Weak Completion Semantics 2

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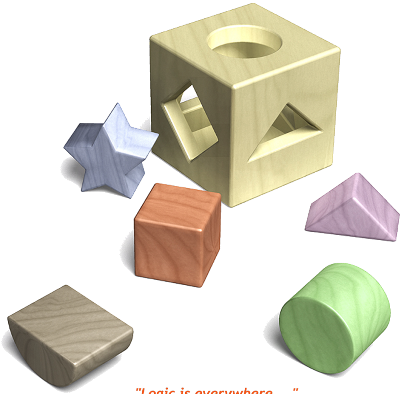
- Suppression Task
- Selection Task



"Logic is everywhere ..."

The Suppression Task

- Byrne: Suppressing Valid Inferences with Conditionals
  Cognition 31, 61-83: 1989
- If she has an essay to write then she will study late in the library
  She has an essay to write
  - Will she study late in the library? ☐ yes ☐ no ☐ I don't know
  - 96% conlude that she will study late in the library; modus ponens
- If she has an essay to write then she will study late in the library
  She has an essay to write
  If she has a textbook to read she will study late in the library
  - 96% conlude that she will study late in the library; alternative arguments
- If she has an essay to write then she will study late in the library
  She has an essay to write
  If the library stays open she will study late in the library
  - 38% conlude that she will study late in the library.
  - Additional arguments lead to suppression of earlier (correct) conclusions

The Suppression Task (Part II) – Affirmation of the Consequent

- Byrne: Suppressing Valid Inferences with Conditionals
  Cognition 31, 61-83: 1989

- If she has an essay to write then she will study late in the library
  She will study late in the library
  - Has she an essay to write? ☐ yes ☐ no ☐ I don't know
  - 53% of subjects conlude that she has an essay to write

- If she has an essay to write then she will study late in the library
  She will study late in the library
  If she has a textbook to read she will study late in the library
  - 16% of subjects conlude that she has an essay to write

- If she has an essay to write then she will study late in the library
  She will study late in the library
  If the library stays open she will study late in the library
  - 55% of subjects conlude that she has an essay to write

Abduction

- Consider the abductive framework $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$, where
  - $\mathcal{A}_\mathcal{P} = \{A \leftarrow \top \mid def(A, \mathcal{P}) = \emptyset\} \cup \{A \leftarrow \bot \mid def(A, \mathcal{P}) = \emptyset\}$ is the set of abducibles
  - $\mathcal{IC}$ is a finite set of integrity constraints, i.e., expressions of the form $\mathsf{U} \leftarrow B_1 \wedge \ldots \wedge B_n$
- An observation $\mathcal{O}$ is a set of ground literals
  - $\mathcal{O}$ is explainable in $\langle \mathcal{P}, \mathcal{A}_\mathcal{P}, \mathcal{IC}, \models_{wcs} \rangle$
    iff there exists a minimal $\mathcal{E} \subseteq \mathcal{A}_\mathcal{P}$ called explanation such that $\mathcal{M}_{\mathcal{P} \cup \mathcal{E}}$ satisfies $\mathcal{IC}$ and $\mathcal{P} \cup \mathcal{E} \models_{wcs} L$ for each $L \in \mathcal{O}$
  - F follows creduluously from $\mathcal{P}$ and $\mathcal{O}$
    iff there exists an explanantion $\mathcal{E}$ such that $\mathcal{P} \cup \mathcal{E} \models_{wcs} F$
  - F follows skeptically from $\mathcal{P}$ and $\mathcal{O}$
    iff for all explanantions $\mathcal{E}$ we find $\mathcal{P} \cup \mathcal{E} \models_{wcs} F$

Human Reasoning – Modus Ponens and AC

- If she has an essay to write then she will study late in the library
  She will study late in the library
  - Byrne 1989 53% of subjects conlude that she has an essay to write
- We obtain

$$\begin{array}{rcl} \mathcal{P} & = & \{\ell \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \bot\} \\ \mathcal{A} & = & \{e \leftarrow \top,\ e \leftarrow \bot\} \\ \mathcal{O} & = & \ell \end{array}$$

- Thus,

$$\begin{array}{rcl} \mathcal{M}_{\mathcal{P}} & = & \langle \emptyset, \{ab_1\}\rangle \\ \mathcal{M}_{\mathcal{P}\cup\{e\leftarrow\top\}} & = & \langle \{e, \ell\}, \{ab_1\}\rangle \\ \mathcal{M}_{\mathcal{P}\cup\{e\leftarrow\bot\}} & = & \langle \emptyset, \{e, \ell, ab_1\}\rangle \end{array}$$

- Hence, $\{e \leftarrow \top\}$ is the only minimal explanation for $\mathcal{O}$
  - e follows skeptically as well as creduluously from $\mathcal{P}$ and $\mathcal{O}$

Human Reasoning – Alternative Arguments and AC

- **If she has an essay to write then she will study late in the library**
  She will study late in the library
  If she has a textbook to read she will study late in the library
  - Byrne 1989 **16% of subjects conlude that she has an essay to write**
- **We obtain**

$$\begin{array}{lcl} \mathcal{P} & = & \{\ell \leftarrow e \wedge \neg ab_1,\ ab_1 \leftarrow \bot,\ \ell \leftarrow t \wedge \neg ab_2,\ ab_2 \leftarrow \bot\} \\ \mathcal{A} & = & \{e \leftarrow \top,\ e \leftarrow \bot,\ t \leftarrow \top,\ e \leftarrow \bot\} \\ \mathcal{O} & = & \ell \end{array}$$

- **Thus,**

$$\begin{array}{rcl} \mathcal{M}_{\mathcal{P}} & = & \langle \emptyset, \{ab_1, ab_2\} \rangle \\ \mathcal{M}_{\mathcal{P} \cup \{e \leftarrow \top\}} & = & \langle \{e, \ell\}, \{ab_1, ab_2\} \rangle \\ \mathcal{M}_{\mathcal{P} \cup \{e \leftarrow \bot\}} & = & \langle \emptyset, \{e, ab_1, ab_2\} \rangle \\ \mathcal{M}_{\mathcal{P} \cup \{t \leftarrow \top\}} & = & \langle \{t, \ell\}, \{ab_1, ab_2\} \rangle \\ \mathcal{M}_{\mathcal{P} \cup \{t \leftarrow \bot\}} & = & \langle \emptyset, \{t, ab_1, ab_2\} \rangle \end{array}$$

- **Hence, $\{e \leftarrow \top\}$ and $\{t \leftarrow \top\}$ are minimal explanations for $\mathcal{O}$**
  - **e follows creduluously (but not skeptically) from $\mathcal{P}$ and $\mathcal{O}$**

The Selection Task – Abstract Case

- Wason: Reasoning about a Rule
  The Quarterly Journal of Experimental Psychology 20, 273-281: 1968
- **Consider cards which have a letter on one side and a number on the other side**

D	**F**	**3**	**7**

- **Consider the rule**

  if there is a D on one side, then there is a 3 on the other side

- **Which cards do you have to turn in order to show that the rule holds?**
  - **Only 10% of the subjects give the logically correct solutions**

An Analysis

- Almost everyone (89%) correctly selects D
  - Corresponds to modus ponens in classical logic
- Almost everyone (84%) correctly does not select F
  - *Because the condition does not mention F*
- Many (62%) incorrectly select 3
  - *If there is a 3 on one side, then there is a D on the other side*
  - Converse of the given conditional
- Only a small percentage of subjects (25%) correctly selects 7
  - *If the number on one side is not 3, then the letter on the other side is not D*
  - Contrapositive of the given conditional

The Selection Task – Social Case

- Griggs, Cox: The elusive thematic materials effect in the Wason selection task British Journal of Psychology 73, 407-420: 1982
- Consider cards which have a person's age on the one side and a drink on the other side

beer	coke	22yrs	16yrs

- Consider the rule

  If a person is drinking beer, then the person must be over 19 years of age

- Which cards do you have to turn in order to show that the rule holds?
  - Most people solve this variant correctly

Formalizing the Social Case

- The conditional is viewed as a social constraint
- Let o and a be propositional variables denoting that the person is older than 19 years and is drinking alcohol, respectively
- The rule is encoded by $\mathcal{C} = \{o \leftarrow a \wedge \neg ab\}$
- Consider the four cases

case	$\mathcal{P}$	$\mathcal{M}_{\mathcal{P}}$				
beer	$\{a \leftarrow \top,\ ab \leftarrow \bot\}$	$\langle\{a\},\{ab\}\rangle$	$\not\models_{3Ł}$	$\mathcal{C}$	$\leadsto$	check
22yrs	$\{o \leftarrow \top,\ ab \leftarrow \bot\}$	$\langle\{o\},\{ab\}\rangle$	$\models_{3Ł}$	$\mathcal{C}$	$\leadsto$	no check
coke	$\{a \leftarrow \bot,\ ab \leftarrow \bot\}$	$\langle\emptyset,\{a, ab\}\rangle$	$\models_{3Ł}$	$\mathcal{C}$	$\leadsto$	no check
16yrs	$\{o \leftarrow \bot,\ ab \leftarrow \bot\}$	$\langle\emptyset,\{o, ab\}\rangle$	$\not\models_{3Ł}$	$\mathcal{C}$	$\leadsto$	check

Formalizing the Abstract Case

- The conditional is viewed as a belief
- Let $D, F, 3, 7$ be propositional variables denoting that the corresponding symbol is on one side
- Consider $\mathcal{P} = \{3 \leftarrow D \wedge \neg ab,\ ab \leftarrow \bot\}$ with $\mathcal{M}_\mathcal{P} = \langle\emptyset, \{ab\}\rangle$
- $\langle\emptyset, \{ab\}\rangle$ does not explain any letter on a card
- The set of abducibles is $\{D \leftarrow \top, D \leftarrow \bot, F \leftarrow \top, F \leftarrow \bot, 7 \leftarrow \top, 7 \leftarrow \bot\}$
- Consider the four cases

$\mathcal{O}$	$\mathcal{E}$	$\mathcal{M}_{\mathcal{P}\cup\mathcal{E})}$		
D	$\{D \leftarrow \top\}$	$\langle\{D, 3\}, \{ab\}\rangle$	$\leadsto$	turn
F	$\{F \leftarrow \top\}$	$\langle\{F\}, \{ab\}\rangle$	$\leadsto$	no turn
3	$\{D \leftarrow \top\}$	$\langle\{D, 3\}, \{ab\}\rangle$	$\leadsto$	turn
7	$\{7 \leftarrow \top\}$	$\langle\{7\}, \{ab\}\rangle$	$\leadsto$	no turn

A Computational Logic Approach to the Selection Task

- **The computational logic approach to model human reasoning can be extended to adequately handle the selection task**
  - **if the social case is understood as a social constraint and**
  - **if the abstract case is understood as a belief**

- Kowalski: Computational Logic and Human Life: How to be Artificially Intelligent. Cambridge University Press: 2011
- Dietz, H., Ragni: A Computational Logic Approach to the Abstract and the Social Case of the Selection Task. In: Proceedings of the 11th International Symposium on Logic Formalizations of Commonsense Reasoning: 2013