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Complexity Theory

Exercise 7: Diagonalisation and Alternation

Exercise 7.1. Show that Cook-reducibility is transitive. In other words, show that if **A** is Cook-reducible to **B** and **B** is Cook-reducible to **C**, then **A** is Cook-reducible to **C**.

Exercise 7.2. Show that there exists an oracle **C** such that $\mathrm{NP}^{\mathbf{C}} \neq \mathrm{coNP}^{\mathbf{C}}$.

Hint:
What kind of Turing machines exist for languages in coNP? Use the answer to adapt the proof of the Baker-Gill-Solovay Theorem for coNP instead of P.

Exercise 7.3. Describe a polynomial-time ATM solving **EXACT INDEPENDENT SET**:

Input: Given a graph G and some number k.
Question: Does there exists a maximal independent set in G of size exactly k?

Exercise 7.4. Consider the Japanese game *go-moku* that is played by two players X and O on a 19x19 board. Players alternately place markers on the board, and the first one to have five of its markers in a row, column, or diagonal wins.

Consider the generalised version of go-moku on an $n \times n$ board. Say that a *position* of go-moku is a placement of markers on such a board as it could occur during the game. Define

$$\mathbf{GM} = \{\langle B \rangle \mid B \text{ is a position of go-moku where X has a winning strategy}\}.$$

Describe a polynomial-time ATM solving **GM**.

Exercise 7.5. Show that $\mathrm{AExpTime} = \mathrm{ExpSpace}$.

∗ **Exercise 7.6.** Show that Σ_2QBF is complete for Σ_2P. Generalise your argument to show that Σ_iQBF is complete for Σ_iP for all $i \geq 1$.

Exercise 7.7. Show that if P = NP, then P = PH.