PROBLEM SOLVING AND SEARCH IN ARTIFICIAL INTELLIGENCE

Lecture 6 Answer-Set Programming Motivation and Introduction
* slides adapted from Torsten Schaub [Gebser et al.(2012)]

Sarah Gaggl

Dresden, 13th May 2019
Agenda

1. Introduction
2. Constraint Satisfaction (CSP)
3. Uninformed Search versus Informed Search (Best First Search, A* Search, Heuristics)
4. Local Search, Stochastic Hill Climbing, Simulated Annealing
5. Tabu Search
6. Answer-set Programming (ASP)
7. Evolutionary Algorithms/ Genetic Algorithms
8. Structural Decomposition Techniques (Tree/Hypertree Decompositions)
Outline

1 Motivation
   - Declarative Problem Solving
   - ASP in a Nutshell
   - ASP Paradigm

2 Introduction
   - Syntax
   - Semantics
   - Examples
   - Language Constructs
   - Modeling
Informatics

- Problem
- Computer
- Solution
- Output
“What is the problem?” versus “How to solve the problem?”
Traditional programming

“What is the problem?” versus “How to solve the problem?”
Traditional programming

“What is the problem?” versus “How to solve the problem?”

Problem

Program

Solution

Output

Programming

Interpreting

Executing
Declarative problem solving

“What is the problem?” versus “How to solve the problem?”
Declarative problem solving

“What is the problem?” versus “How to solve the problem?”

Problem Representation Solution

Modeling Interpreting

Solving

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Declarative problem solving

- What is the problem?
- How to solve the problem?

Problem → Representation → Solution

Modeling → Representing

Interpreting → Output

Solving
Answer Set Programming
in a Nutshell

- ASP is an approach to **declarative problem solving**, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
Answer Set Programming in a Nutshell

- ASP is an approach to **declarative problem solving**, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
- ASP has its roots in
  - (deductive) databases
  - logic programming (with negation)
  - (logic-based) knowledge representation and (nonmonotonic) reasoning
  - constraint solving (in particular, SATisfiability testing)
Answer Set Programming
in a Nutshell

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- **ASP** allows for solving all search problems in $NP$ (and $NP^{NP}$)
in a uniform way
Answer Set Programming
in a Nutshell

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- ASP is versatile as reflected by the ASP solver **clasp**, winning first places at ASP, CASC, MISC, PB, and SAT competitions
Answer Set Programming
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• ASP allows for solving all search problems in \(NP\) (and \(NP^{NP}\)) in a uniform way

• ASP is versatile as reflected by the ASP solver **clasp**, winning first places at ASP, CASC, MISC, PB, and SAT competitions

• ASP embraces many emerging application areas
Answer Set Programming
in a Hazelnutshell

- ASP is an approach to **declarative problem solving**, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
tailored to **Knowledge Representation and Reasoning**
Answer Set Programming
in a Hazelnutshell

- ASP is an approach to **declarative problem solving**, combining
  - a rich yet simple modeling language
  - with high-performance solving capacities
tailored to Knowledge Representation and Reasoning

\[
\text{ASP} = \text{DB} + \text{LP} + \text{KR} + \text{SAT}
\]
Theorem Proving based approach (eg. Prolog)

1. Provide a representation of the problem
2. A solution is given by a derivation of a query
KR’s shift of paradigm

Theorem Proving based approach (eg. Prolog)
1. Provide a representation of the problem
2. A solution is given by a derivation of a query

Model Generation based approach (eg. SATisfiability testing)
1. Provide a representation of the problem
2. A solution is given by a model of the representation
## Model Generation based Problem Solving

<table>
<thead>
<tr>
<th>Representation</th>
<th>Solution</th>
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<tbody>
<tr>
<td>constraint satisfaction problem</td>
<td>assignment</td>
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SAT
LP-style playing with blocks

Prolog program

on(a,b).
on(b,c).

above(X,Y) :- on(X,Y).
above(X,Y) :- on(X,Z), above(Z,Y).
LP-style playing with blocks

Prolog program

\[
\text{on}(a,b).
\]
\[
\text{on}(b,c).
\]
\[
\text{above}(X,Y) :- \text{on}(X,Y).
\]
\[
\text{above}(X,Y) :- \text{on}(X,Z), \text{above}(Z,Y).
\]

Prolog queries

\[*\text{?-- above}(a,c).\]
\[*\text{true}.\]

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LP-style playing with blocks

Prolog program
on(a,b).
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Prolog queries
?- above(a,c).
true.

?- above(c,a).
no.
LP-style playing with blocks

Prolog program

\[\text{on}(a,b).\]
\[\text{on}(b,c).\]

\[\text{above}(X,Y) :- \text{on}(X,Y).\]
\[\text{above}(X,Y) :- \text{on}(X,Z), \text{above}(Z,Y).\]

Prolog queries (testing entailment)

\[? - \text{above}(a,c).\]
\[\text{true}.\]

\[? - \text{above}(c,a).\]
\[\text{no}.\]
LP-style playing with blocks

Shuffled Prolog program

\[
\begin{align*}
on(a, b). \\
on(b, c).
\end{align*}
\]

\[
\begin{align*}
\text{above}(X, Y) & :\quad \text{above}(X, Z), \ on(Z, Y). \\
\text{above}(X, Y) & :\quad \text{on}(X, Y).
\end{align*}
\]
LP-style playing with blocks

Shuffled Prolog program

\[
on(a, b).
on(b, c).
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\[
above(X, Y) :- above(X, Z), on(Z, Y).
above(X, Y) :- on(X, Y).
\]

Prolog queries

?– above(a, c).
LP-style playing with blocks

Shuffled Prolog program

```prolog
on(a,b).
on(b,c).

above(X,Y) :- above(X,Z), on(Z,Y).
above(X,Y) :- on(X,Y).
```

Prolog queries (answered via fixed execution)

```prolog
?- above(a,c).
```

Fatal Error: local stack overflow.
SAT-style playing with blocks

Formula

\[
\begin{align*}
on(a, b) \\
\land on(b, c) \\
\land (on(X, Y) \rightarrow above(X, Y)) \\
\land (on(X, Z) \land above(Z, Y) \rightarrow above(X, Y))
\end{align*}
\]
SAT-style playing with blocks

**Formula**

\[
\begin{align*}
on(a, b) \\
\wedge & \quad on(b, c) \\
\wedge & \quad (on(X, Y) \rightarrow above(X, Y)) \\
\wedge & \quad (on(X, Z) \wedge above(Z, Y) \rightarrow above(X, Y))
\end{align*}
\]

**Herbrand model**

\[
\begin{align*}
\{ & \quad on(a, b), \quad on(b, c), \quad on(a, c), \quad on(b, b), \\
& \quad above(a, b), \quad above(b, c), \quad above(a, c), \quad above(b, b), \quad above(c, b) \}
\end{align*}
\]
SAT-style playing with blocks

Formula

\[
\text{on}(a, b) \\
\land \text{on}(b, c) \\
\land (\text{on}(X, Y) \rightarrow \text{above}(X, Y)) \\
\land (\text{on}(X, Z) \land \text{above}(Z, Y) \rightarrow \text{above}(X, Y))
\]

Herbrand model (among 426!)

\[
\{ \text{on}(a, b), \quad \text{on}(b, c), \quad \text{on}(a, c), \quad \text{on}(b, b), \quad \text{above}(a, b), \quad \text{above}(b, c), \quad \text{above}(a, c), \quad \text{above}(b, b), \quad \text{above}(c, b) \}
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SAT-style playing with blocks

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\end{align*}
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Herbrand model (among 426!)

\[
\{ \begin{array}{c}
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above(a, b), \quad above(b, c), \quad above(a, c), \quad above(b, b), \quad above(c, b)
\end{array} \}
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SAT-style playing with blocks

**Formula**

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on(a, b) \land on(b, c) \land (on(X, Y) \rightarrow above(X, Y)) \land (on(X, Z) \land above(Z, Y) \rightarrow above(X, Y))\]

**Herbrand model (among 426!)**

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KR’s shift of paradigm

Theorem Proving based approach (eg. Prolog)

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Answer Set Programming (ASP)
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## Answer Set Programming at large

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### Answer Set Programming in practice

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<td>first-order programs</td>
<td>stable Herbrand models</td>
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ASP-style playing with blocks

### Logic program

- `on(a, b).
- `on(b, c).

- `above(X, Y) :- on(X, Y).
- `above(X, Y) :- on(X, Z), above(Z, Y).`
ASP-style playing with blocks

Logic program

\[
on(a,b).
\]
\[
on(b,c).
\]
\[
\text{above}(X,Y) :- \text{on}(X,Y).
\]
\[
\text{above}(X,Y) :- \text{on}(X,Z), \text{above}(Z,Y).
\]

Stable Herbrand model

\{ \text{on}(a,b), \text{on}(b,c), \text{above}(b,c), \text{above}(a,b), \text{above}(a,c) \}
ASP-style playing with blocks

Logic program

\[
\text{on}(a, b) . \\
\text{on}(b, c) . \\
\text{above}(X, Y) :- \text{on}(X, Y) . \\
\text{above}(X, Y) :- \text{on}(X, Z), \text{above}(Z, Y) .
\]

Stable Herbrand model (and no others)

\{ \text{on}(a, b), \text{on}(b, c), \text{above}(b, c), \text{above}(a, b), \text{above}(a, c) \}
ASP-style playing with blocks

Logic program

```
on(a, b).
on(b, c).

above(X, Y) :- above(Z, Y), on(X, Z).
above(X, Y) :- on(X, Y).
```

Stable Herbrand model (and no others)

```
{ on(a, b), on(b, c), above(b, c), above(a, b), above(a, c) }
```
### ASP versus LP

<table>
<thead>
<tr>
<th>ASP</th>
<th>Prolog</th>
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<tbody>
<tr>
<td>Model generation</td>
<td>Query orientation</td>
</tr>
<tr>
<td>Bottom-up</td>
<td>Top-down</td>
</tr>
<tr>
<td>Modeling language</td>
<td>Programming language</td>
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**Rule-based format**

<table>
<thead>
<tr>
<th>Instantiation</th>
<th>Unification</th>
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<tr>
<td>Flat terms</td>
<td>Nested terms</td>
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\[(Turing +) NP^{NP} \]

Turing
## ASP versus SAT

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<tr>
<td><strong>Bottom-up</strong></td>
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<tr>
<td><strong>Constructive Logic</strong></td>
<td><strong>Classical Logic</strong></td>
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<tr>
<td><strong>Closed (and open) world reasoning</strong></td>
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<tr>
<td><strong>Modeling language</strong></td>
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<tr>
<td><strong>Complex reasoning modes</strong></td>
<td><strong>Satisfiability testing</strong></td>
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<td>Satisfiability</td>
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<td>Enumeration/Projection</td>
<td>Satisfiability</td>
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<td>Optimization</td>
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<td>Intersection/Union</td>
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<td><em>(Turing +) $NP^{NP}$</em></td>
<td>$NP$</td>
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ASP solving

Problem

Logic Program

Grounder

Solver

Stable Models

Solution

Modeling

Solving

Interpreting

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PSSAI

slide 48 of 141
SAT solving

Programming

Problem

Formula (CNF)

Solving

Solver

Solution

Classical Models

Interpreting
Rooting ASP solving

Problem

Modeling

Logic Program

Solving

Grounder

Solver

Stable Models

Solution

Interpreting

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Rooting ASP solving

Problem

Modeling
KR

Logic Program
LP

Grounder
DB
Solving

Solver
SAT

Solution

Stable Models
DB+KR+LP

Interpreting
Two sides of a coin

- **ASP as High-level Language**
  - Express problem instance(s) as sets of facts
  - Encode problem (class) as a set of rules
  - Read off solutions from stable models of facts and rules

- **ASP as Low-level Language**
  - Compile a problem into a logic program
  - Solve the original problem by solving its compilation
What is ASP good for?

- Combinatorial search problems in the realm of $P$, $NP$, and $NP^{NP}$
  (some with substantial amount of data), like

  - Automated Planning
  - Code Optimization
  - Composition of Renaissance Music
  - Database Integration
  - Decision Support for NASA shuttle controllers
  - Model Checking
  - Product Configuration
  - Robotics
  - System Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more
What is ASP good for?

- Combinatorial search problems in the realm of $P$, $NP$, and $NP^{NP}$ (some with substantial amount of data), like
  - Automated Planning
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  - System Biology
  - System Synthesis
  - (industrial) Team-building
  - and many many more
What does ASP offer?

- Integration of DB, KR, and SAT techniques
- Succinct, elaboration-tolerant problem representations
  - Rapid application development tool
- Easy handling of dynamic, knowledge intensive applications
  - including: data, frame axioms, exceptions, defaults, closures, etc
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**ASP = DB+LP+KR+SAT**
Agenda

1 Motivation
   - Declarative Problem Solving
   - ASP in a Nutshell
   - ASP Paradigm

2 Introduction
   - Syntax
   - Semantics
   - Examples
   - Language Constructs
   - Modeling
Problem solving in ASP: Syntax

- Problem
- Logic Program
  - Modeling
- Solution
  - Interpreting
- Stable Models
  - Solving
Normal logic programs

- A (normal) logic program over a set $\mathcal{A}$ of atoms is a finite set of rules
- A (normal) rule, $r$, is of the form

$$a_0 \leftarrow a_1, \ldots, a_m, \text{not } a_{m+1}, \ldots, \text{not } a_n$$

where $0 \leq m \leq n$ and each $a_i \in \mathcal{A}$ is an atom for $0 \leq i \leq n$
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- Notation

$\text{head}(r) = a_0$
$\text{body}(r) = \{a_1, \ldots, a_m, \text{not } a_{m+1}, \ldots, \text{not } a_n\}$
$\text{body}(r)^+ = \{a_1, \ldots, a_m\}$
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- \( \text{body}(r)^+ = \{a_1, \ldots, a_m\} \)
- \( \text{body}(r)^- = \{a_{m+1}, \ldots, a_n\} \)

A program is called positive if \( \text{body}(r)^- = \emptyset \) for all its rules.
We sometimes use the following notation interchangeably in order to stress the respective view:

<table>
<thead>
<tr>
<th>source code logic program</th>
<th>true, false</th>
<th>if</th>
<th>and</th>
<th>or</th>
<th>iff</th>
<th>not</th>
<th>¬</th>
</tr>
</thead>
<tbody>
<tr>
<td>formula</td>
<td>:- ,</td>
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Problem solving in ASP: Semantics

Problem → Logic Program → Stable Models

Modeling → Logic Program

Solution → Interpreting

Solving
Formal Definition

Stable models of positive programs

• A set of atoms $X$ is closed under a positive program $P$ iff for any $r \in P$, $\text{head}(r) \in X$ whenever $\text{body}(r)^+ \subseteq X$
  – $X$ corresponds to a model of $P$ (seen as a formula)
Formal Definition

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- A set of atoms $X$ is **closed under** a positive program $P$ iff for any $r \in P$, $\text{head}(r) \in X$ whenever $\text{body}(r)^+ \subseteq X$
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- The **smallest** set of atoms which is closed under a positive program $P$ is denoted by $Cn(P)$
  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of $P$ (ditto)
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  - $Cn(P)$ corresponds to the $\subseteq$-smallest model of $P$ (ditto)

- The set $Cn(P)$ of atoms is the **stable model** of a positive program $P$
Some “logical” remarks

- Positive rules are also referred to as **definite clauses**
  - Definite clauses are disjunctions with **exactly one positive atom**:
    
    \[ a_0 \lor \neg a_1 \lor \cdots \lor \neg a_m \]

  - A set of definite clauses has a (unique) smallest model
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- **Horn clauses** are clauses with at most one positive atom
  - Every definite clause is a Horn clause but not vice versa
  - Non-definite Horn clauses can be regarded as integrity constraints
  - A set of Horn clauses has a smallest model or none
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- This smallest model is the intended semantics of such sets of clauses
  - Given a positive program \( P \), \( Cn(P) \) corresponds to the smallest model of the set of definite clauses corresponding to \( P \)
Basic idea

Consider the logical formula $\Phi$ and its three (classical) models:

$$\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}$$

$$\Phi \equiv q \land (q \land \neg r \rightarrow p)$$
Basic idea

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Basic idea

Consider the logical formula \( \Phi \) and its three (classical) models:

\[
\{p, q\}, \{q, r\}, \text{ and } \{p, q, r\}
\]

\[
\begin{align*}
p & \leftrightarrow 1 \\
q & \leftrightarrow 1 \\
r & \leftrightarrow 0
\end{align*}
\]

\[
\Phi \quad q \land (q \land \neg r \rightarrow p)
\]
Basic idea

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Informally, a set $X$ of atoms is a stable model of a logic program $P$

- if $X$ is a (classical) model of $P$ and
- if all atoms in $X$ are justified by some rule in $P$

(rooted in intuitionistic logics HT (Heyting, 1930) and G3 (Gödel, 1932))
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Formal Definition

Stable model of normal programs

- The **Gelfond-Lifschitz Reduct** [Gelfond and Lifschitz (1991)], $P^X$, of a program $P$ relative to a set $X$ of atoms is defined by

\[ P^X = \{ \text{head}(r) \leftarrow \text{body}(r)^+ \mid r \in P \text{ and } \text{body}(r)^- \cap X = \emptyset \} \]
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- A set \( X \) of atoms is a stable model of a program \( P \), if \( Cn(P^X) = X \)
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- A set $X$ of atoms is a stable model of a program $P$, if $Cn(P^X) = X$

- Note: $Cn(P^X)$ is the $\subseteq$-smallest (classical) model of $P^X$
- Note: Every atom in $X$ is justified by an “applying rule from $P$”
A closer look at $P^X$

- In other words, given a set $X$ of atoms from $P$,

$P^X$ is obtained from $P$ by deleting

1. each rule having $\text{not } a$ in its body with $a \in X$
   and then
2. all negative atoms of the form $\text{not } a$
   in the bodies of the remaining rules
A closer look at $P^X$

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  1. each rule having $\text{not } a$ in its body with $a \in X$
     and then
  2. all negative atoms of the form $\text{not } a$
     in the bodies of the remaining rules

- Note: Only negative body literals are evaluated w.r.t. $X$
A first example

\[ P = \{ p \leftarrow p, \ q \leftarrow \text{not } p \} \]
A first example

\[ P = \{ p \leftarrow p, \ q \leftarrow \text{not } p \} \]

<table>
<thead>
<tr>
<th>( X )</th>
<th>( Cn(P^X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \emptyset )</td>
<td></td>
</tr>
<tr>
<td>( {p} )</td>
<td></td>
</tr>
<tr>
<td>( {q} )</td>
<td></td>
</tr>
<tr>
<td>( {p, q} )</td>
<td></td>
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\[ P = \{ p \leftarrow p, \ q \leftarrow \text{not } p \} \]

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<tr>
<td>(\emptyset)</td>
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<td>({q})</td>
</tr>
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<td>(\emptyset)</td>
</tr>
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<td>({p, q})</td>
<td>(p \leftarrow p)</td>
<td>(\emptyset)</td>
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A first example

\[ P = \{ p \leftarrow p, \ q \leftarrow not p \} \]

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<tr>
<td>( \emptyset )</td>
<td>( p \leftarrow p )</td>
<td>( { q } ) ( \times )</td>
</tr>
<tr>
<td>( { p } )</td>
<td>( p \leftarrow p )</td>
<td>( \emptyset )</td>
</tr>
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A first example

\[ P = \{ p \leftrightarrow p, \ q \leftrightarrow \text{not } p \} \]

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<tr>
<td>(\emptyset)</td>
<td>(p \leftarrow p), (q \leftarrow )</td>
<td>{q}, (\times)</td>
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<tr>
<td>{p}</td>
<td>(p \leftarrow p), (q \leftarrow )</td>
<td>(\emptyset), (\times)</td>
</tr>
<tr>
<td>{q}</td>
<td>(p \leftarrow p), (q \leftarrow )</td>
<td>{q}, (\checkmark)</td>
</tr>
<tr>
<td>{p, q}</td>
<td>(p \leftarrow p), (q \leftarrow )</td>
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\[ P = \{ p \leftarrow p, \; q \leftarrow \text{not} \; p \} \]

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<td>( p \leftarrow p ) ; ( q \leftarrow )</td>
<td>( { q } ) ( \times )</td>
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Some properties

- A logic program may have zero, one, or multiple stable models!
Some properties

- A logic program may have zero, one, or multiple stable models!
- If $X$ is a stable model of a logic program $P$, then $X$ is a model of $P$ (seen as a formula)
- If $X$ and $Y$ are stable models of a normal program $P$, then $X \subsetneq Y$
Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$
Programs with Variables

Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of variable-free terms (also called Herbrand universe)
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$ (also called alphabet or Herbrand base)
Programs with Variables

Let $P$ be a logic program

- Let $\mathcal{T}$ be a set of (variable-free) terms
- Let $\mathcal{A}$ be a set of (variable-free) atoms constructable from $\mathcal{T}$

- **Ground Instances** of $r \in P$: Set of variable-free rules obtained by replacing all variables in $r$ by elements from $\mathcal{T}$:

  $$\text{ground}(r) = \{ r\theta \mid \theta : \text{var}(r) \rightarrow \mathcal{T}, \text{var}(r\theta) = \emptyset \}$$

where $\text{var}(r)$ stands for the set of all variables occurring in $r$; $\theta$ is a (ground) substitution
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- **Ground Instances of $r \in P$:** Set of variable-free rules obtained by replacing all variables in $r$ by elements from $\mathcal{T}$:

$$ground(r) = \{r\theta \mid \theta : var(r) \rightarrow \mathcal{T}, var(r\theta) = \emptyset\}$$

where $var(r)$ stands for the set of all variables occurring in $r$; $\theta$ is a (ground) substitution

- **Ground Instantiation of $P$:**

$$ground(P) = \bigcup_{r \in P} ground(r)$$
An example

\[ P = \{ r(a, b) \leftarrow, r(b, c) \leftarrow, t(X, Y) \leftarrow r(X, Y) \} \]

\[ T = \{ a, b, c \} \]

\[ A = \left\{ r(a, a), r(a, b), r(a, c), r(b, a), r(b, b), r(b, c), r(c, a), r(c, b), r(c, c), t(a, a), t(a, b), t(a, c), t(b, a), t(b, b), t(b, c), t(c, a), t(c, b), t(c, c) \right\} \]
An example

\[ P = \{ \text{r}(a, b) \leftarrow, \text{r}(b, c) \leftarrow, \text{t}(X, Y) \leftarrow \text{r}(X, Y) \} \]

\[ \mathcal{T} = \{a, b, c\} \]

\[ \mathcal{A} = \left\{ \text{r}(a, a), \text{r}(a, b), \text{r}(a, c), \text{r}(b, b), \text{r}(b, c), \text{r}(c, a), \text{r}(c, b), \text{r}(c, c), \text{t}(a, a), \text{t}(a, b), \text{t}(a, c), \text{t}(b, a), \text{t}(b, b), \text{t}(b, c), \text{t}(c, a), \text{t}(c, b), \text{t}(c, c) \right\} \]

\[ \text{ground}(P) = \left\{ \begin{array}{l}
\text{r}(a, b) \leftarrow, \\
\text{r}(b, c) \leftarrow, \\
\text{t}(a, a) \leftarrow \text{r}(a, a), \text{t}(b, a) \leftarrow \text{r}(b, a), \text{t}(c, a) \leftarrow \text{r}(c, a), \\
\text{t}(a, b) \leftarrow \text{r}(a, b), \text{t}(b, b) \leftarrow \text{r}(b, b), \text{t}(c, b) \leftarrow \text{r}(c, b), \\
\text{t}(a, c) \leftarrow \text{r}(a, c), \text{t}(b, c) \leftarrow \text{r}(b, c), \text{t}(c, c) \leftarrow \text{r}(c, c)
\end{array} \right\} \]
An example

\[ P = \{ r(a, b) \leftarrow, r(b, c) \leftarrow, t(X, Y) \leftarrow r(X, Y) \} \]

\[ T = \{ a, b, c \} \]

\[ A = \{ r(a, a), r(a, b), r(a, c), r(b, a), r(b, b), r(b, c), r(c, a), r(c, b), r(c, c), t(a, a), t(a, b), t(a, c), t(b, a), t(b, b), t(b, c), t(c, a), t(c, b), t(c, c) \} \]

\[ \text{ground}(P) = \{ \]
\[ \begin{align*}
    r(a, b) & \leftarrow, \\
    r(b, c) & \leftarrow, \\
    t(a, b) & \leftarrow r(a, b), \\
    t(b, c) & \leftarrow r(b, c), 
\end{align*} \]
\[ \} \]

- Intelligent Grounding aims at reducing the ground instantiation
Stable models of programs with Variables

Let $P$ be a normal logic program with variables.
Stable models of programs with Variables

Let $P$ be a normal logic program with variables

- A set $X$ of (ground) atoms is a stable model of $P$, if $Cn(ground(P)^X) = X$
Problem solving in ASP: Extended Syntax

Diagram:
- **Problem**
  - Modeling
  - Logic Program
- **Solution**
  - Interpreting
- **Stable Models**
  - Solving
Language Constructs

• Variables (over the Herbrand Universe)
  \[ p(X) :- q(X) \]
  over constants \{ a, b, c \}
  stands for
  \[ p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \]

• Conditional Literals
  \[ p :- q(X) : r(X) \]
given \[ r(a), r(b), r(c) \]
stands for
  \[ p :- q(a), q(b), q(c) \]

• Disjunction
  \[ p(X) | q(X) :- r(X) \]

• Integrity Constraints
  \[ :- q(X), p(X) \]

• Choice
  \[ 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \]

• Aggregates
  \[ s(Y) :- r(Y), 2 \#count \{ p(X,Y) : q(X) \} 7 \]

  – also:
    \#sum, \#avg, \#min, \#max, \#even, \#odd
Language Constructs

- Variables (over the Herbrand Universe)
  - \( p(X) : - q(X) \) over constants \( \{a, b, c\} \) stands for
    - \( p(a) : - q(a), p(b) : - q(b), p(c) : - q(c) \)

- Conditional Literals
  - \( p : - q(X) : r(X) \)
  - Given \( r(a), r(b), r(c) \) stands for
    - \( p : - q(a), q(b), q(c) \)

- Disjunction
  - \( p(X) \land q(X) : - r(X) \)

- Integrity Constraints
  - : - q(X), p(X)

- Choice
  - \( 2 \{ p(X,Y) : q(X) \} 7 : - r(Y) \)

- Aggregates
  - \( s(Y) : - r(Y), 2 \#count \{ p(X,Y) : q(X) \} 7 \)
  - also: \#sum, \#avg, \#min, \#max, \#even, \#odd
Language Constructs

- **Conditional Literals**

  - `p :- q(X) : r(X)` given `r(a), r(b), r(c)` stands for:
    - `p :- q(a), q(b), q(c)`
Language Constructs

- **Disjunction**
  \[
  p(X) \lor q(X) : r(X)
  \]
Language Constructs

- Variables (over the Herbrand Universe)
  - \( p(X) \leftarrow q(X) \) over constants \{a, b, c\}
    stands for
    \( p(a) \leftarrow q(a), p(b) \leftarrow q(b), p(c) \leftarrow q(c) \)

- Conditional Literals
  - \( p \leftarrow q(X) : r(X) \) given \( r(a), r(b), r(c) \)
    stands for
    \( p \leftarrow q(a), q(b), q(c) \)

- Disjunction
  - \( p(X) \lor q(X) \leftarrow r(X) \)

- Integrity Constraints
  - \( \leftarrow q(X), p(X) \)

- Choice
  - \( 2 \{ p(X,Y) : q(X) \} 7 \leftarrow r(Y) \)

- Aggregates
  - \( s(Y) \leftarrow r(Y), 2 \# \text{count} \{ p(X,Y) : q(X) \} 7 \)
  - also: \#\text{sum}, \#\text{avg}, \#\text{min}, \#\text{max}, \#\text{even}, \#\text{odd}
Language Constructs

- **Variables** (over the Herbrand Universe)
  
  \[ p(X) :- q(X) \]
  
  over constants \{ a, b, c \} stands for
  
  \[ p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \]

- **Conditional Literals**
  
  \[ p :- q(X) : r(X) \]
  
  given \[ r(a), r(b), r(c) \] stands for
  
  \[ p :- q(a), q(b), q(c) \]

- **Disjunction**
  
  \[ p(X) | q(X) :- r(X) \]

- **Integrity Constraints**
  
  \[ :- q(X), p(X) \]

- **Choice**
  
  \[ 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \]

- **Aggregates**
  
  \[ s(Y) :- r(Y), 2 \# \text{count} \{ p(X,Y) : q(X) \} 7 \]
  
  also: \#\text{sum}, \#\text{avg}, \#\text{min}, \#\text{max}, \#\text{even}, \#\text{odd}
Language Constructs

- **Variables (over the Herbrand Universe)**
  
  - \( p(X) :- q(X) \) over constants \( \{ a, b, c \} \) stands for \( p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \)

- **Conditional Literals**
  
  - \( p :- q(X) : r(X) \) given \( r(a), r(b), r(c) \) stands for \( p :- q(a), q(b), q(c) \)

- **Disjunction**
  
  - \( p(X) \mid q(X) :- r(X) \)

- **Integrity Constraints**
  
  - \( :- q(X), p(X) \)

- **Choice**
  
  - \( 2 \{ p(X,Y) : q(X) \} 7 :- r(Y) \)

- **Aggregates**
  
  - \( s(Y) :- r(Y), 2 \#count \{ p(X,Y) : q(X) \} 7 \)
  
  - also: \#sum, \#avg, \#min, \#max, \#even, \#odd
Language Constructs

- **Variables** (over the Herbrand Universe)
  - \( p(X) :- q(X) \) over constants \( \{a, b, c\} \) stands for
  - \( p(a) :- q(a), p(b) :- q(b), p(c) :- q(c) \)

- **Conditional Literals**
  - \( p :- q(X) : r(X) \) given \( r(a), r(b), r(c) \) stands for
  - \( p :- q(a), q(b), q(c) \)

- **Integrity Constraints**
  - \( :- q(X), p(X) \)

- **Choice**
  - \( 2 \{ p(X, Y) : q(X) \} 7 :- r(Y) \)

- **Aggregates**
  - \( s(Y) :- r(Y), 2 \#count \{ p(X, Y) : q(X) \} 7 \)
  - also: \#sum, \#avg, \#min, \#max, \#even, \#odd
Modeling

• For solving a problem class $C$ for a problem instance $I$, encode

  1. the problem instance $I$ as a set $P_I$ of facts and
  2. the problem class $C$ as a set $P_C$ of rules

such that the solutions to $C$ for $I$ can be (polynomially) extracted from the stable models of $P_I \cup P_C$
Modeling

• For solving a problem class $\mathbf{C}$ for a problem instance $\mathbf{I}$, encode
  1. the problem instance $\mathbf{I}$ as a set $P_I$ of facts and
  2. the problem class $\mathbf{C}$ as a set $P_C$ of rules
such that the solutions to $\mathbf{C}$ for $\mathbf{I}$ can be (polynomially) extracted from the stable models of $P_I \cup P_C$

• $P_I$ is (still) called problem instance
• $P_C$ is often called the problem encoding
Modeling

• For solving a problem class $\mathbf{C}$ for a problem instance $\mathbf{I}$, encode
  1. the problem instance $\mathbf{I}$ as a set $P_I$ of facts and
  2. the problem class $\mathbf{C}$ as a set $P_C$ of rules
  such that the solutions to $\mathbf{C}$ for $\mathbf{I}$ can be (polynomially) extracted from the stable models of $P_I \cup P_C$

• $P_I$ is (still) called problem instance
• $P_C$ is often called the problem encoding

• An encoding $P_C$ is uniform, if it can be used to solve all its problem instances
  That is, $P_C$ encodes the solutions to $\mathbf{C}$ for any set $P_I$ of facts
Example 3-Colorability

- Vertices are represented with predicates $\text{node}(X)$;
- Edges are represented with predicates $\text{edge}(X, Y)$.

Question: Is there a valid assignment of three colors for an input graph $G$ such that no two adjacent vertices have the same color?
Graph coloring

node(1..6).
Graph coloring

node(1..6).

edge(1, 2). edge(1, 3). edge(1, 4).
edge(2, 4). edge(2, 5). edge(2, 6).
edge(3, 1). edge(3, 4). edge(3, 5).
edge(4, 1). edge(4, 2).
edge(5, 3). edge(5, 4). edge(5, 6).
edge(6, 2). edge(6, 3). edge(6, 5).
Graph coloring

```
node(1..6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).
```
Graph coloring

node(1..6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).
Graph coloring

node(1..6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

1 { color(X,C) : col(C) } 1 :- node(X).
Graph coloring

node(1..6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
Graph coloring

node(1..6).

edge(1,2).  edge(1,3).  edge(1,4).
edge(2,4).  edge(2,5).  edge(2,6).
edge(3,1).  edge(3,4).  edge(3,5).
edge(4,1).  edge(4,2).
edge(5,3).  edge(5,4).  edge(5,6).
edge(6,2).  edge(6,3).  edge(6,5).

col(r).  col(b).  col(g).

1 { color(X,C) : col(C) } 1 :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
Graph coloring

node(1..6).
edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

1 { color(X,C) : col(C) } 1 :- node(X).
:- edge(X,Y), color(X,C), color(Y,C).
node(1..6).

edge(1,2). edge(1,3). edge(1,4).
edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5).
edge(4,1). edge(4,2).
edge(5,3). edge(5,4). edge(5,6).
edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

l { color(X,C) : col(C) } l :- node(X).

:- edge(X,Y), color(X,C), color(Y,C).
ASP solving process

- Problem
  - Modeling
    - Logic Program
  - Grounder
  - Solver
  - Stable Models

- Solution
  - Interpreting

TU Dresden, 13th May 2019
Graph coloring: Grounding

$ gringo --text color.lp
Graph coloring: Grounding

$\text{gringo --text color.lp}$

```
node(1). node(2). node(3). node(4). node(5). node(6).
edge(1,2). edge(1,3). edge(1,4). edge(2,4). edge(2,5). edge(2,6).
edge(3,1). edge(3,4). edge(3,5). edge(4,1). edge(4,2). edge(5,3).
edge(5,4). edge(5,6). edge(6,2). edge(6,3). edge(6,5).

col(r). col(b). col(g).

1 {color(1,r), color(1,b), color(1,g)} 1.
1 {color(2,r), color(2,b), color(2,g)} 1.
1 {color(3,r), color(3,b), color(3,g)} 1.
1 {color(4,r), color(4,b), color(4,g)} 1.
1 {color(5,r), color(5,b), color(5,g)} 1.
1 {color(6,r), color(6,b), color(6,g)} 1.

:- color(1,r), color(2,r). :- color(2,g), color(5,g). ... :- color(6,r), color(2,r).
:- color(1,b), color(2,b). :- color(2,r), color(6,r). :- color(6,g), color(2,g).
:- color(1,g), color(2,g). :- color(2,b), color(6,b). :- color(6,r), color(3,r).
:- color(1,r), color(3,r). :- color(2,b), color(6,g). :- color(6,b), color(3,3).
:- color(1,b), color(3,b). :- color(3,r), color(1,r). :- color(6,b), color(3,b).
:- color(1,g), color(3,g). :- color(3,b), color(1,b). :- color(6,b), color(3,g).
:- color(1,r), color(4,r). :- color(3,g), color(1,g). :- color(6,r), color(5,r).
:- color(1,b), color(4,b). :- color(3,r), color(4,r). :- color(6,b), color(5,5).
:- color(1,g), color(4,g). :- color(3,b), color(4,b). :- color(6,b), color(5,5).
:- color(2,r), color(4,r). :- color(3,g), color(4,g).
:- color(2,b), color(4,b). :- color(3,r), color(5,r).
:- color(2,g), color(4,g). :- color(3,b), color(5,g).
:- color(2,r), color(5,r). :- color(3,g), color(5,g).
:- color(2,b), color(5,b). :- color(4,r), color(1,r).
```
ASP solving process

1. Problem
2. Logic Program
3. Grounder
4. Solver
5. Stable Models
6. Solution

Modeling → Grounder → Solver → Interpreting

Solving
Graph coloring: Solving

$ gringo color.lp | clasp 0

SATISFIABLE
Models: 6
Time: 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time: 0.000s
$ \texttt{gringo color.lp | clasp 0}

\begin{verbatim}
clasp version 2.1.0
Reading from stdin
Solving...
Answer: 1
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,g) color(4,b) color(3,r) color(2,r) color(1,g)
Answer: 2
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,g) color(4,r) color(3,b) color(2,b) color(1,g)
Answer: 3
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,b) color(4,g) color(3,r) color(2,r) color(1,b)
Answer: 4
edge(1,2) ... col(r) ... node(1) ... color(6,r) color(5,b) color(4,r) color(3,g) color(2,g) color(1,b)
Answer: 5
edge(1,2) ... col(r) ... node(1) ... color(6,g) color(5,r) color(4,g) color(3,b) color(2,b) color(1,r)
Answer: 6
edge(1,2) ... col(r) ... node(1) ... color(6,b) color(5,r) color(4,b) color(3,g) color(2,g) color(1,r)
SATISFIABLE
\end{verbatim}

Models : 6
Time : 0.002s (Solving: 0.00s 1st Model: 0.00s Unsat: 0.00s)
CPU Time : 0.000s
Problem solving in ASP: Reasoning Modes

Problem

Logic Program

Modeling

Solving

Solution

Interpreting

Stable Models
Reasoning Modes

- Satisfiability
- Enumeration†
- Projection†
- Intersection‡
- Union‡
- Optimization
- and combinations of them

† without solution recording
‡ without solution enumeration
References

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Michael Gelfond and Vladimir Lifschitz.
Classical negation in logic programs and disjunctive databases.

- See also: http://potassco.sourceforge.net