

## Introduction to Formal Concept Analysis

### Exercise Sheet 6, Winter Semester 2017/18

**Definition** (closure system and closure operator).

(a) A set  $\mathfrak{A} \subseteq \mathfrak{P}(M)$  is a closure system on the set  $M$ , iff  $M \in \mathfrak{A}$  and  $\mathfrak{X} \subseteq \mathfrak{A} \implies \bigcap \mathfrak{X} \in \mathfrak{A}$ .

(b) A closure operator  $\varphi$  on  $M$  is a map  $\varphi$  which maps each subset  $X \subseteq M$  onto the corresponding closure  $\varphi(X) \subseteq M$  such that

$$1) X \subseteq \varphi(X) \quad (\text{extensive})$$

$$2) X \subseteq Y \implies \varphi(X) \subseteq \varphi(Y) \quad (\text{monotone})$$

$$3) \varphi(\varphi(X)) = \varphi(X) \quad (\text{idempotent})$$

holds.

#### Exercise 1

Prove the following statements:

- For any closure system  $\mathfrak{A}$  on some set  $M$ , the mapping  $\varphi_{\mathfrak{A}} : X \mapsto \bigcap_{X \subseteq Y \in \mathfrak{A}} Y$  is a closure operator on  $M$ .
- For any closure operator  $\varphi$  on some set  $M$ , the family  $\mathfrak{A}_{\varphi} := \{\varphi(X) \mid X \subseteq M\}$  is a closure system on  $M$ .

#### Exercise 2 (closure system)

Regard the “family context”  $\mathbb{K} := (\{\text{father, mother, daughter, son}\}, \{\text{old, young, male, female}\}, \{(\text{father, old}), (\text{father, male}), (\text{mother, old}), (\text{mother, female}), (\text{daughter, young}), (\text{daughter, female}), (\text{son, young}), (\text{son, male})\})$ .

- Explicitly list the elements of the map  $\varphi : \mathfrak{P}(M) \rightarrow \mathfrak{P}(M)$  with  $\varphi : B \mapsto B''$  and verify that  $\varphi$  is a closure operator.
- Verify that the set of all concept intents of the family context is a closure system.
- Draw a line diagram of the powerset of  $\{\text{father, mother, daughter, son}\}$  and highlight the sets that have the same closure. Compare the diagram with the diagram of the concept lattice of the family context.

**Exercise 3** (Next-Closure)

	old (1)	young (2)	male (3)	female (4)
father	×		×	
mother	×			×
son		×	×	
daughter		×		×

Compute all concept intents of the above “family context” using the Next-Closure algorithm. Compare your result with the concept intents from Exercise 2.

A	$i$	$(A \cap \{1, 2, \dots, i-1\}) \cup \{i\}$ $A + i$	$(A+i)''$ $A \oplus i$	$A <_i A \oplus i?$	new intent