets} \leftarrow \text{rain} \wedge \neg \text{ab}_1, \text{umbrella} \leftarrow \text{rain} \wedge \neg \text{ab}_2, \text{ab}_1 \leftarrow \bot, \text{ab}_2 \leftarrow \bot \} \)
- \( E = \{ \text{rain} \leftarrow \bot \} \)
- \( M_{P \cup E} = \langle \emptyset, \{ \text{rain, wet\_streets, umbrella, ab}_1, \text{ab}_2 \} \rangle \)
- The conditional is true
Examples: revisited (Factual and sufficiency)

▶ If I did not take my umbrella, then it did not rain
  ▶ \textit{cond}(\{\neg\text{umbrella}\}, \{\neg\text{rain}\})
  ▶ \( M_P = \langle \emptyset, \{ab_1, ab_2\} \rangle \)
  ▶ \( \mathcal{A}_P = \{\text{rain} \leftarrow \top, \text{rain} \leftarrow \bot, \text{umbrella} \leftarrow \top, ab_2 \leftarrow \top\} \)
  ▶ \( \mathcal{P} = \{\text{wet\_streets} \leftarrow \text{rain} \land \neg ab_1, \text{umbrella} \leftarrow \text{rain} \land \neg ab_2, ab_1 \leftarrow \bot, ab_2 \leftarrow \bot\} \)

\[
\begin{array}{ccc}
E & M_{P \cup E} & \mathcal{P} \cup E \models_{\text{wcs}} \neg \text{rain} \\
\{\text{rain} \leftarrow \bot\} & \langle \emptyset, \{\text{rain, wet\_streets, umbrella, ab}_1, ab_2\} \rangle & \mathcal{P} \cup E \nsubseteq_{\text{wcs}} \neg \text{rain} \\
\{ab_2 \leftarrow \top\} & \langle \{ab_2\}, \{\text{umbrella, ab}_1\} \rangle &
\end{array}
\]

▶ The conditional is unknown
Examples: revisited (Obligation and necessity)

- If the streets are wet, then it rained
  \[ \text{cond} \left( \{ \text{wet\_streets} \}, \{ \text{rain} \} \right) \]

- \[ M_P = \langle \emptyset, \{ ab_1, ab_2 \} \rangle \]

- \[ A_P = \{ \text{rain} \leftarrow \top, \text{rain} \leftarrow \bot, \text{umbrella} \leftarrow \top, ab_2 \leftarrow \top \} \]

- \[ P = \{ \text{wet\_streets} \leftarrow \text{rain} \land \neg ab_1, \]
  \[ \text{umbrella} \leftarrow \text{rain} \land \neg ab_2, \]
  \[ ab_1 \leftarrow \bot, \]
  \[ ab_2 \leftarrow \bot \} \]

- \[ E = \{ \text{rain} \leftarrow \top \} \]

- \[ M_P \cup E = \langle \{ \text{rain, wet\_streets, umbrella} \}, \{ ab_1, ab_2 \} \rangle \]

- The conditional is true
Examples: revisited (Factual and sufficiency)

- If I took my umbrella, then it rained
  - $\text{cond}({\text{umbrella}}, {\text{rain}})$
  - $M_P = \langle \emptyset, \{ab_1, ab_2\} \rangle$
  - $A_P = \{\text{rain} \leftarrow T, \text{rain} \leftarrow \bot, \text{umbrella} \leftarrow T, ab_2 \leftarrow T\}$
  - $P = \{\text{wet\_streets} \leftarrow \text{rain} \land \neg ab_1, \text{umbrella} \leftarrow \text{rain} \land \neg ab_2, ab_1 \leftarrow \bot, ab_2 \leftarrow \bot\}$

<table>
<thead>
<tr>
<th>$\mathcal{E}$</th>
<th>$M_{P \cup \mathcal{E}}$</th>
<th>$P \cup \mathcal{E} \models_{wcs} \text{rain}$</th>
<th>$P \cup \mathcal{E} \not\models_{wcs} \text{rain}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>${\text{rain} \leftarrow T}$</td>
<td>$\langle {\text{rain, wet_streets, umbrella}}, {ab_1, ab_2} \rangle$</td>
<td></td>
<td>$P \cup \mathcal{E} \not\models_{wcs} \text{rain}$</td>
</tr>
<tr>
<td>${\text{umbrella} \leftarrow T}$</td>
<td>$\langle {\text{umbrella}}, {ab_1, ab_2} \rangle$</td>
<td></td>
<td>$P \cup \mathcal{E} \models_{wcs} \text{rain}$</td>
</tr>
</tbody>
</table>

- The conditional is unknown
Examples: revisited

▶ Background knowledge
  ▶ If it rains, then the streets are wet and I take my umbrella

▶ Obligation conditionals with necessary conditions: Evaluated as true
  ▶ If the streets are not wet, then it did not rain
  ▶ If the streets are wet, then it rained

▶ Factual conditionals with sufficient conditions: Evaluated as unknown
  ▶ If I did not take my umbrella, then it did not rain
  ▶ If I took my umbrella, then it rained
The Wason Selection Task: Abstract Case (Wason 1968)

- Consider cards with a letter on one side and a number on the other side
- Given the conditional
  
  If there is a D on one side of the card, then there is a 3 on the other side

- Which cards must be turned to show that the conditional holds?

<table>
<thead>
<tr>
<th></th>
<th>D</th>
<th>F</th>
<th>3</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>89%</td>
<td>16%</td>
<td>62%</td>
<td>25%</td>
<td></td>
</tr>
</tbody>
</table>
The Wason Selection Task: Social Case (Griggs and Cox 1982)

- Consider cards with a person’s age on one side and what the person is drinking on the other side

- Given the conditional

  If a person is drinking beer, then the person must be over 19 years of age

- Which cards must be turned to show that the conditional holds?

  beer  coke  22yrs  16yrs
  95%  0.025%  0.025%  80%
Modelling the Wason Selection Task

- Consider cards with X on one side and Y on the other side
- Conditional: If X, then Y
- $\mathcal{P} = \{ Y \leftarrow X \land \neg ab, \quad ab \leftarrow \bot \}$
- Set $\mathcal{A}_\mathcal{P}$ of abducibles will depend on the type of the conditional
- Assume $\mathcal{O} = \{X\}$ st X can be negative or positive
- Turn the card, when
  - X and Y follow skeptically from $\mathcal{P}$ and $\mathcal{O}$, or
  - Y is the head of a rule in each $\mathcal{E} \in \{ \mathcal{E} \mid \mathcal{P} \cup \mathcal{E} \models X \}$
The Wason Selection Task: Abstract Case (Wason 1968)

- If there is a D on one side of the card, then there is a 3 on the other side
  - Defined as a belief by Kowalski
  - Factual conditional and necessity

- \( \mathcal{P} = \{ 3 \leftarrow D \land \neg ab, \ ab \leftarrow \bot \} \)

- \( M_\mathcal{P} = \langle \emptyset, \{ ab \} \rangle \)

- \( \mathcal{A}_\mathcal{P} = \{ D \leftarrow T, D \leftarrow \bot, ab \leftarrow T \} \)

<table>
<thead>
<tr>
<th>( \mathcal{O} )</th>
<th>( \mathcal{E} )</th>
<th>( M_{\mathcal{P} \cup \mathcal{E}} )</th>
<th>Experimental results</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>{ D \leftarrow T }</td>
<td>\langle { D, 3 }, { ab } \rangle \xrightarrow{\text{turn}}</td>
<td>89%</td>
</tr>
<tr>
<td>F (\neg D)</td>
<td>{ D \leftarrow \bot }</td>
<td>\langle \emptyset, { D, 3, ab } \rangle</td>
<td>16%</td>
</tr>
<tr>
<td>3</td>
<td>{ D \leftarrow T }</td>
<td>\langle { D, 3 }, { ab } \rangle \xrightarrow{\text{turn}}</td>
<td>62%</td>
</tr>
<tr>
<td>7 (\neg 3)</td>
<td>{ D \leftarrow \bot }</td>
<td>\langle \emptyset, { D, 3, ab } \rangle</td>
<td>25%</td>
</tr>
<tr>
<td></td>
<td>{ ab \leftarrow T }</td>
<td>\langle { ab }, { 3 } \rangle</td>
<td></td>
</tr>
</tbody>
</table>
The Wason Selection Task: Social Case (Griggs and Cox 1982)

- If a person is drinking beer, then the person must be over 19 years of age
  - Defined as a goal by Kowalski
  - Obligation conditional and sufficiency

- $\mathcal{P} = \{\text{over} \_19 \leftarrow \text{beer} \land \neg \text{ab}, \text{ab} \leftarrow \bot\}$

- $M_\mathcal{P} = \langle \emptyset, \{\text{ab}\} \rangle$

- $A_\mathcal{P} = \{\text{beer} \leftarrow \top, \text{beer} \leftarrow \bot, \text{over} \_19 \leftarrow \top\}$

<table>
<thead>
<tr>
<th>$\mathcal{O}$</th>
<th>$\mathcal{E}$</th>
<th>$M_\mathcal{P} \cup \mathcal{E}$</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>beer</td>
<td>${\text{beer} \leftarrow \top}$</td>
<td>$\langle{\text{beer, over} _19}, {\text{ab}}\rangle \rightsquigarrow \text{turn} \quad 95%$</td>
<td></td>
</tr>
<tr>
<td>coke (\neg \text{beer})</td>
<td>${\text{beer} \leftarrow \bot}$</td>
<td>$\langle\emptyset, {\text{beer, over} _19, \text{ab}}\rangle \quad 0.025%$</td>
<td></td>
</tr>
<tr>
<td>22yrs (\text{over} _19)</td>
<td>${\text{beer} \leftarrow \top}$</td>
<td>$\langle{\text{beer, over} _19}, {\text{ab}}\rangle \quad 0.025%$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>${\text{over} _19 \leftarrow \top}$</td>
<td>$\langle{\text{over} _19}, {\text{ab}}\rangle$</td>
<td></td>
</tr>
<tr>
<td>16yrs (\neg \text{over} _19)</td>
<td>${\text{beer} \leftarrow \bot}$</td>
<td>$\langle\emptyset, {\text{beer, over} _19, \text{ab}}\rangle \rightsquigarrow \text{turn} \quad 80%$</td>
<td></td>
</tr>
</tbody>
</table>
Summary

- MRFA is not sufficient to evaluate all types of conditionals
- Proposed modifications solve the problem
  - Change the set of abducibles
  - Reason skeptically
- Solve the Wason Selection Task
- Future work
  - More categories of conditionals
  - Analyze more examples
  - Experimental data