Dynamic Bayesian Description Logics

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1 Introduction

It is well known that many artificial intelligence applications need to represent and reason with knowledge that is not fully certain. This has motivated the study of many knowledge representation formalisms that can effectively handle uncertainty, and in particular probabilistic description logics (DLs) [7–9]. Although these logics are encompassed under the same umbrella, they differ greatly in the way they interpret the probabilities (e.g. statistical vs. subjective), their probabilistic constructors (i.e., probabilistic axioms or probabilistic concepts and roles), their semantics, and even their probabilistic independence assumptions. A recent example of probabilistic DLs are the *Bayesian DLs*, which can express both logical and probabilistic dependencies between axioms [2–4].

One common feature among most of these probabilistic DLs is that they consider the uncertainty degree (i.e., the probability) of the different events to be fixed and static through time. However, this assumption is still too strong for many application scenarios. Consider for example a situation where a grid of sensors is collecting knowledge that is then fed into an ontology to reason about the situation of a large system. Since the sensors might perform an incorrect reading, this knowledge and the consequences derived from it can only be guaranteed to hold with some probability. However, the failure rate of a sensor is not static over time; as the sensor ages, its probability of failing increases. Moreover, the speed at which each sensor ages may also be influenced by other external factors like the weather at the place it is located, or the amount of use it is given.

We propose to extend the formalism of Bayesian DLs to *dynamic* Bayesian DLs, in which the probabilities of the axioms to hold are updated over discrete time steps following the principles of dynamic Bayesian networks. Using this principle, we can not only reason about the probabilistic entailments at every point in time, but also reason about future events given some evidence at different times. This work presents the first steps towards probabilistic reasoning about complex events over time.

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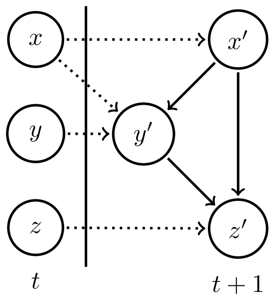

		x'
x		.4
$\neg x$		.4

			y'
x	y	x'	.9
x	y	$\neg x'$	.5
x	$\neg y$	x'	.8
x	$\neg y$	$\neg x'$	.4
$\neg x$	y	x'	.8
$\neg x$	y	$\neg x'$	.4
$\neg x$	$\neg y$	x'	.7
$\neg x$	$\neg y$	$\neg x'$	.1

			z'
z	x'	y'	.7
z	x'	$\neg y'$	.2
z	$\neg x'$	y'	.4
z	$\neg x'$	$\neg y'$	1
$\neg z$	x'	y'	.6
$\neg z$	x'	$\neg y'$	.1
$\neg z$	$\neg x'$	y'	.3
$\neg z$	$\neg x'$	$\neg y'$	1

Fig. 1. The TBN $\mathcal{B}_\rightarrow$ over the variables $V = \{x, y, z\}$

2 Formalism

Following the ideas presented in [2], a dynamic Bayesian ontology is an ontology from an arbitrary (but fixed) DL $\mathcal{L}$, whose axioms are annotated with a context that expresses when they are considered to hold. The difference is that these contexts are now related via a dynamic Bayesian network.

A *Bayesian network* (BN) [5] is a pair $\mathcal{B} = (G, \Phi)$, where $G = (V, E)$ is a finite DAG and Φ contains, for every $x \in V$, a conditional probability distribution $P_\mathcal{B}(x \mid \pi(x))$ of x given its parents $\pi(x)$. Dynamic BNs (DBNs) [6,10] extend BNs to provide a compact representation of evolving joint probability distributions for a fixed set of random variables. Updates of the JPD are expressed through a two-slice BN, which expresses the probabilities at the next point in time, given the current context. A *two-slice BN* (TBN) is a pair (G, Φ), where $G = (V \cup V', E)$ is a DAG containing no edges between elements of V, $V' = \{x' \mid x \in V\}$, and Φ contains for every $x' \in V'$ a conditional probability distribution $P(x' \mid \pi(x'))$ of x' given its parents $\pi(x')$ (see Figure 1). A *dynamic Bayesian network* (DBN) is a pair $\mathcal{D} = (\mathcal{B}_1, \mathcal{B}_\rightarrow)$ where $\mathcal{B}_1$ is a BN and $\mathcal{B}_\rightarrow$ is a TBN. Using the Markov property: the probability of the future state is independent from the past, given the present state, the DBN $\mathcal{D} = (\mathcal{B}_1, \mathcal{B}_\rightarrow)$ defines, for every $t \geq 1$, a probability distribution $P_\mathcal{B}(V_t) = \prod_{i=2}^{t} \prod_{x \in V} P_{\mathcal{B}_\rightarrow}(x_i \mid \pi(x_i)_{i-1}) P_{\mathcal{B}_1}(V_1)$. This distribution is defined by unraveling the DBN starting from $\mathcal{B}_1$, using $\mathcal{B}_\rightarrow$ until t copies of V have been created. This produces a new BN $\mathcal{B}_{1:t}$ encoding the distribution over time of the variables. Figure 2 depicts $\mathcal{B}_{1:3}$ for the DBN $(\mathcal{B}_1, \mathcal{B}_\rightarrow)$ where $\mathcal{B}_\rightarrow$ is the TBN from Figure 1. The conditional probability tables of each node given its parents (not depicted) are those of $\mathcal{B}_1$ for the nodes in V_1, and of $\mathcal{B}_\rightarrow$ for nodes in $V_2 \cup V_3$. Notice that $\mathcal{B}_{1:t}$ has t copies of each random variable in V.

A *V-context* is a consistent set of literals over V. A *V-axiom* is of the form $\langle \alpha : \kappa \rangle$ where $\alpha \in \mathfrak{A}$ is an axiom and κ is a V-context. A *V-ontology* is a finite set O of V-axioms, from the DL $\mathcal{L}$. A *$\mathcal{DBL}$ knowledge base* (KB) over V is a pair $\mathcal{K} = (\mathsf{O}, \mathcal{D})$ where $\mathcal{D}$ is a DBN over V and O is a V-ontology. The semantics of this logic is defined by producing for every point in time, a multiple-world interpretation, where each world is associated to a probability that is compatible with the probability distribution defined by the DBN at that point in time.

3 Reasoning

The main reasoning task that we consider is to compute the probability of observing a consequence at different points in time. We consider three variants of this problem; namely, the probability of observing the consequence (i) *exactly* at time $t \geq 0$, (ii) *at most* at time t, or (iii) at *any* point in time. Combining methods from DBNs and context-based reasoning [1], we show that all these reasoning problems can be solved effectively.

The main idea is based on the unraveling of the DBN to time t. Using this unraveling, we readily know the probability of each context at every point in time between the present state and t. The logical element of the problem (i.e., knowing which contexts entail the consequence under consideration) is handled through the computation of a so-called *context formula*, which intuitively summarizes all the logical causes for the consequence to follow. Importantly, this context formula can be computed once for each consequence, and used for many different tests while reasoning. This unraveling and context formula can be used to solve the problems (i) and (ii) introduced above, with the help of a standard BN inference engine. Moreover, it is possible to add evidence observed at different points in time into the computation. This additional evidence does not yield any technical difficulties to our techniques, although may cause an increase in complexity, depending on the type and frequency of observations.

Clearly, the unraveling method cannot be used to compute the probability of *eventually* observing the consequence, as described by the problem (iii) above: one would potentially need to unravel the DBN to an infinite time, yielding a structure for which no effective reasoning methods exist. Instead, we identify some conditions under which this probability is easy to compute. Overall, this does not yield a full reasoning mechanism, but provides a good approximation in several meaningful cases.

As mentioned before, this work provides only the first steps towards a formalism for reasoning about events with evolving uncertainty. The following step is to be able to handle more complex time expressions and evidence.

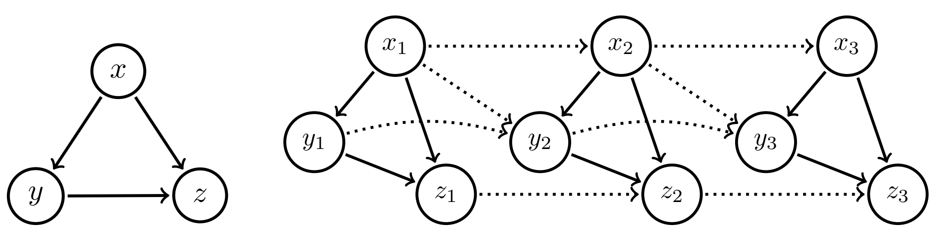

Fig. 2. $\mathcal{B}_1$ and the three step unraveling $\mathcal{B}_{1:3}$ of $(\mathcal{B}_1, \mathcal{B}_\rightarrow)$

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