Exercise 2.1:
Use the Martelli-Montanari algorithm step by step to unify the following pairs of terms with variables $x$, $y$, and $z$. For each step indicate which rule you have used.

a) $f(g(x), g(c), y)$ and $f(g(g(y)), x, a)$

b) $f(b, x, x, y)$ and $f(b, g(y), g(g(z)), g(a))$

c) $f(x, g(z), g(z))$ and $f(h(y), y, g(h(x)))$

Give the corresponding most general unifier (mgu) or give the reason why the terms are not unifiable.

Exercise 2.2:
Consider the following program
\[
p(X) :\leftarrow q(X), r(X).
\]
\[
q(f(X)).
\]
\[
r(f(a)).
\]

a) Give an SLD-derivation $\xi$ for the query $?\leftarrow p(X)$ that uses the Prolog selection rule.

b) For each derivation step of $\xi$, give the resultant that is associated with this step (Slide 18, Lecture 3).

c) Give the resultants of every level $i$ of $\xi$ (Slide 19, Lecture 3).

Exercise 2.3:
Consider the query $?\leftarrow fact(0,Y), fact(Y,s(0))$. together with the program
\[
fact(0,s(0)).
\]
\[
fact(s(N),F) :\leftarrow fact(N,G), mul(s(N),G,F).
\]

a) Give an SLD-derivation using the Prolog selection rule (you don’t have to show the multiplication in detail). Give the substitutions and the CAS.
b) Show that the Switching Lemma (Slide 26, Lecture 3) holds for the initial query (i.e., for $n = 0$).

*Hint:* Give a second SLD-derivation selecting the second atom at the beginning and using the Prolog selection rule afterwards. Show the correspondence of both derivations.

**Exercise 2.4:**

Give the SLD-tree for the query `?- p(X,Y).` and the following program. Use Prolog’s selection and computation rule.

```prolog
p(X,Y) :- q(X,Y), r(Y,X).
q(X,a) :- s(X).
q(X,c) :- s(X).
r(X,b) :- t(X).
s(a).
s(b).
s(c).
t(a).
t(c).
```